

$$\begin{aligned} \pi^*(\alpha_1^*) &= \alpha_1^* \\ \pi^*(\alpha_2^*) &= \alpha_2^* \\ \pi^*(\alpha_3^*) &= 0 \end{aligned}$$

$$\begin{aligned} (1,0,0) &= \alpha_1^* \\ (0,1,0) &= \alpha_2^* \\ (0,0,1) &= \alpha_3^* \end{aligned}$$

$$(ab, ab, ab) = 1$$

$$\begin{aligned} \ker \pi^* &= \text{lin}(\alpha_3^*) \\ \text{im} \pi^* &= \text{lin}(\alpha_1^*, \alpha_2^*) \end{aligned}$$

W base $\alpha_1^*, \alpha_2^*, \alpha_3^*$
W base $\alpha_1^*, \alpha_2^*, \alpha_3^*$

$$\begin{aligned} \ker \pi^* &= \{(0,0,1)\} \\ \text{im} \pi^* &= \{(1,0,0), (0,1,0)\} \end{aligned}$$

iii) $\pi^*(f) = f \circ \pi$
 $f(x,y,z) = x+y+z$

$$\pi(x_1, x_2, x_3) = (x_1, x_2, x_2 - x_1)$$

$$\pi^*(f)(x_1, x_2, x_3) = x_1 + x_2 + x_2 - x_1 = 2x_2$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &\sim \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{array} \right] \xrightarrow{R_2 - R_1, R_3 - R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 3 & 2 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{array} \right] \\ &\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 3 & 2 & 0 & -1 & 1 \end{array} \right] \xrightarrow{R_3 - 3R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & -1 & -3 & 2 & 1 \end{array} \right] \\ &\xrightarrow{R_3 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -3 & 2 & 1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -3 & 3 & 1 \\ 0 & 0 & -1 & -3 & 2 & 1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{array} \right] \\ &\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & -6 & 5 & 2 \\ 0 & 0 & -1 & -3 & 2 & 1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -9 & 7 & 3 \\ 0 & 0 & -1 & -3 & 2 & 1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{array} \right] \\ &\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & -12 & 9 & 4 \\ 0 & 0 & -1 & -3 & 2 & 1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{array} \right] \end{aligned}$$