The streaming *k*-mismatch problem

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UNIVERSITY OF WARSAW

Tradition of Excellence Bar-Ilan University

SODA 2019

San Diego, California, January 7, 2019

Pattern matching

Exact pattern matching

Given two strings: a **pattern** P (of length m) and a **text** T (of length n), find all fragments of T **matching** P.

P

b b a a b b b

T

a b b a a b b b a a b b b b b b a a b b b b a a

Classic algorithms

Knuth, Morris, Pratt 1978, SIAM J. Comput.	$\mathcal{O}(n+m)$ time	$\mathcal{O}(m)$ space
Galil, Seiferas 1983, J. Comput. Syst. Sci.	$\mathcal{O}(n+m)$ time	$\mathcal{O}(1)$ space[1]

[1] Does not include read-only random access to P and T.

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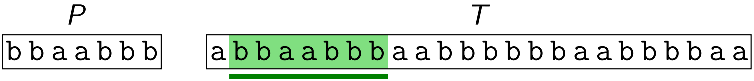

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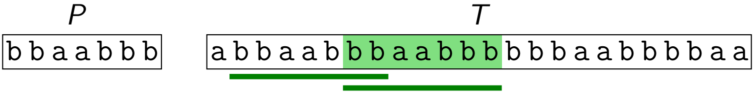

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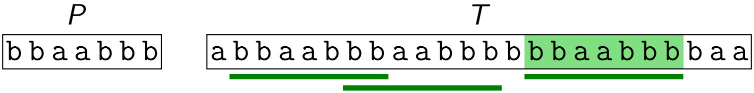

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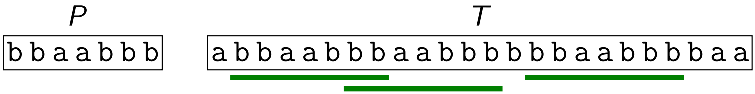

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Streaming pattern matching

Data stream model:

- single sequential scan of the input data,
- online (partial answers after processing each symbol),
- small working space,
- real-time (worst-case per symbol processing time).

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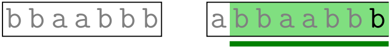

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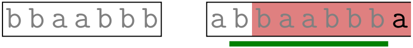

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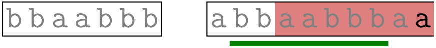

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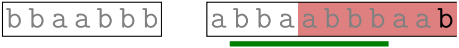

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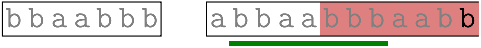

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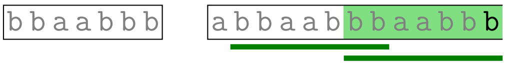

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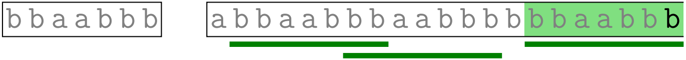

bbaabbb

abbaabbbaabbbbbbaabbb

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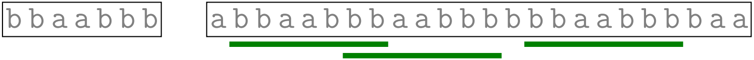

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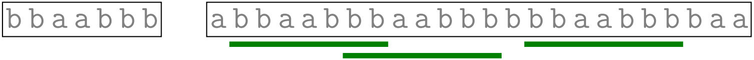


Lower bounds

- deterministic: $\Omega(m \log \sigma)$ bits,
- randomized: $\Omega(\log m)$ bits.

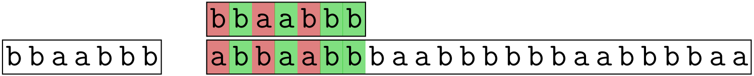
Randomized algorithms

Porat, Porat FOCS 2009	$\mathcal{O}(\log m)$ time	$\mathcal{O}(\log^2 m)$ bits
Breslauer, Galil 2014, ACM Trans. Algorithms	$\mathcal{O}(1)$ time	$\mathcal{O}(\log^2 m)$ bits

Approximate pattern matching

Pattern matching with mismatches

Given a pattern P of length m and a text T of length n, compute the **Hamming distances** between P and all length-m fragments of T.



Output: 3

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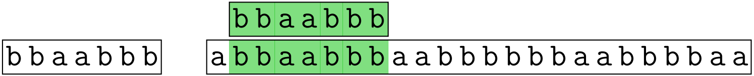

b b a a b b b

b b a a b b b

a b b a a b b b a a b b b b b b a a b b b b a a

Output: 3 0

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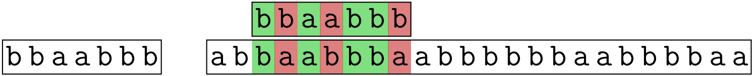

b b a a b b b

b b a a b b b

a b b a a b b b a a b b b b b b a a b b b b a a

Output: 3 0 3

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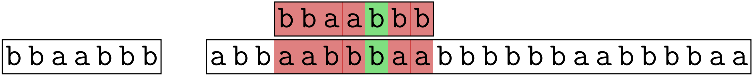

Output: 3 0 3 6

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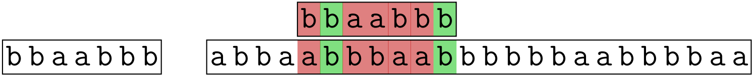

b b a a b b b

b b a a b b b

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Output: 3 0 3 6 5

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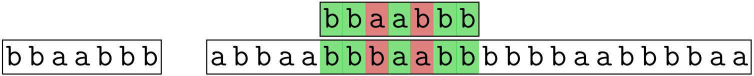

Output: 3 0 3 6 5 2

Approximate pattern matching

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```
b b a a b b b     a b b a a b b b a a b b b b b b a a b b b b a a
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Output: 3 0 3 6 5 2 0 2 4 3 3 4 4 2 0 2 5 5

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Output: 3 0 3 6 5 2 0 2 4 3 3 4 4 2 0 2 5 5

Algorithms

Fischer, Patterson
1973, Complex. Comput. $\mathcal{O}(n\sigma \log m)$ time

Abrahamson
1987, SIAM J. Comput. $\mathcal{O}(n\sqrt{m \log m})$ time

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Lower bound

- no $\mathcal{O}(nm^{0.5-\varepsilon})$-time **combinatorial** algorithms, conditioned on BMM

The k-mismatch problem

Problem

Given a pattern P, a text T, and a **threshold** k, find all fragments of the text T at Hamming distance **at most** k from P (along with the distances).

$\mathbf{k = 3}$

b b a a b b b

a b b a a b b b a a b b b b b b a a b b b b a a

3 0 3 6 5 **2 0 2** 4 **3 3** 4 4 **2 0 2** 5 5

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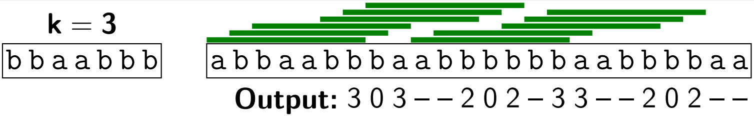

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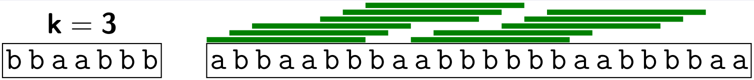

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Output: 3 0 3 – – 2 0 2 – 3 3 – – 2 0 2 – –

Algorithms

Landau, Vishkin 1986, Theor. Comput. Sci.	$\mathcal{O}(nk)$ time
Amir, Lewenstein, Porat 2004, J. Algorithms	$\mathcal{O}(n\sqrt{k \log k})$ time $\mathcal{O}(n + nk^3 \log k/m)$ time
Clifford et al. SODA 2016	$\widetilde{\mathcal{O}}(n + nk^2/m)$ time
Gawrychowski, Uznański ICALP 2018	$\widetilde{\mathcal{O}}(n + nk/\sqrt{m})$ time **Tight** for combinatorial algorithms (from BMM).

The k-mismatch problem: online and streaming algorithms

Algorithms	Time per symbol	Space in bits

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Our main result

Theorem (This work)

There is a streaming k-mismatch algorithm which uses $\mathcal{O}(k \log m \log \frac{m}{k})$ *bits of space and takes* $\mathcal{O}((\sqrt{k \log k} + \log^3 m) \log \frac{m}{k})$ *time per symbol.*

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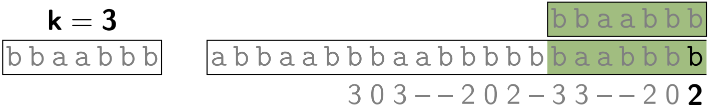

Extra features of the new algorithm:

- For each reported occurrence, the **mismatch information** can be computed on demand in $\mathcal{O}(k)$ time.

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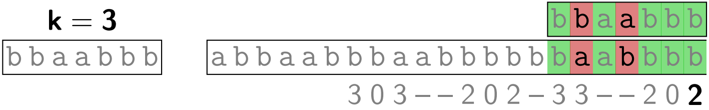

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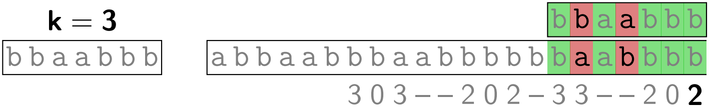

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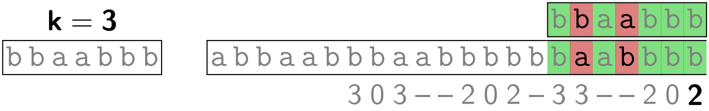

$\mathbf{k = 3}$

b b a a b b b

b b a a b b b

a b b a a b b b a a b b b b b b a a b b b b

3 0 3 − − 2 0 2 − 3 3 − − 2 0 **2**

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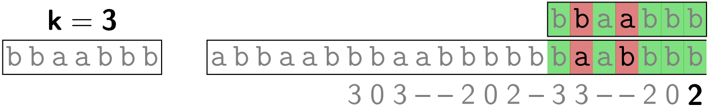
- Pattern preprocessing under the same bounds on space and time.

Our main result

Theorem (This work)

There is a streaming k-mismatch algorithm which uses $\mathcal{O}(k \log m \log \frac{m}{k})$ *bits of space and takes* $\mathcal{O}((\sqrt{k \log k} + \log^3 m) \log \frac{m}{k})$ *time per symbol.*



Extra features of the new algorithm:

- For each reported occurrence, the **mismatch information** can be computed on demand in $\mathcal{O}(k)$ time.
  The only previous streaming algorithm computing mismatch information: Radoszewski, Starikovskaya (DCC 2017): $\widetilde{\mathcal{O}}(k)$ time per symbol, $\widetilde{\mathcal{O}}(k^2)$ space.
- Pattern preprocessing under the same bounds on space and time.
  All previous algorithms require non-streaming preprocessing.

Outline of the talk

Introduction

Exact streaming pattern matching

Our streaming k-mismatch algorithm

Conclusions and open problems

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Karp–Rabin fingerprints

Assign $\mathcal{O}(\log m)$-bit integer **fingerprints** $\Psi(\cdot)$ to strings of length up to m so that if $X \neq Y$, then $\Pr[\Psi(X) = \Psi(Y)] \leq m^{-\Theta(1)}$.

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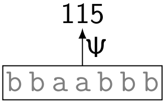

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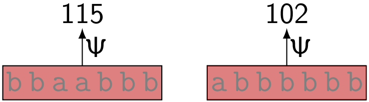

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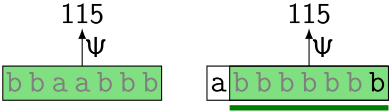

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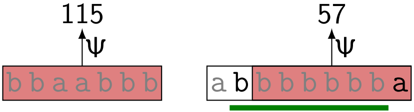

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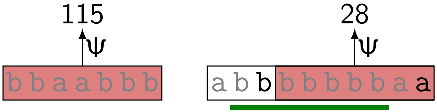

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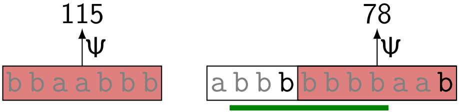

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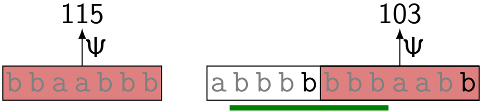

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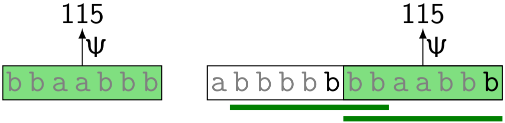

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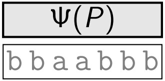

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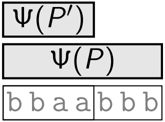

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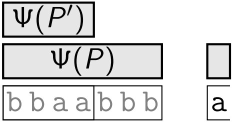

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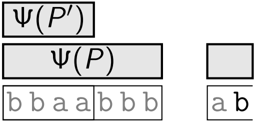

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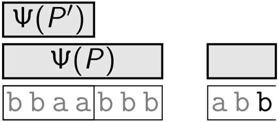

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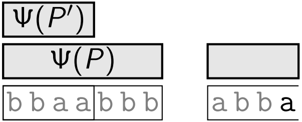

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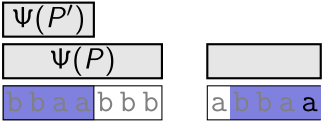

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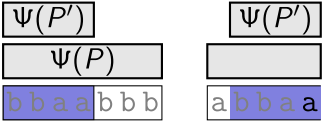

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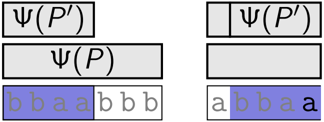

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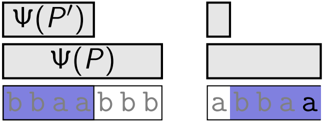

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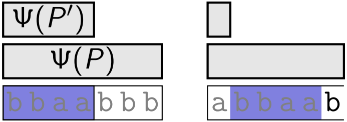

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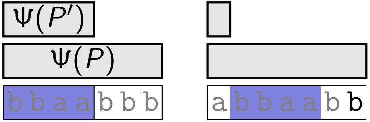

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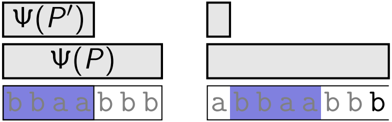

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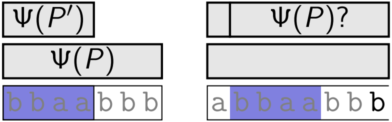

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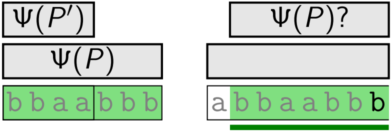

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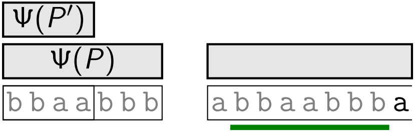

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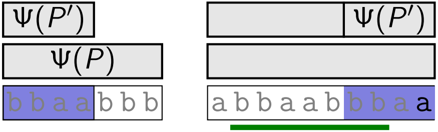

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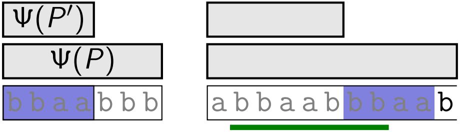

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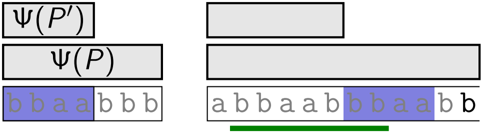

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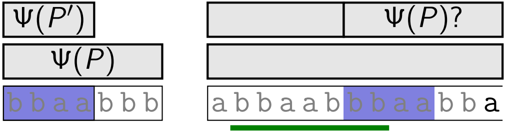

Streaming pattern matching in $\mathcal{O}(\log^2 n)$ bits

Porat and Porat (FOCS 2009), Breslauer and Galil (2014, ACM Trans. Algorithms)

Issue: Rabin–Karp algorithm needs $T[i-m]$ to process $T[i]$.

How to avoid accessing this character?

1. Recursively look for the occurrences of $P' := P[1\mathinner{.\,.}\lceil m/2\rceil]$.
2. Maintain $\Psi(T[1\mathinner{.\,.}i])$.
3. If P' is detected at position j, retrieve and store $\Psi(T[1\mathinner{.\,.}j-1])$.
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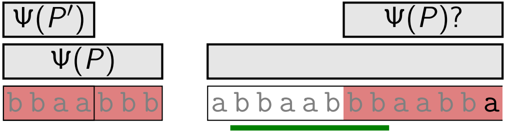

Structure of the viable occurrences of P'

Porat and Porat (FOCS 2009), Breslauer and Galil (2014, ACM Trans. Algorithms)

Viable occurrences of P'

Occurrences of P' in T starting at positions $j \in \{i-m, \ldots, i-|P'|\}$.

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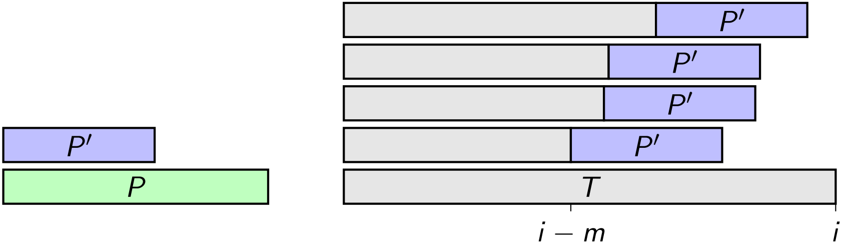

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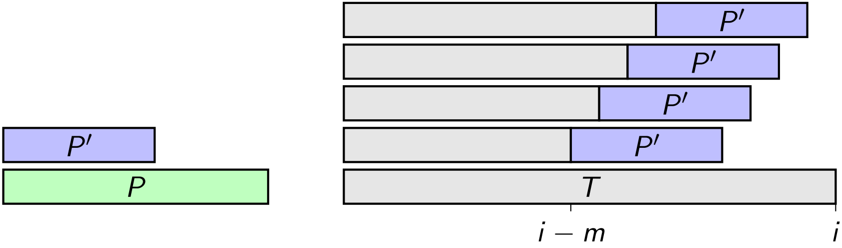

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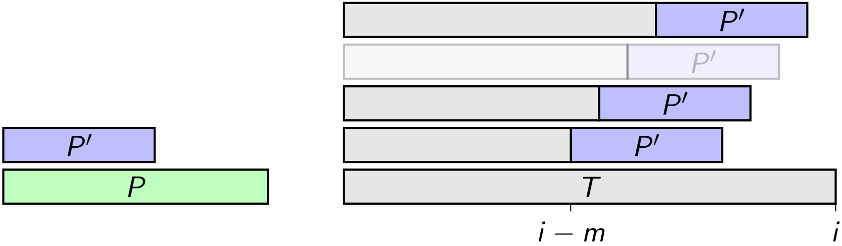

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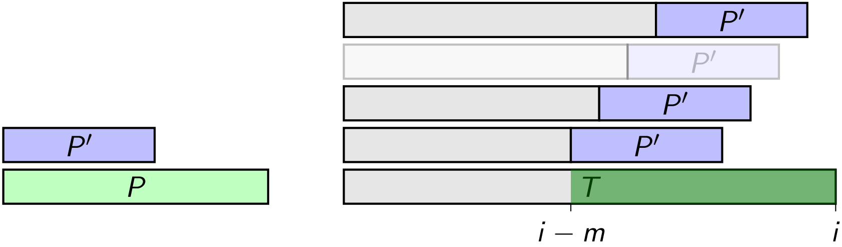

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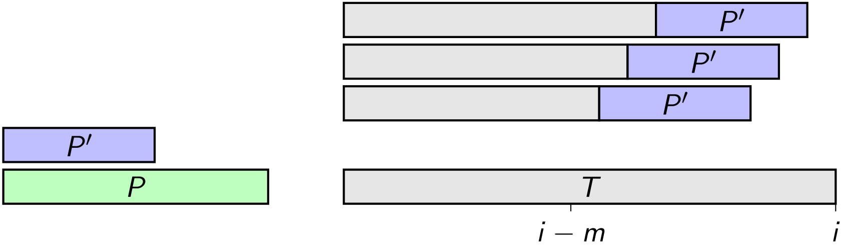

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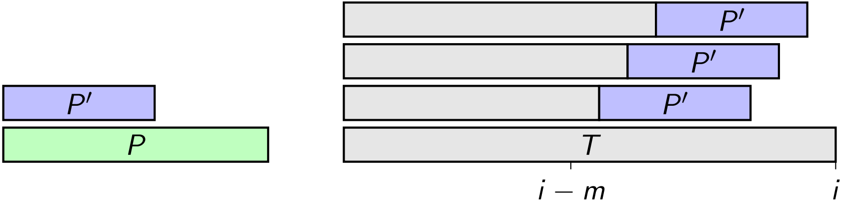

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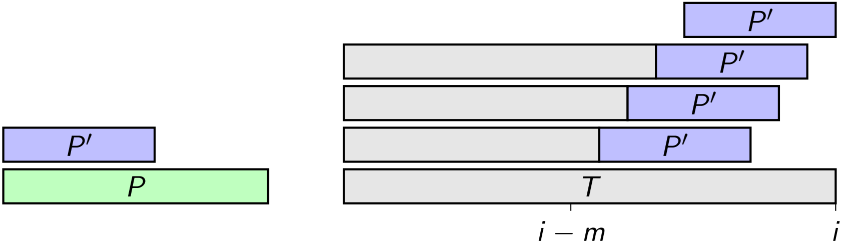

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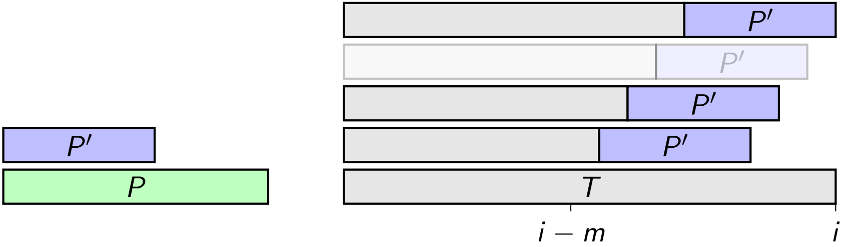

Outline of the talk

Introduction

Exact streaming pattern matching

Our streaming k-mismatch algorithm

Conclusions and open problems

Contribution #1: Rolling k-mismatch sketches

The k-mismatch sketches

A **sketch function** sk_k mapping words X, $|X| \le n$, to $\mathcal{O}(k \log n)$-bit values $\mathrm{sk}_k(X)$ designed so that $\mathrm{sk}_k(X)$ and $\mathrm{sk}_k(Y)$ are sufficient to:

- decide whether $\mathrm{HD}(X, Y) \le k$

X `baabbba`
Y `bbaabbb`

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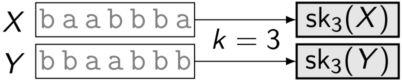

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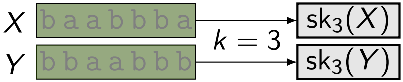

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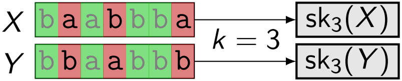

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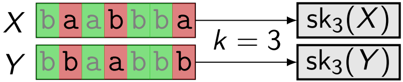

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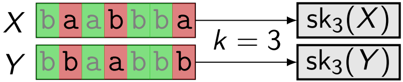

X b a a b b b a $\to$ $\mathsf{sk}_3(X)$

$k = 3$

Y b b a a b b b $\to$ $\mathsf{sk}_3(Y)$

Manipulation in $\widetilde{\mathcal{O}}(k)$ time:

- concatenation,
- prefix and suffix removal,
- appending $\mathcal{O}(k)$ chars,
- $\mathcal{O}(k)$ substitutions.

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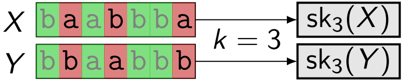

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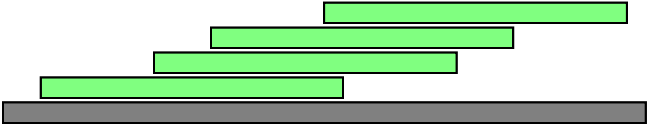
Techniques:

- Reed–Solomon error correcting codes,
- Karp–Rabin fingerprints,
- polynomial factorization, evaluation, and interpolation.

Contribution #2: Encoding viable k-mismatch occurrences

Theorem

The k-mismatch occurrences of a length-m pattern in a length-$2m$ text, each with the mismatch information (MI), can be encoded in $\widetilde{\mathcal{O}}(k)$ bits.



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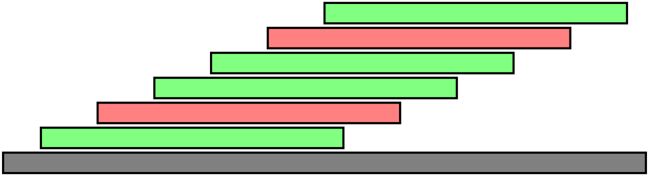

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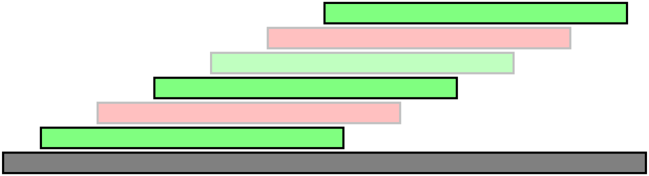

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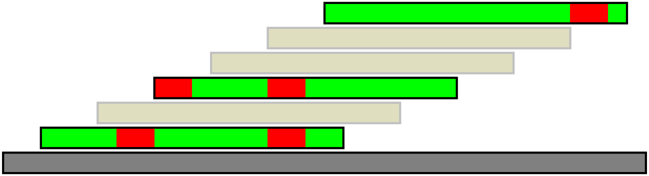

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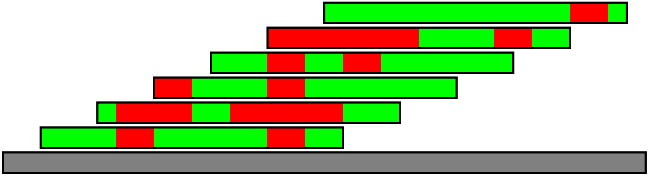

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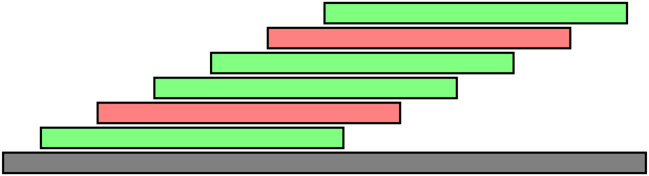

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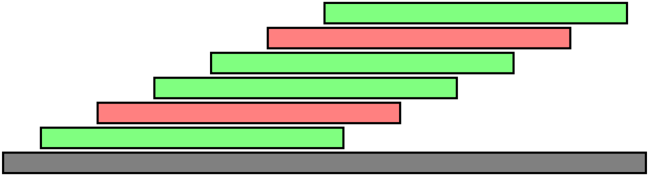

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Consequence

A k-mismatch streaming algorithm with $\widetilde{\mathcal{O}}(k)$ space and time per symbol.

Contribution #3: New solution for nearly periodic patterns

Bottleneck: manipulation of sketches at every position.

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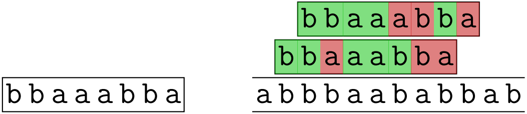
Hope: sketches needed only when a viable occurrence is processed.



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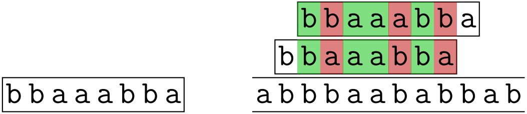

Approximate period

An integer p is a d-**period** of P if $\mathrm{HD}(P[1\mathinner{.\,.}m-p], P[p+1\mathinner{.\,.}m]) \le d$.

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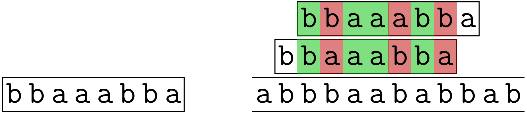

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Improved specialized algorithm

A **deterministic** streaming k-mismatches algorithm for patterns P with an $\mathcal{O}(k)$-period $\mathcal{O}(k)$. Complexity: $\widetilde{\mathcal{O}}(k)$ bits and $\widetilde{\mathcal{O}}(\sqrt{k})$ time per symbol.

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Possible directions for further research:

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3. Improved **amortized** running time?
4. Any $\omega(k + \log n)$ lower bounds?
    - $\Omega(k \log n)$ might be feasible.
    - $\omega(\log n)$ for constant k might give hints for exact matching.

Questions?

Thank you for your attention!

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