On the Solvability of Ill-Posed Problems in two Models of Computation

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Can a computational problem given by an unbounded linear transformation be solvable? Pour-El and Richards showed that the answer is no in the framework of computability theory, and Werschulz showed that the answer is no in the worst case setting in Information-Based Complexity. In fact, Pour-El and Richards showed that there exist partial differential equations with computable initial conditions that have non-computable solutions. We explain this phenomenon and related results by Brattka and by Weihrauch and Zhong. On the other hand, due to work by Kon, Ritter, and Werschulz, as well as Vakhania, it is known that every measurable unbounded linear operator is bounded on the average, for all Gaussian measures. Hence, due to a result by Werschulz, every linear ill-posed problem is solvable in the average case setting in Information-Based Complexity, for any Gaussian measure. We discuss these results from the viewpoint of computability theory. We also discuss the solvability of other ill-posed problems.