Zadanie 5 (1 pkt) Kiszel Michał

$$so(n) = \{ A \in M_{n \times n}(\mathbb{R}) : -A = A^T \}$$

I)
$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$-A = A^T \Rightarrow -a = a \wedge -b = d \wedge -c = g \wedge -e = e \wedge -i = i \wedge -f = h$$

Na podstawie powyższego:

$$A = \begin{bmatrix} 0 & b & c \\ -b & 0 & f \\ -c & -f & 0 \end{bmatrix}$$

Jeśli $b = 0 \vee c = 0 \vee f = 0$, to $r(A) \leq 2$.

Niech więc $b \neq 0 \wedge c \neq 0 \wedge f \neq 0$

$$A = \begin{bmatrix} 0 & b & c \\ -b & 0 & f \\ c & -f & 0 \end{bmatrix} \begin{matrix} \cdot f \\ \cdot c \\ \cdot b \end{matrix} \equiv \begin{bmatrix} 0 & bf & cf \\ -bc & 0 & fc \\ cb & -fb & 0 \end{bmatrix} {\scriptstyle +w_1+w_2} \equiv \begin{bmatrix} 0 & bf & cf \\ 0 & 0 & 0 \\ cb & -fb & 0 \end{bmatrix} \equiv \begin{bmatrix} cb & -fb & 0 \\ 0 & bf & cf \\ 0 & 0 & 0 \end{bmatrix}$$

Tak więc $(b \neq 0 \wedge c \neq 0 \wedge f \neq 0) \Rightarrow r(A) = 2$ ✓

II) Niech

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{bmatrix}, \text{ wówczas } A^T = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = -A, \text{ tak więc } A \in so(4)$$

$$\begin{bmatrix} 0 & +1 & +1 & +1 \\ 1 & 0 & +1 & +1 \\ -1 & -1 & 0 & +1 \\ -1 & -1 & -1 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} \begin{matrix} \\ \\ -(w_1+w_2) \\ -(w_1+w_2) \end{matrix} \equiv \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{matrix} +w_3-w_4 \\ -w_3+w_4 \\ \\ \\ \end{matrix} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Na podstawie powyższego $r(A) = 4$ ✓