Zadanie 4

$$16 z^4 \bar{z}^2 = i\,|z|^2$$

Z faktu: $\bar{z} \cdot z = |z|^2$

Otrzymujemy

$$16 z^4 \bar{z}^2 = 16 z^2 |z|^4 = i|z|^2$$

$$16 z^2 |z|^4 - i|z|^2 = |z|^2\left(16 z^2 |z|^2 - i\right) = 0$$

$$|z|^2\left(16 z^2 |z|^2 - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)^2\right) = 0$$

$$|z|^2\left(\sqrt{16|z|^2 z^2} - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)\right)\left(\sqrt{16|z|^2 z^2} + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)\right) = 0$$

Zatem

i) $|z|^2 = 0 \iff z = 0$

ii) $\sqrt{16|z|^2 z^2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i = 0 \iff 4|z|z - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i = 0$

iii) $\sqrt{16|z|^2 z^2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i = 0 \iff 4|z|z + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i = 0$

Ad ii) $z = a + bi \qquad a, b \in \mathbb{R}$

$$4(a+bi)\sqrt{a^2+b^2} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$$

$$(a+bi)\sqrt{a^2+b^2} = \frac{\sqrt{2}}{8} + \frac{\sqrt{2}}{8} i$$

$$a\sqrt{a^2+b^2} + b\sqrt{a^2+b^2}\, i = \frac{\sqrt{2}}{8} + \frac{\sqrt{2}}{8} i$$

$$\begin{cases} a\sqrt{a^2+b^2} = \frac{\sqrt{2}}{8} \\ b\sqrt{a^2+b^2} = \frac{\sqrt{2}}{8} \end{cases} \Rightarrow \begin{matrix} a > 0 \\ b > 0 \end{matrix}$$

Dzieląc stronami (Można bo nie zachodzi jednocześnie $a = 0 \wedge b = 0$)

$$\frac{a}{b} = 1 \iff a = b$$

Zatem $a\sqrt{a^2+a^2} = \frac{\sqrt{2}}{8}$

$$|a|\, \cancel{a} \sqrt{2} = \frac{\sqrt{2}}{8} \Rightarrow a = \frac{\sqrt{2}}{4} \vee a = -\frac{\sqrt{2}}{4} \text{ ale } a > 0$$

Zatem $a = b = \frac{\sqrt{2}}{4}$