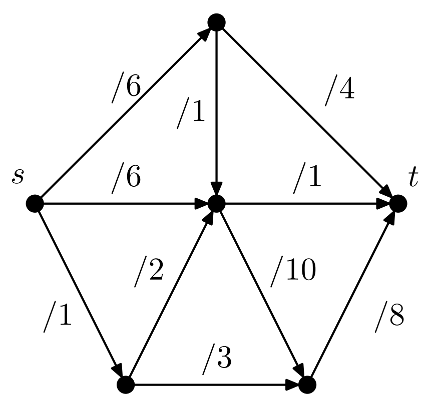
Algorithmics - final exam, 11.06.2014

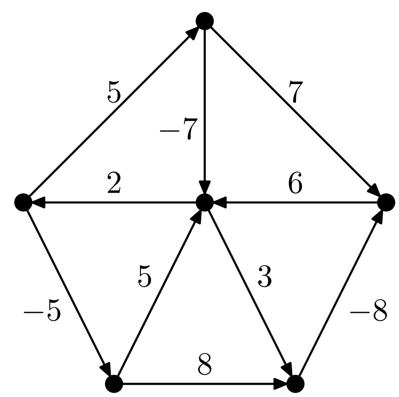
part I (90 minutes)

Note: In questions 5–9, give a short explanation of your answer (not a formal proof!). Also note, that in question 3, you get negative points for an incorrect answer.

1. (**0...4 pt.**)
   - (**0...1 pt.**) Find any maximum flow in the network below. Put the flow values on the edges.
   - (**0...1 pt.**) Give any minimum cut in this network.
   - (**0..2 pt.**) How many iterations can Dinic's algorithm make in this network?



2. (**0...2 pt.**) Find any potential function for the network below. Put the potential values on the edges.



3. (**-6...6 pt.**) Are the following problems NP-hard (assuming P≠NP)? Answer YES/NO.
   - (**-2...2 pt.**) Given a planar graph G and $k \in \mathbb{N}$, decide whether G contains a clique of size k....
   - (**-2...2 pt.**) Given an n-vertex graph G, decide whether G contains a simple cycle of length $\lfloor \sqrt{n} \rfloor$....
   - (**-2...2 pt.**) Let $S \subseteq \mathbb{N}$, $|S| = n$ and $T \in \mathbb{N}$ s.t. $\forall_{s \in S} s \leq 1.01T/n$. Decide whether S contains a subset whose elements sum up to T....

4. (**0...4 pt.**) Let $G = (V, E)$ be a directed graph with non-negative edge weights $w : E \to \mathbb{R}_{\geq 0}$. The linear program below models the shortest s-t-path problem in G for $s, t \in V$. Give a dual of this program. What problem is modelled by this dual?

$$\begin{array}{llcll} \text{maximize} & d_t & & & \\ \text{with} & d_v - d_u & \leq & d_{(u,v)} & \forall_{(u,v) \in E} \\ & d_u & = & 0 & \end{array}$$

5. **(0..2 pt.)** Consider the primal-dual 2-approximation algorithm for the VERTEX-COVER problem. Can every vertex cover be returned by this algorithm?

6. **(0..3 pt.)** Consider the $\frac{3}{2}$-approximation algorithm for the TSP. What is the approximation ratio of this algorithm for metrics where all distances are 1 or 2?

7. **(0..2 pt.)** Give the smallest graph (minimize the number of vertices) for which Karger's min-cut algorithm might fail to find the min-cut.

8. **(0...1 pt.)** How many multiplications does the Strassen's algorithm perform when multiplying two 8x8 matrices?

9. **(0...4 pt.)** We are given a set S of n points in the plane represented by $x[1,\ldots,n]$ and $y[1,\ldots,n]$. We want to compute a boolean array $CH[1,\ldots,n]$, s.t. $CH[i] = \mathit{true}$ iff the i-th point lies on the boundary of the convex hull of S. Give a PRAM CRCW (common) algorithm for this problem.

   Note: A correct time-optimal algorithm with $poly(n)$ processors is worth 3 points. Optimizing the number of processors is worth the remaining 1 point.

10. **(0..1 pt.)** What is the most common score (after possible extra-time and penalties) in the history of the World Cup finals in football?