

Zadanie 4.

$\varphi: V \rightarrow W$ - izomorfizm

$\varphi^*: W^* \rightarrow V^*$ gdzie $W^* = L(W, \mathbb{R})$ $V^* = L(V, \mathbb{R})$

$$\varphi^*(f) = f \circ \varphi \quad \text{gdzie } f \in W^*$$

1) Niech $g \in V^*$

$$\begin{aligned} \varphi^* \circ (\varphi^{-1})^*(g) &= \varphi^* \left((\varphi^{-1})^*(g) \right) = \varphi^* \left(g \circ (\varphi^{-1}) \right) = \\ &= g \circ (\varphi^{-1}) \circ \varphi = g \circ (\varphi^{-1} \circ \varphi) = g \end{aligned}$$

Zatem $\varphi^* \circ (\varphi^{-1})^* = \text{id}$

2) Niech $f \in W^*$

$$\begin{aligned} (\varphi^{-1})^* \circ \varphi^*(f) &= (\varphi^{-1})^* \circ (\varphi^*(f)) = (\varphi^{-1})^* (f \circ \varphi) = f \circ \varphi \circ (\varphi^{-1}) = \\ &= f \circ (\varphi \circ \varphi^{-1}) = f \end{aligned}$$

Zatem $(\varphi^{-1})^* \circ \varphi^* = \text{id}$.