Bi-reachability in Petri nets with data

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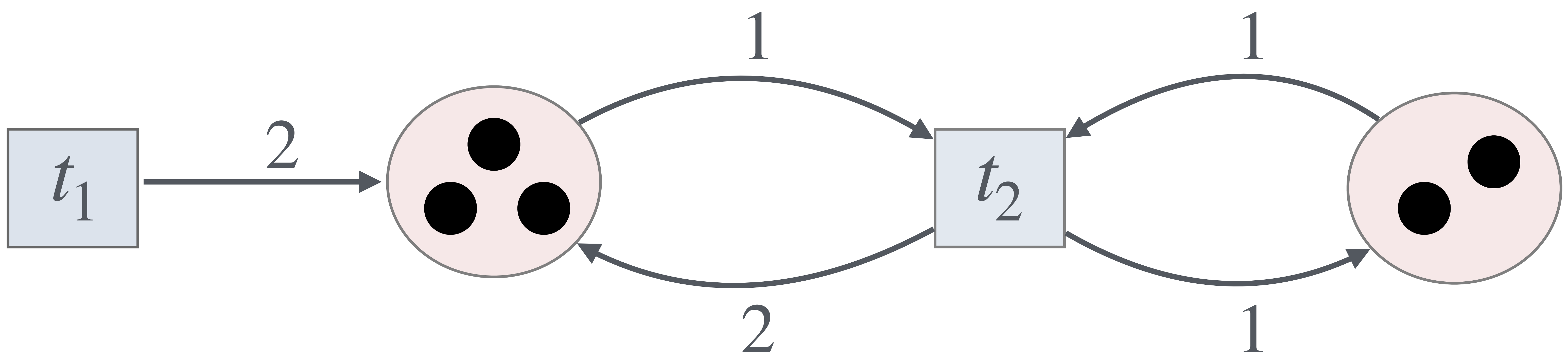
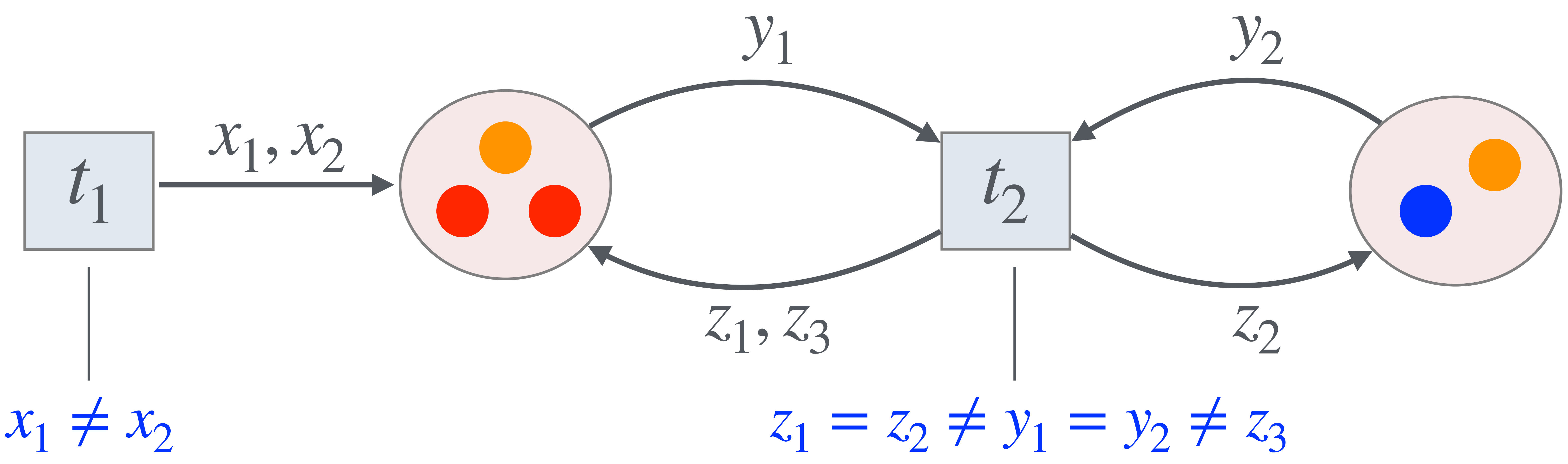
Petri nets



Petri nets with equality data

Let $\mathbb{A} = \{$, , , , …$\}$ be an infinite set of data.



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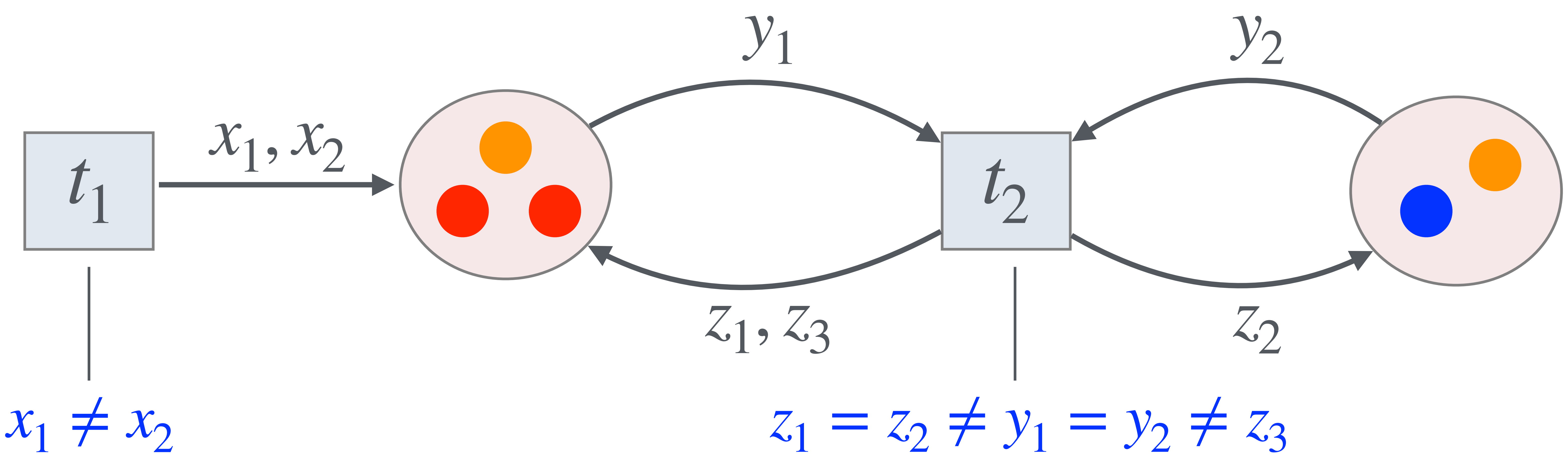
Let $\mathbb{A} = \{ \bullet, \bullet, \bullet, \bullet, \ldots \}$ be an infinite set of data.

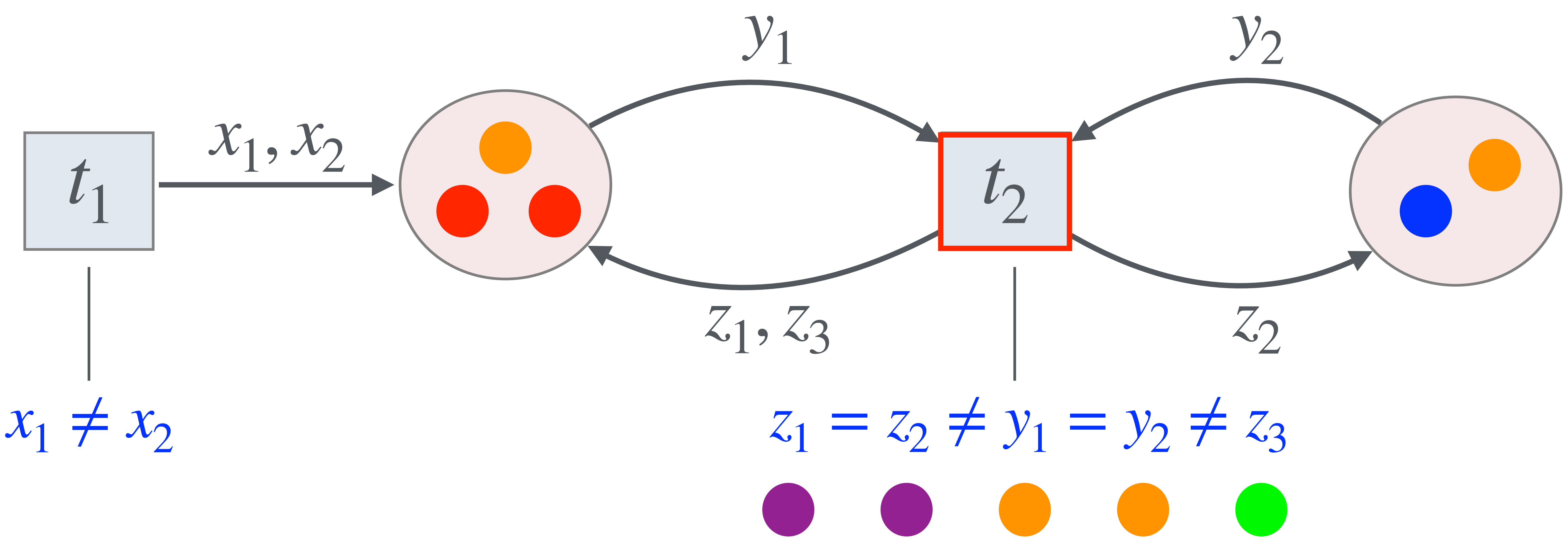


Configuration = distribution of tokens over the places

Formally, a function $P \times \mathbb{A} \to \mathbb{N}$ with finite suppot.

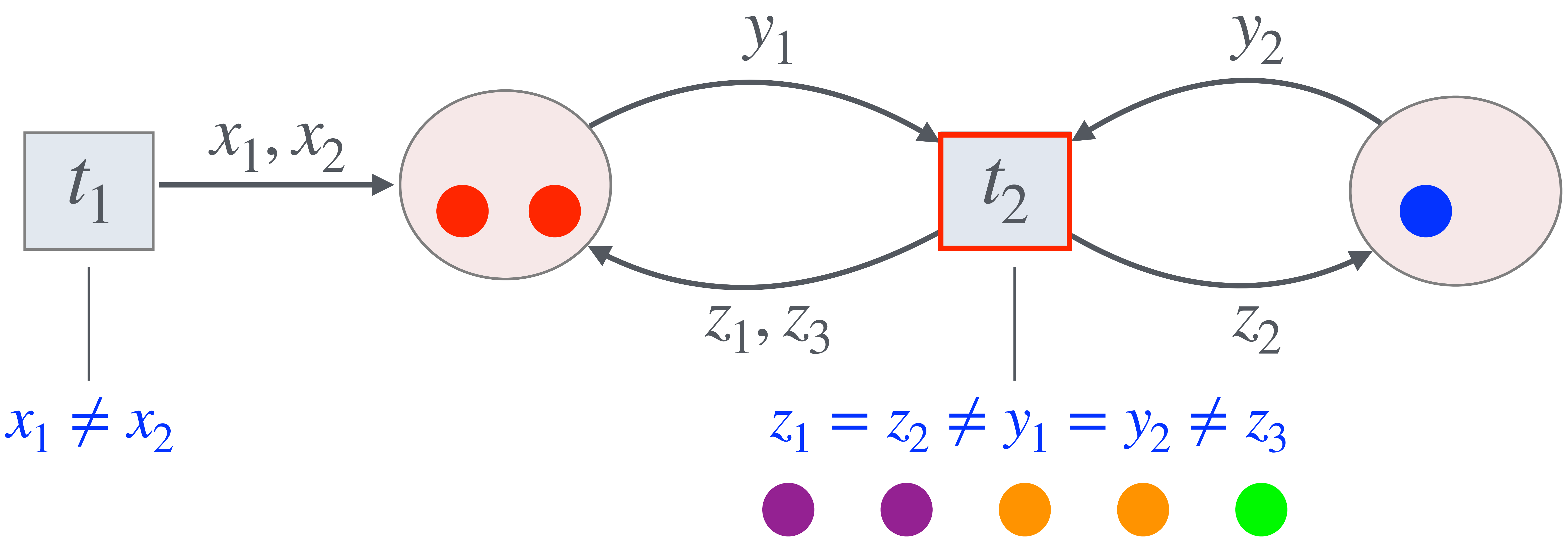
Petri nets with equality data

Let $\mathbb{A} = \{$ ●, ●, ●, ●, $\dots\}$ be an infinite set of data.



Petri nets with equality data

Let $\mathbb{A}$ = { ● , ● , ● , ● , …} be an infinite set of data.



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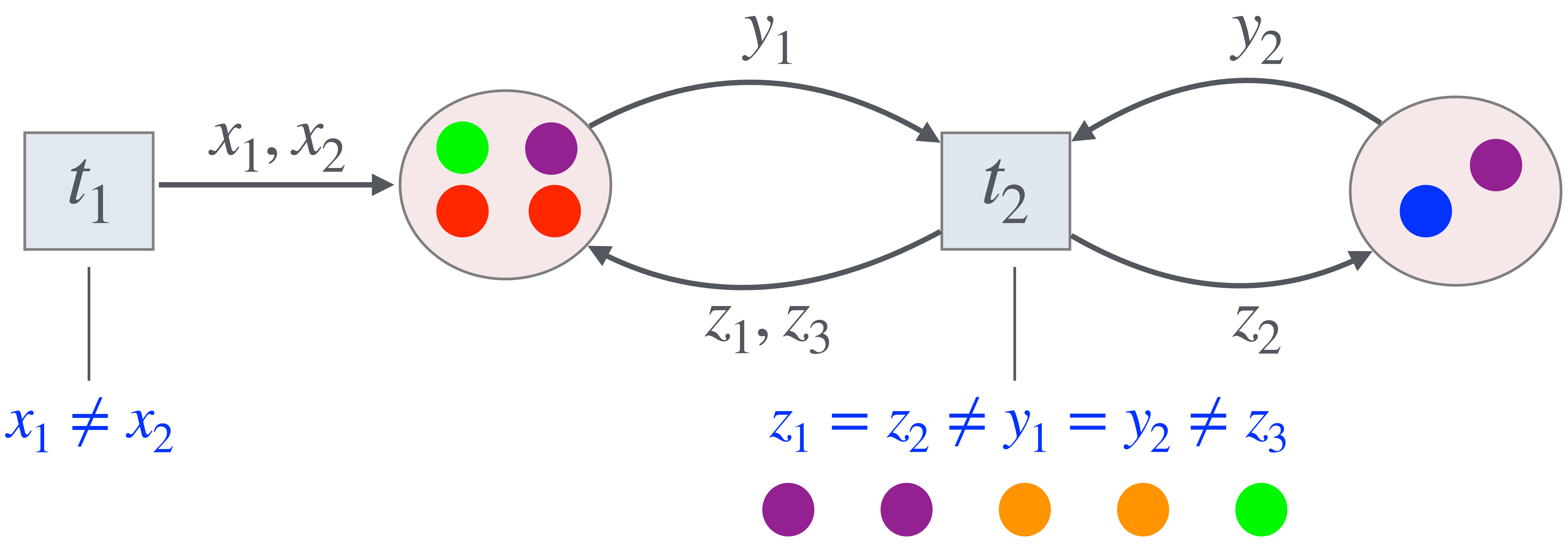

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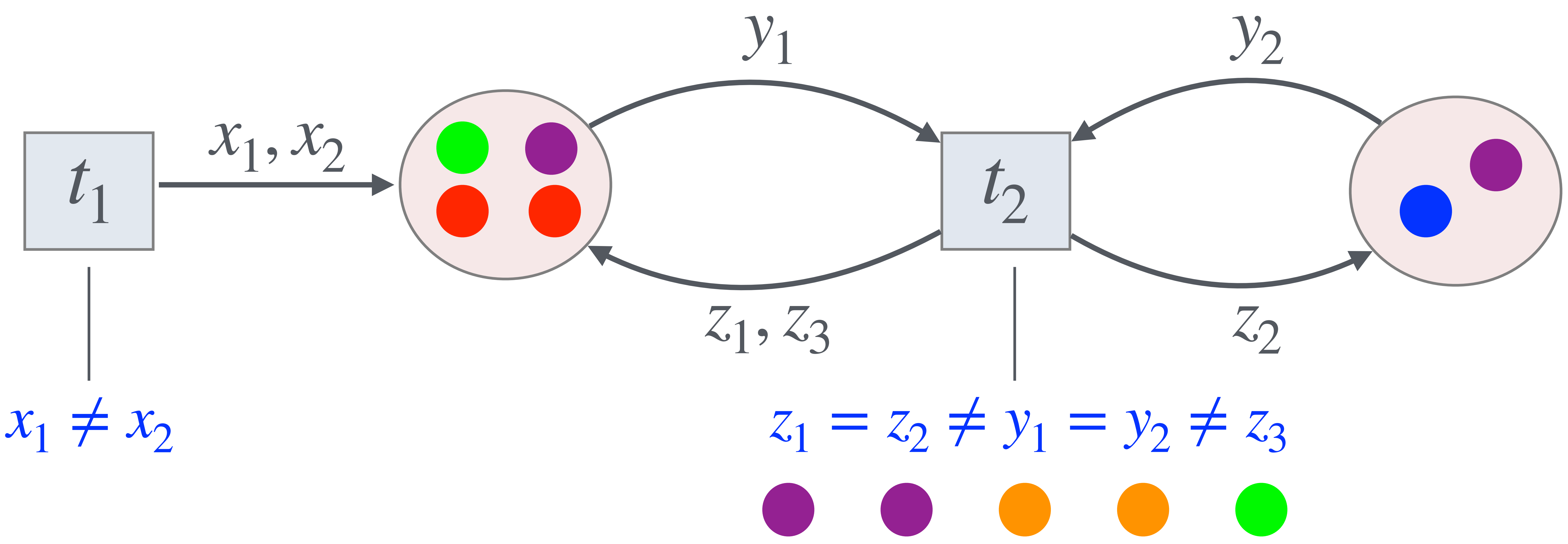

Consider a permutation σ that swaps

and

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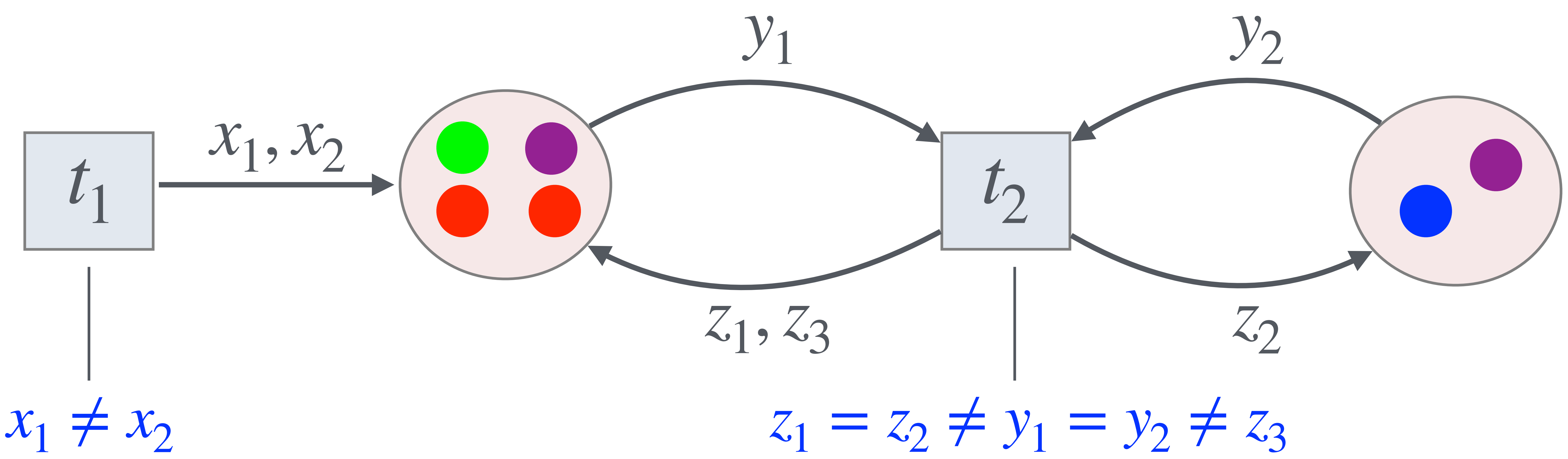

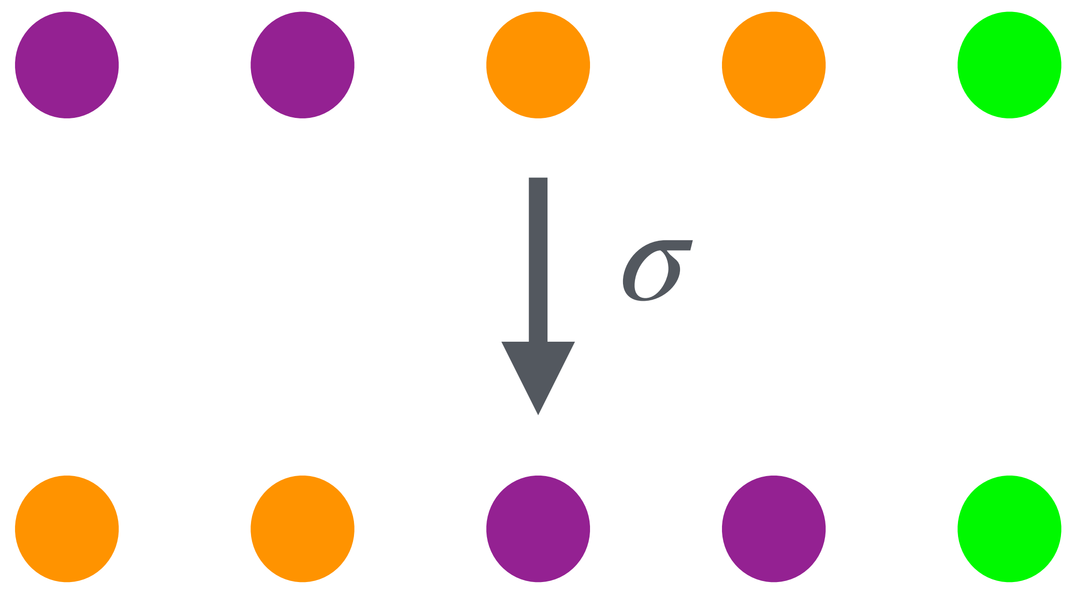

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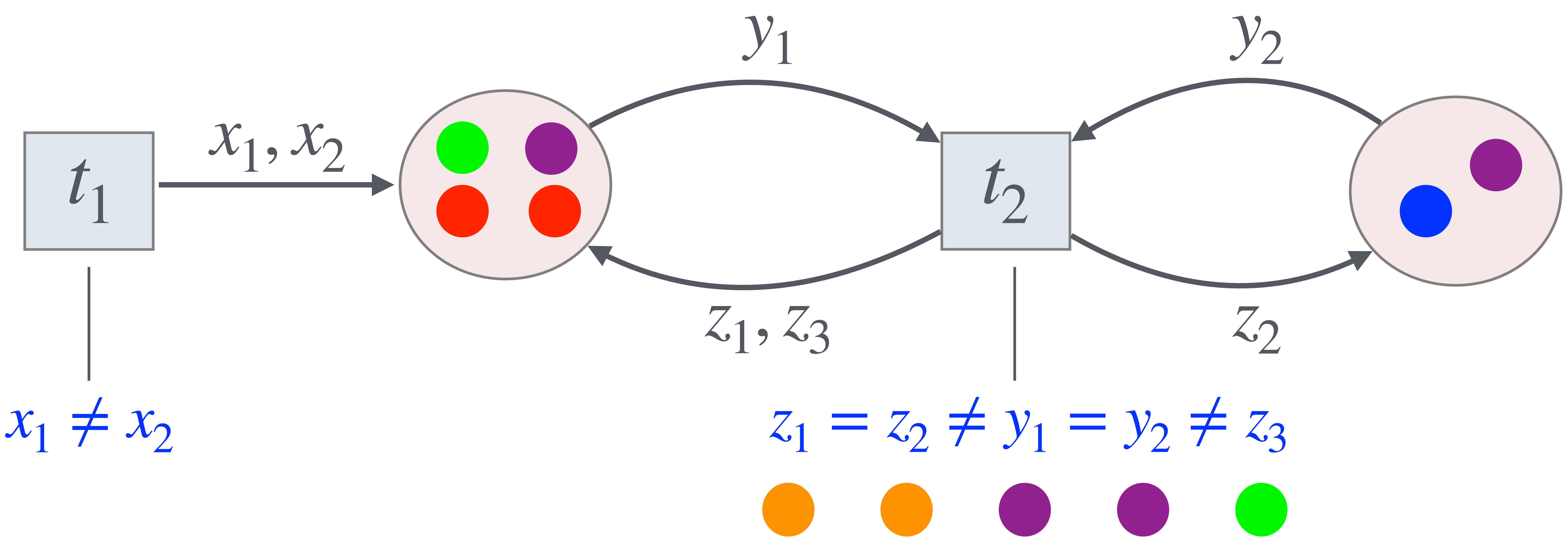

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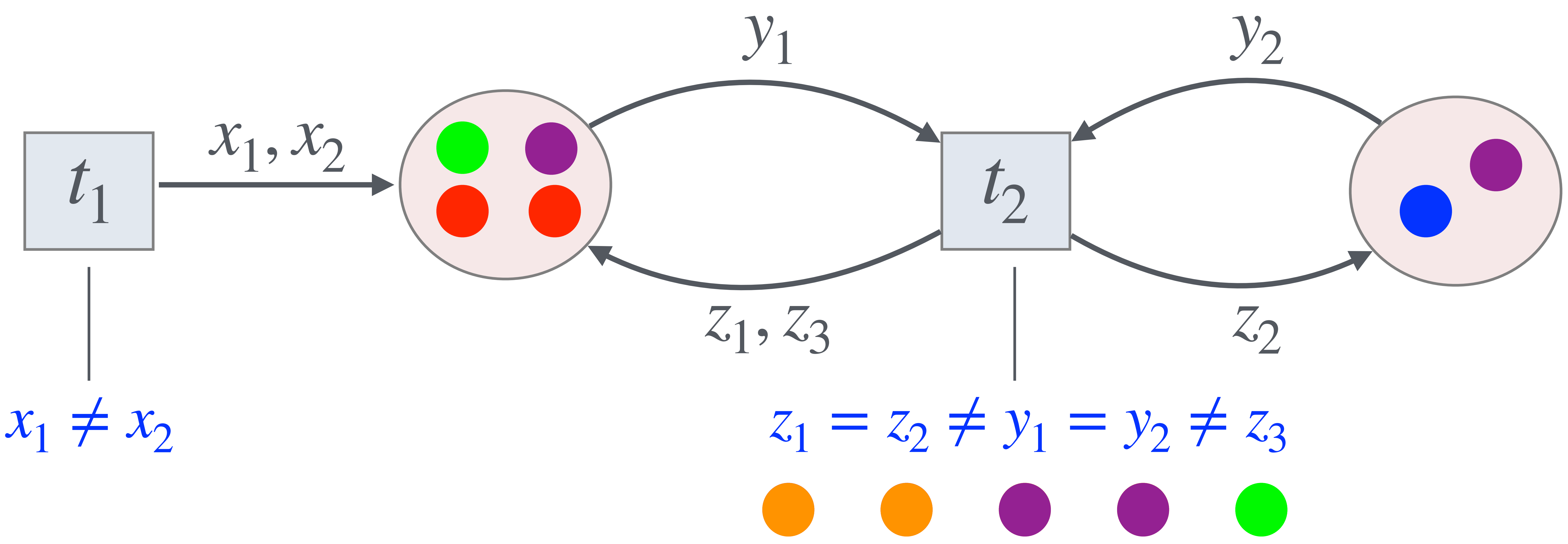

Orbit of a transition = equivalence class w.r.t. actions of permutations

Petri nets with equality data

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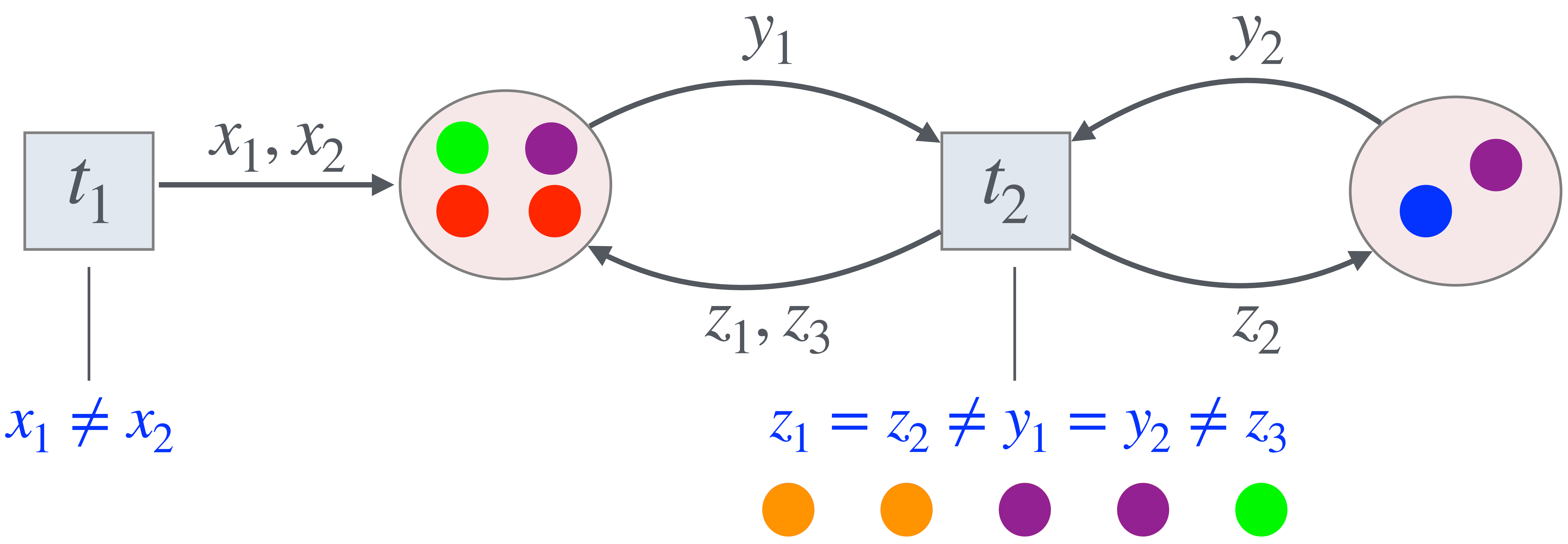

Orbit of a transition = equivalence class w.r.t. actions of permutations

The set of transitions is infinite, but **orbit-finite**

[Bojańczyk, Klin, Lasota, Toruńczyk, '13]
[Bojańczyk, Klin, Lasota, '14]

Petri nets with equality data

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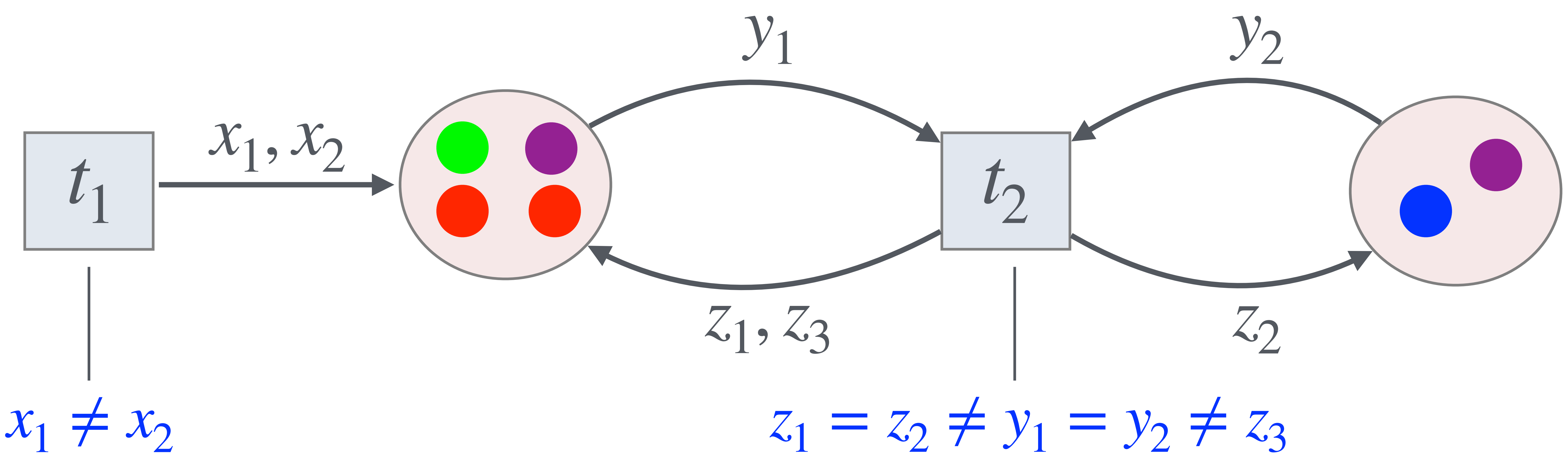


High-level Petri nets
[Genrich, Lautenbach, '81]

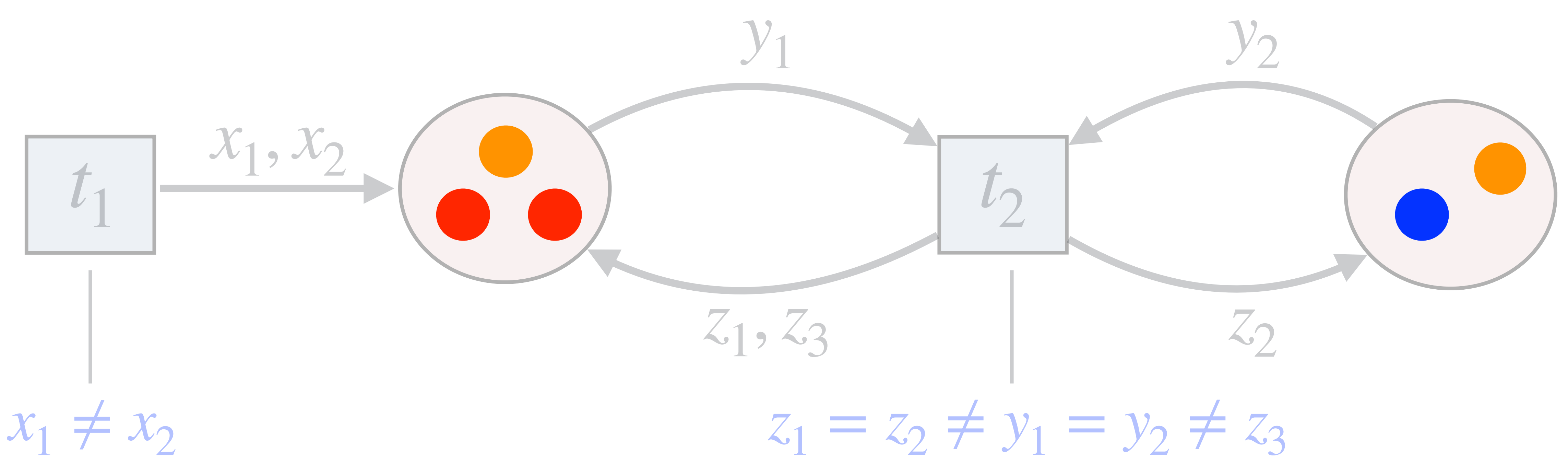
Coloured Petri nets
[Jensen, '81]

Constrained multiset rewriting
[Cervesato, Durgin, Lincoln, Mitchell, Scedrov, '99]
[Delzanno, '05]

Petri nets with data
[Lazic, Newcomb, Ouaknine, Roscoe, Worrell, '07]
[Rosa-Velardo, Frutos-Escrig, '11]
[Lasota, '16]

Runs

Source



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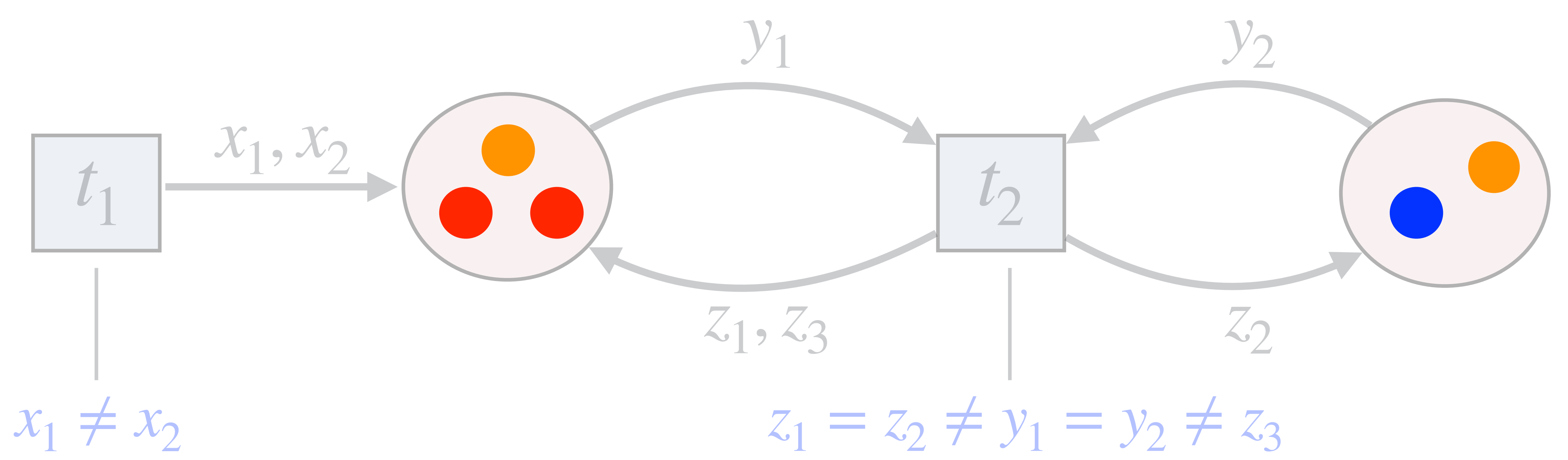
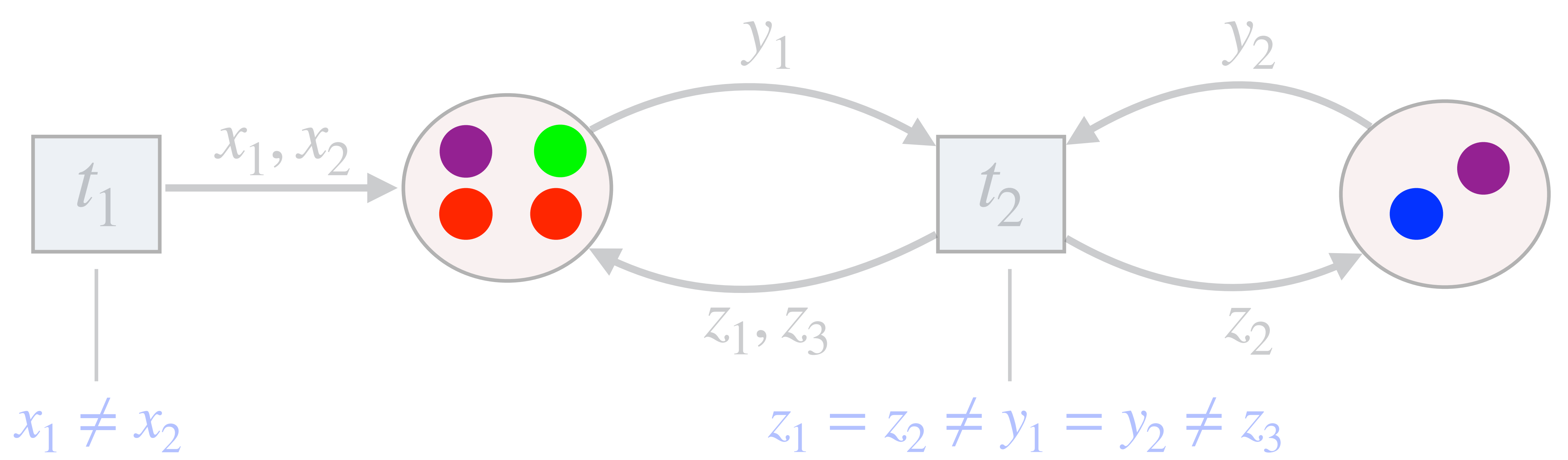


t_1
x_1, x_2
y_1
y_2
t_2
z_1, z_3
z_2
$x_1 \neq x_2$
$z_1 = z_2 \neq y_1 = y_2 \neq z_3$



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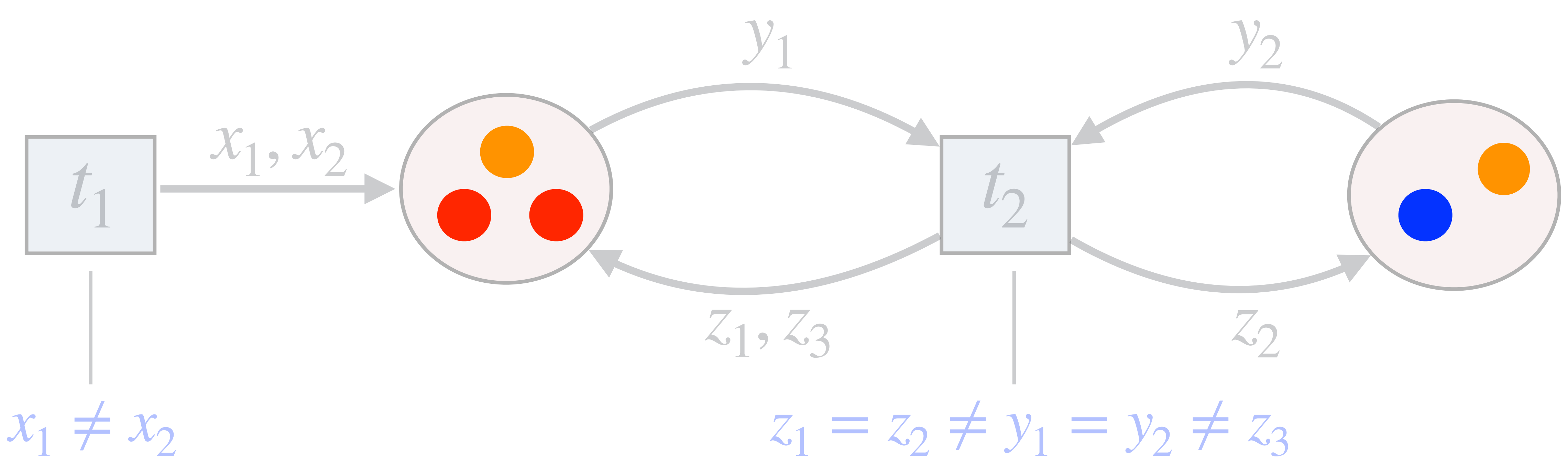
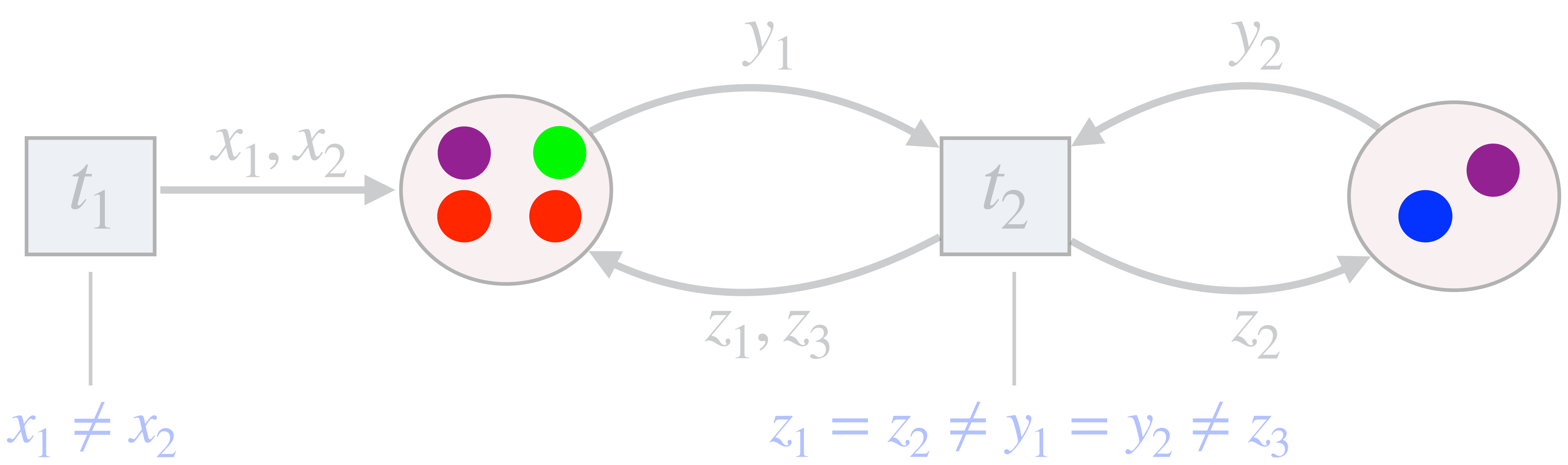
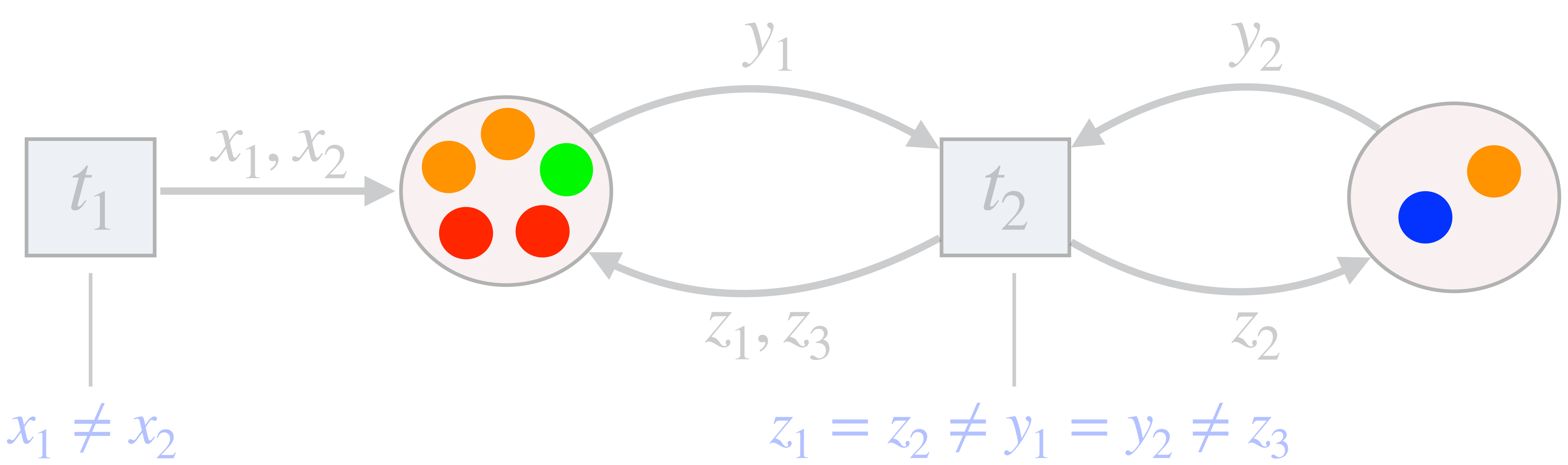




Target



Decision problems

***Input**: a (data) Petri net, source and target configurations*

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Reachability

Is there a run from source to target?

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Is there a run from source to target?

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Is there a run from source to target or a greater configuration?

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Bi-reachability (mutual reachability)

Are there two runs, from source to target, and from target to source?

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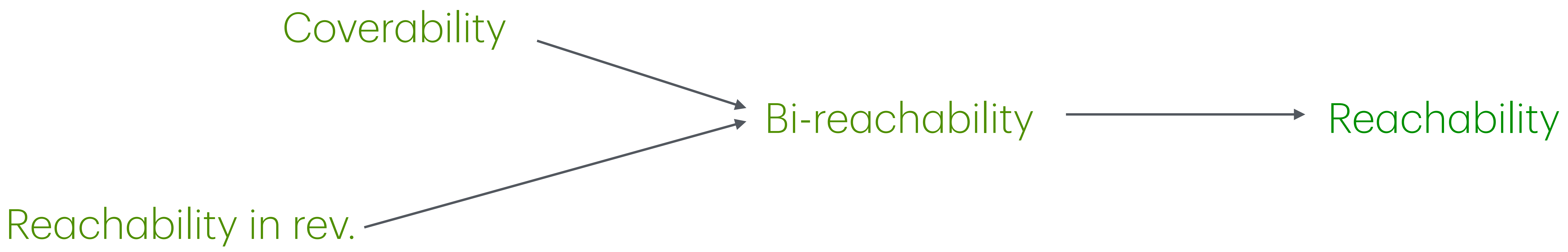
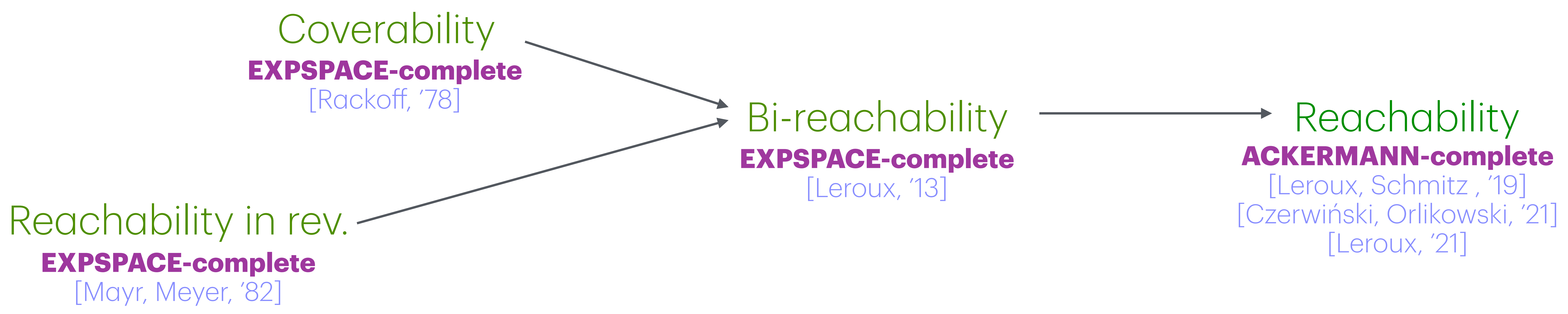


State of the art

Petri nets



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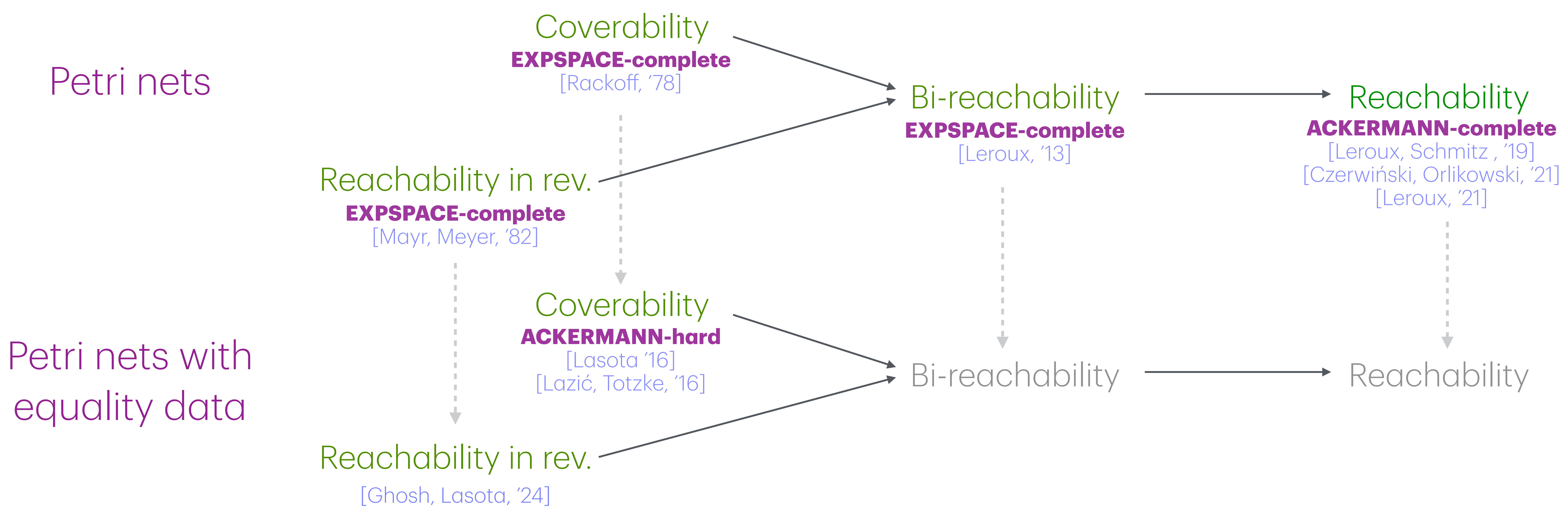

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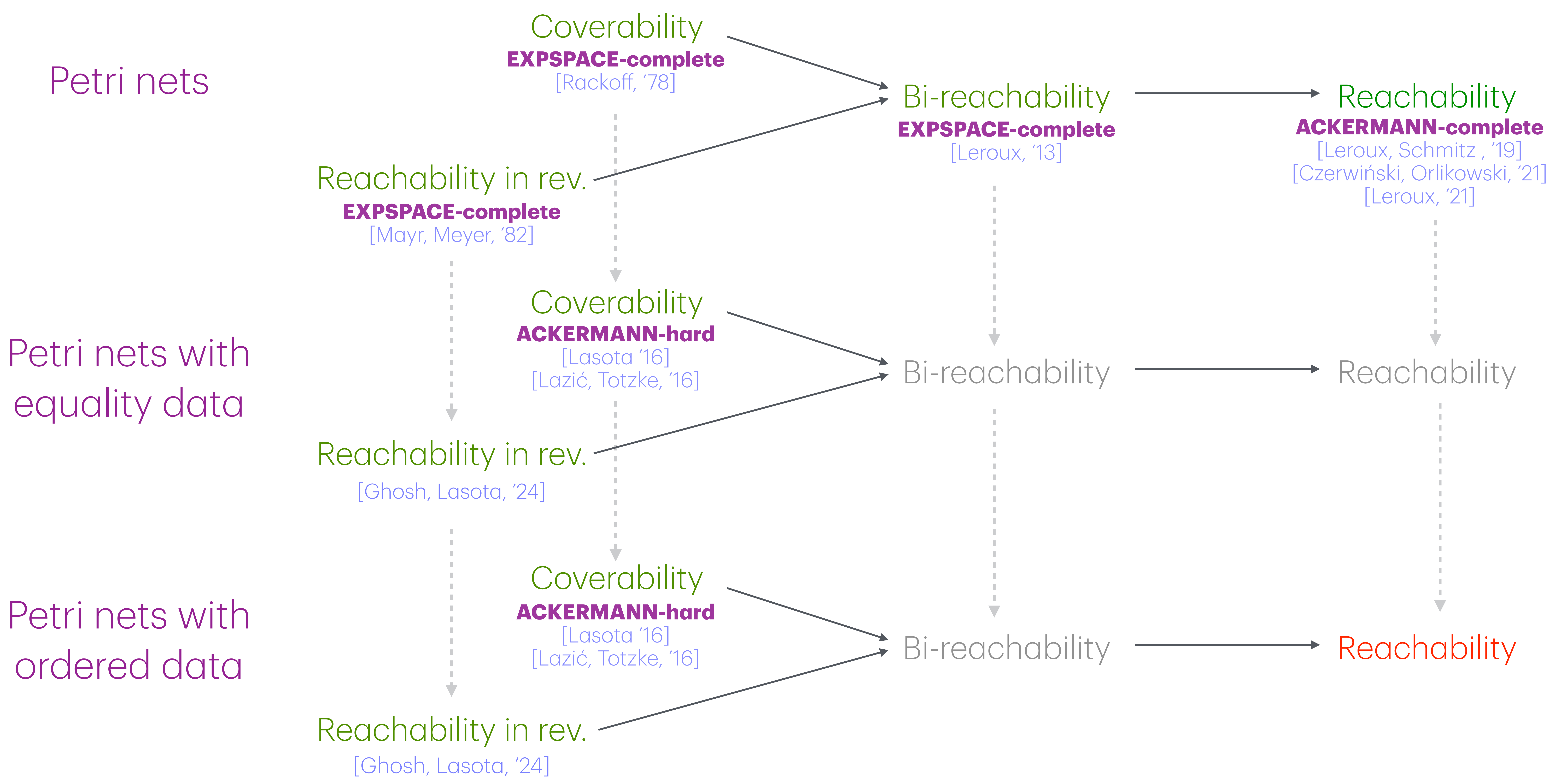
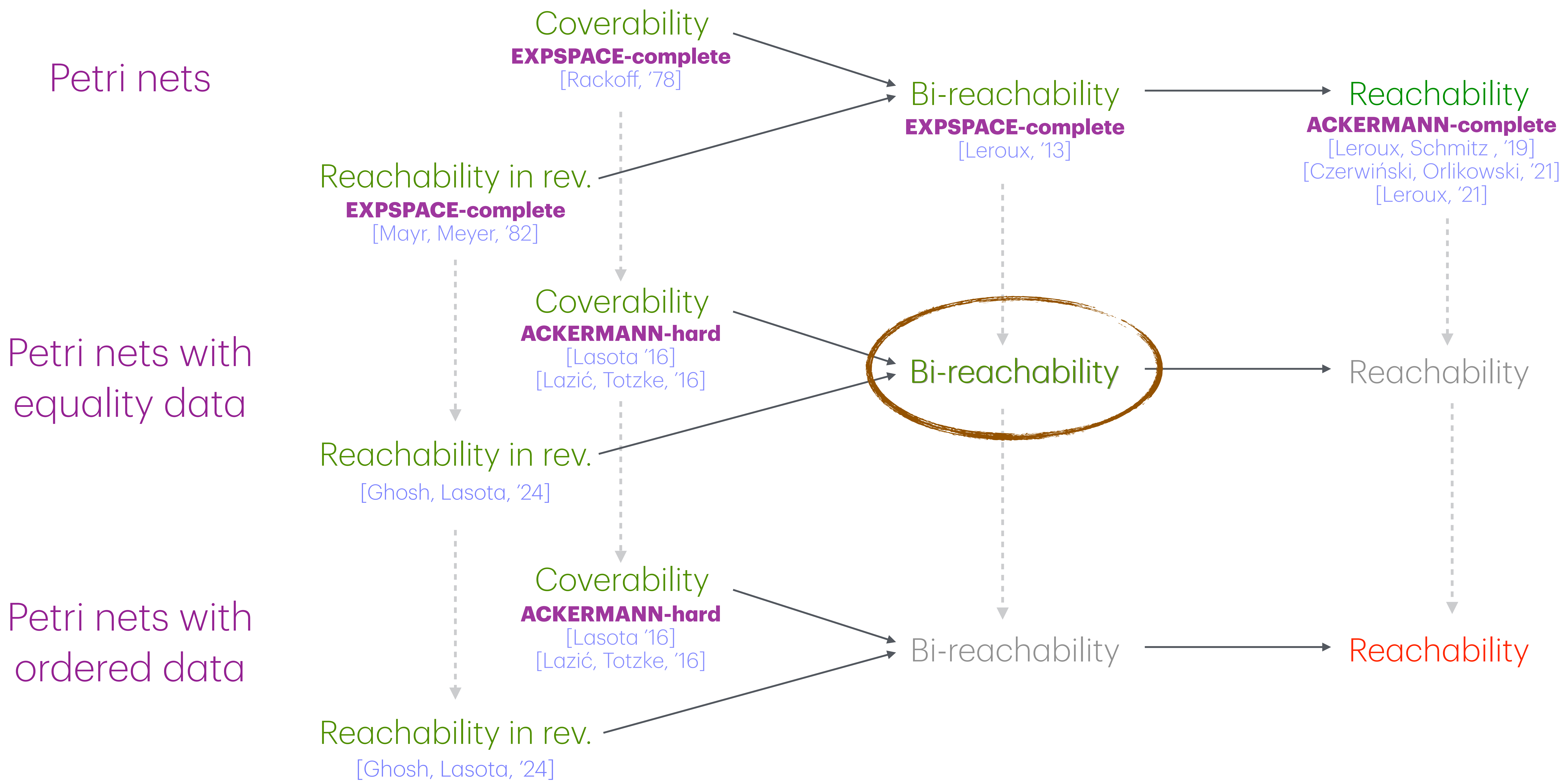
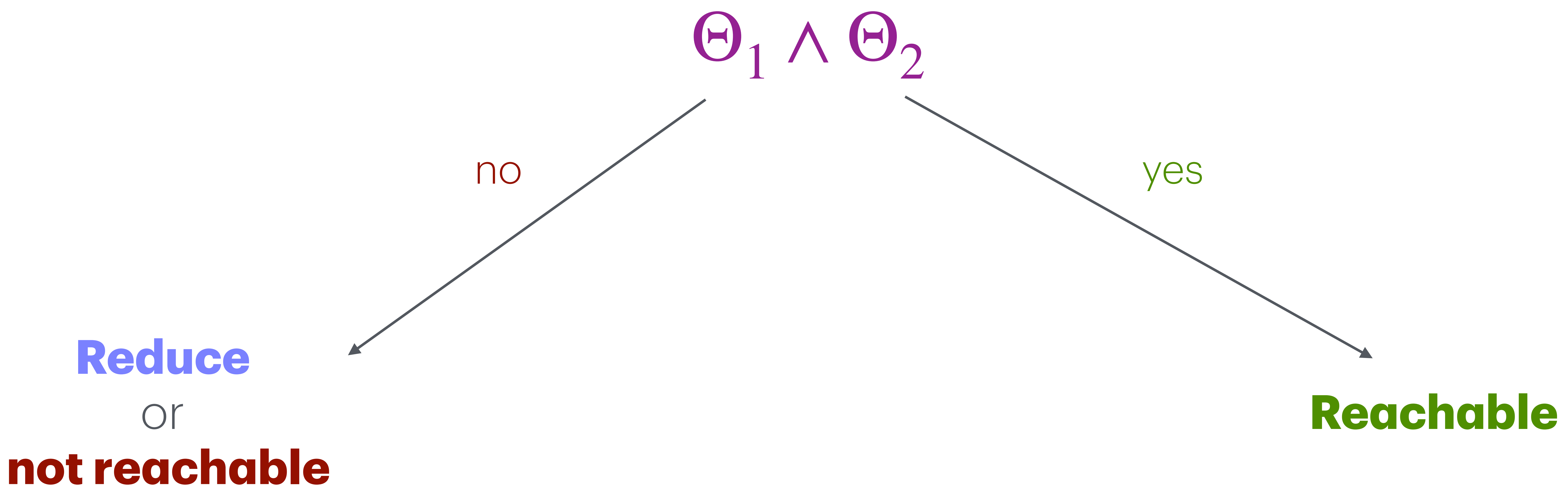


State of the art and the result



Decomposition algorithm



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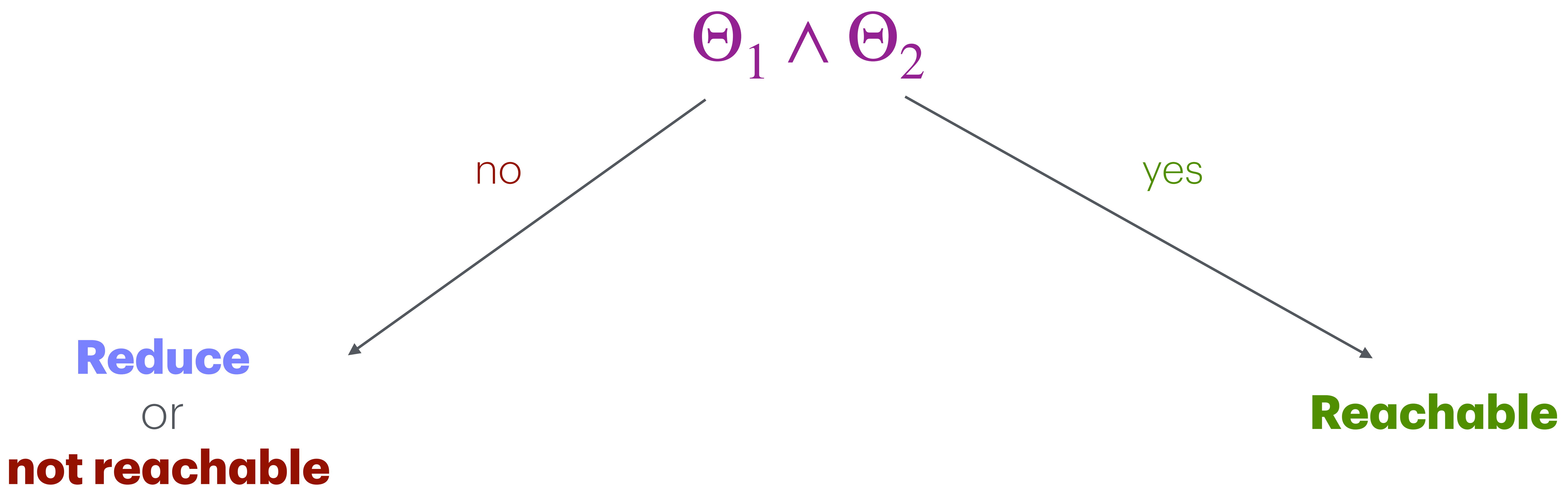

[Kosaraju, '82]

Sufficient condition

Θ_1 : For every $m \in \mathbb{N}$ there is a pseudo-run

$$q \dashrightarrow q'$$

that uses every transition at least m times.

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We allow negative numbers of tokens

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For some vectors $\Delta, \Delta' \gg \mathbf{0}$ there are runs

$$q \rightarrow q + \Delta \qquad q' + \Delta' \rightarrow q'$$

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Sufficient condition

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Θ_1 : ~~For every $m \in \mathbb{N}$~~ there ~~is a~~ **are two** pseudo-runs

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some transition from every orbit.

Θ_2 : For some vectors $\Delta, \Delta' \gg \mathbf{0}$ there are runs

$$q \to q + \Delta \qquad q' + \Delta' \to q'$$

Sufficient condition

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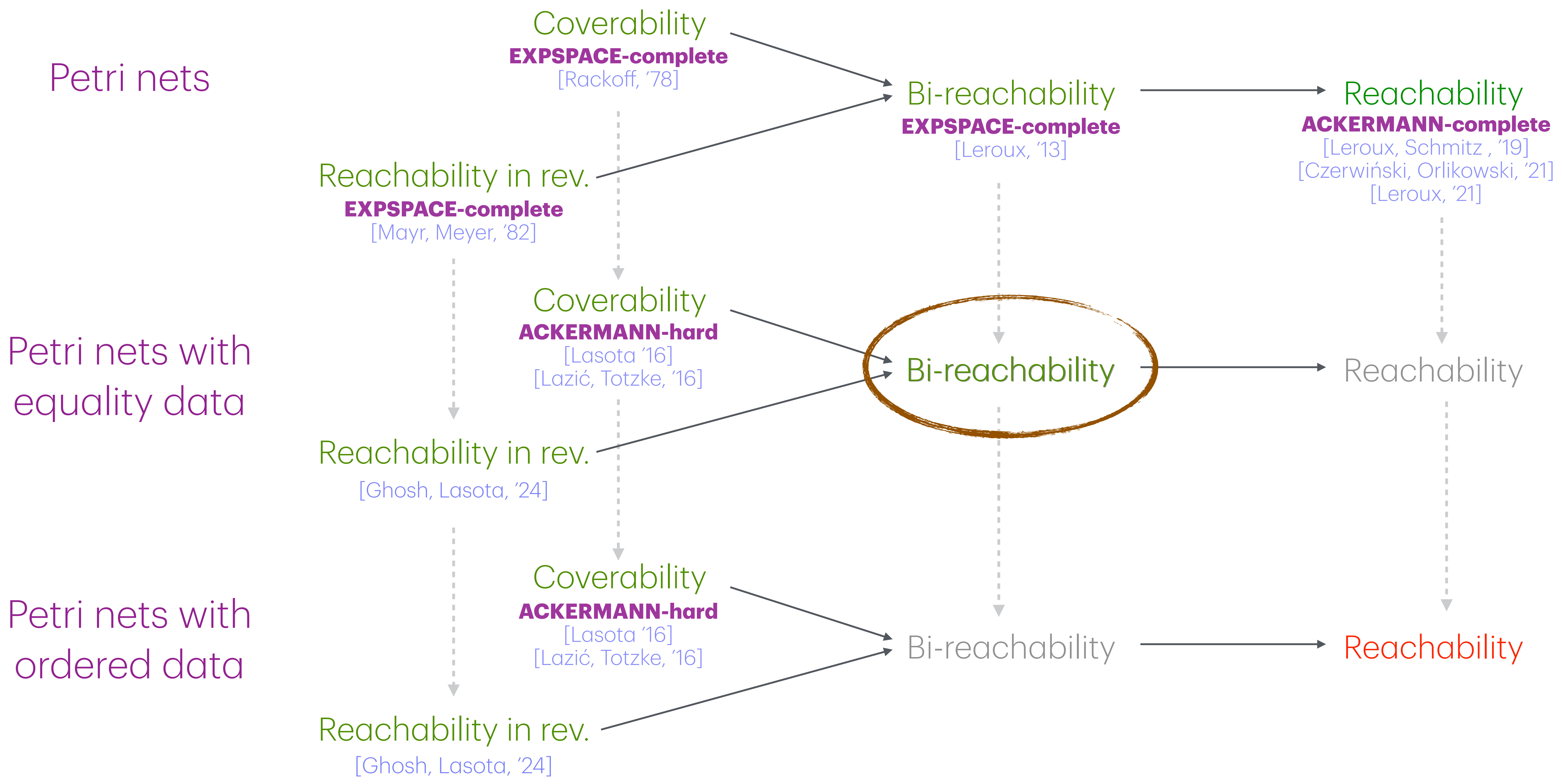
$$q \rightarrow q + \Delta \qquad q' + \Delta' \rightarrow q'$$

and similarly for the second run

Thank you!


Petri nets
Coverability
EXPSPACE-complete
[Rackoff, '78]
Bi-reachability
EXPSPACE-complete
[Leroux, '13]
Reachability
ACKERMANN-complete
[Leroux, Schmitz , '19]
[Czerwiński, Orlikowski, '21]
[Leroux, '21]
Reachability in rev.
EXPSPACE-complete
[Mayr, Meyer, '82]
Petri nets with
equality data
Coverability
ACKERMANN-hard
[Lasota '16]
[Lazić, Totzke, '16]
Bi-reachability
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Reachability in rev.
[Ghosh, Lasota, '24]
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