Meta-algorithms on graphs — problem batch 2

Cliquewidth, locality for fo, deadline: 21.05.2017, 23:59 CET (extended)

Problem 1. Prove that if a class of graphs $\mathcal{C}$ has bounded cliquewidth and is closed under taking subgraphs, then $\mathcal{C}$ has bounded treewidth.

Note: Class $\mathcal{C}$ has bounded π if there is a universal constant c such that $\pi(G) \leqslant c$ for each $G \in \mathcal{C}$.

Problem 2. A formula $\varphi(x, y)$ of fo on graphs, with two free variables x and y, is said to *induce an order of length k* on a graph G if there are vertices $a_1, a_2, \ldots, a_k$ of G such that $\varphi(a_i, a_j)$ holds if and only if $i \leqslant j$, for all $i, j \in \{1, 2, \ldots, k\}$. Prove that for every positive integer d and every such formula $\varphi(x, y)$, there exists a constant k, depending only on d and φ, such that on graphs of maximum degree d the formula φ does not induce orders longer than k.

Problem 3. A graph G is called a *unit interval graph* if there exists a unit interval model for G defined as follows: one assigns a unit-length closed interval on the real line to each vertex of G so that two vertices are adjacent in G if and only if the intervals assigned to them overlap. Consider the following problem: given a unit interval graph G together with a corresponding unit interval model and a sentence φ of fo on graphs, decide whether $G \models \varphi$. Prove that this problem can be solved in time $f(\|\varphi\|) \cdot n^c$, where f is a computable function, n is the number of vertices of G, and c is a universal constant, independent of φ and G.