Zadanie 4.

$\varphi : V \to W$ – izomorfizm

$\varphi^* : W^* \to V^*$ gdzie $W^* = L(W, \mathbb{R})$ $V^* = L(V, \mathbb{R})$

$\varphi^*(f) = f \circ \varphi$ gdzie $f \in W^*$

1) Niech $g \in V^*$

$$\varphi^* \circ (\varphi^{-1})^*(g) = \varphi^*\left((\varphi^{-1})^*(g)\right) = \varphi^*\left(g \circ (\varphi^{-1})\right) =$$

$$= g \circ (\varphi^{-1}) \circ \varphi = g \circ (\varphi^{-1} \circ \varphi) = g$$

Zatem $\varphi^* \circ (\varphi^{-1})^* = id$

2) Niech $f \in W^*$

$$(\varphi^{-1})^* \circ \varphi^*(f) = (\varphi^{-1})^* \circ (\varphi^*(f)) = (\varphi^{-1})^*(f \circ \varphi) = f \circ \varphi \circ (\varphi^{-1}) =$$

$$= f \circ (\varphi \circ \varphi^{-1}) = f$$

Zatem $(\varphi^{-1})^* \varphi^* = id$.