Internal Pattern Matching Queries in a Text and Applications

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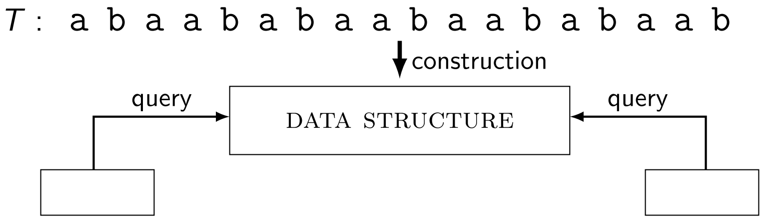
T : a b a a b a b a a b a a b a b a a b

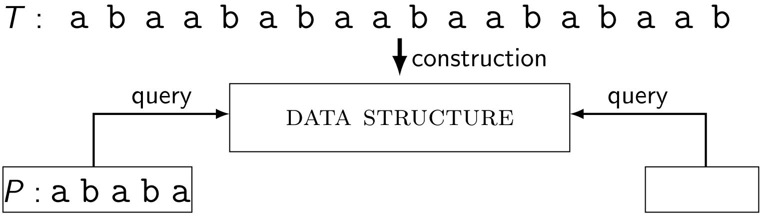
T : `a b a a b a b a a b a a b a b a a b`

↓ construction

DATA STRUCTURE

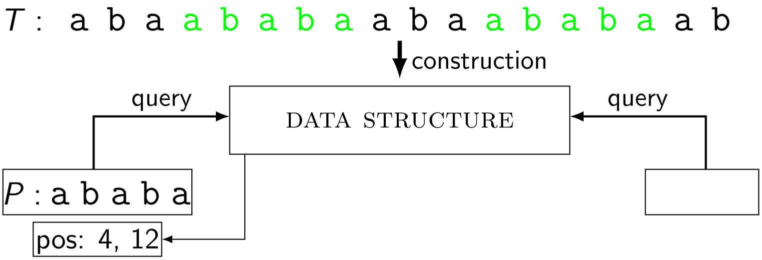
Classic Data Structures for Text Processing





Indexing Subwords

Given a pattern P, find all occurrences of P in T.



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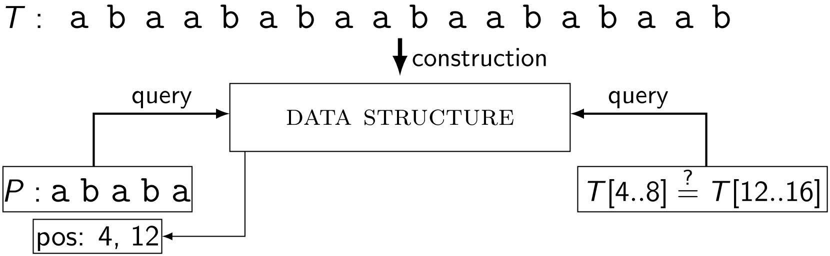

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Given two subwords x, y of T, decide whether $x = y$.

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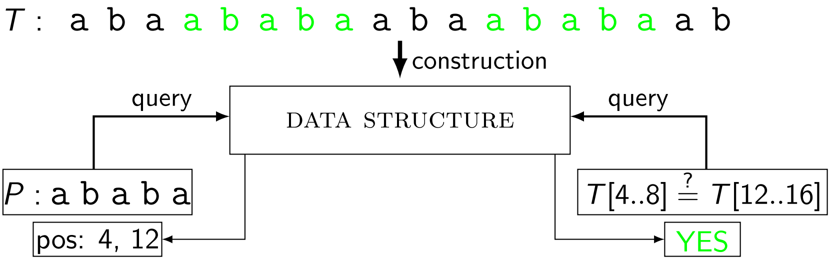

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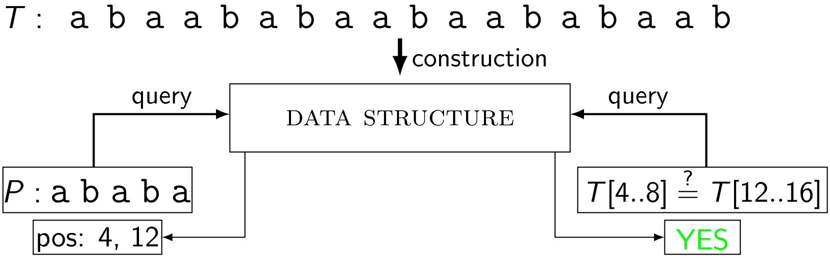

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- concern subwords of T only (identified by positions).

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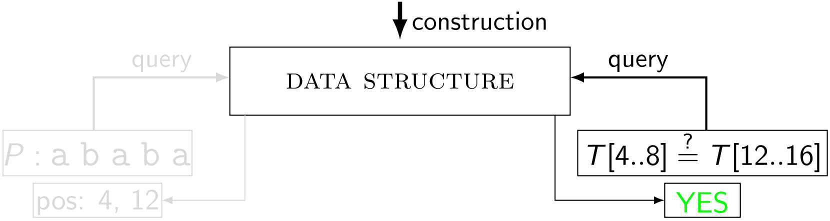

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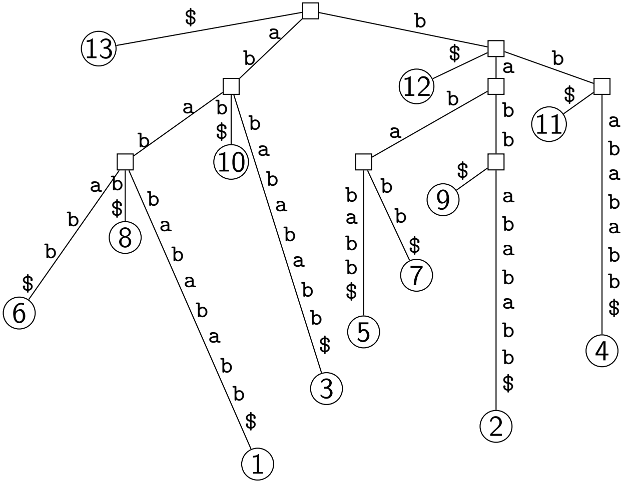
Motivation:

- All input data must be known in advance
- Primitives for algorithms and data structures
  - linear size and $\mathcal{O}(1)$ query time crucial for applicability,
  - efficient construction important for applications in algorithms.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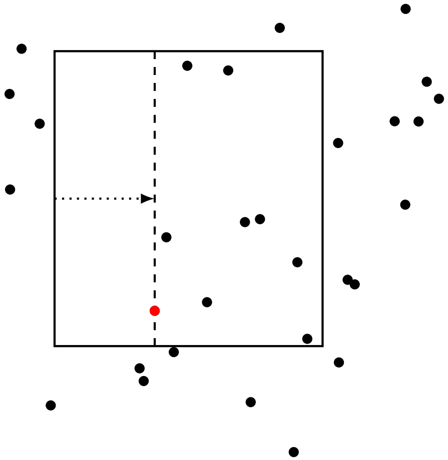
Suffix tree



Range successor queries

Internal Queries: Previous Techniques

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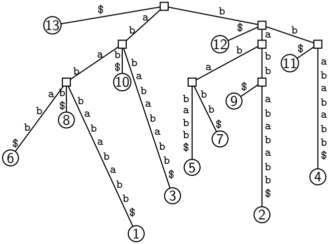

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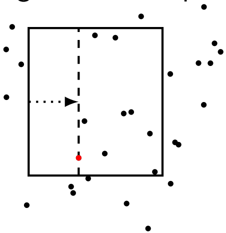

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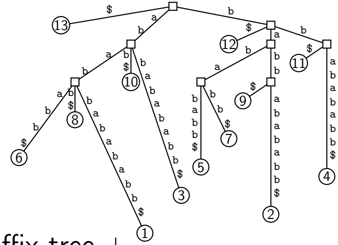

Suffix tree +
problem-specific tools:

- Longest common prefix
- Min/Max suffix

\+ efficient, often $\mathcal{O}(1)$ time queries with $\mathcal{O}(n)$ space,

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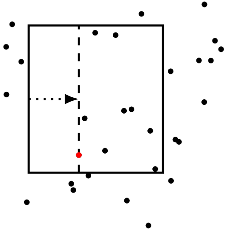

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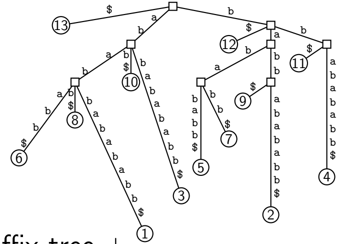

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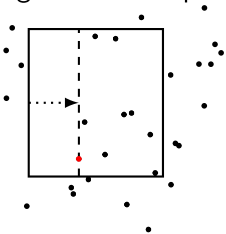

Suffix tree +
orthogonal range queries:

- Pattern matching-related
- Period queries

\+ wide applicability,

– lower bounds for query time, slow construction.

Fact

Let $Occ(x,y)$ be the positions in y where occurrences of x start. $Occ(x,y)$ forms an arithmetic progression if $|y| \le 2|x|$.

x : $\boxed{\texttt{a b a b a b a}}$ y : $\boxed{\texttt{b b a b a b a b a b a b a a}}$

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Problem (INTERNAL PATTERN MATCHING QUERIES)

Given subwords x and y of T with $|y| \le 2|x|$, report all occurrences of x in y (as an arithmetic progression).

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Given subwords x and y of T with $|y| \leq 2|x|$, report all occurrences of x in y (as an arithmetic progression).

- $\Theta\left(\frac{|y|}{|x|}\right)$ space is necessary to encode $Occ(x, y)$,
- $Occ(x, y)$ can be computed with $\Theta\left(\frac{|y|}{|x|}\right)$ IPM Queries.

IPM Queries: Our Results

Problem (Internal Pattern Matching Queries)

Given subwords x and y of T with $|y| \le 2|x|$, report all occurrences of x in y (as an arithmetic progression).

Theorem

IPM Queries *can be answered in $\mathcal{O}(1)$ time by a data structure of size $\mathcal{O}(n)$, which can be constructed in $\mathcal{O}(n)$ time.*

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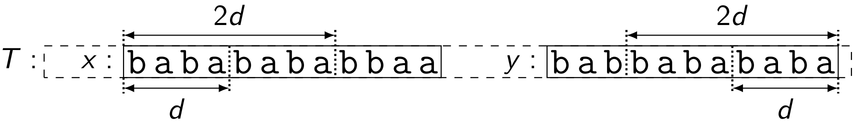
- Construction:
  - randomized (Las Vegas, expected time),

Applications

Prefix-Suffix and Period Queries



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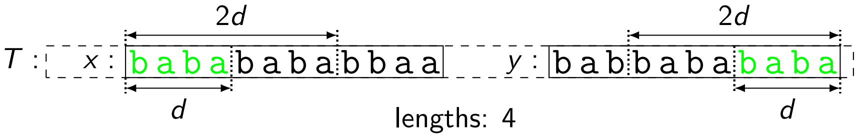

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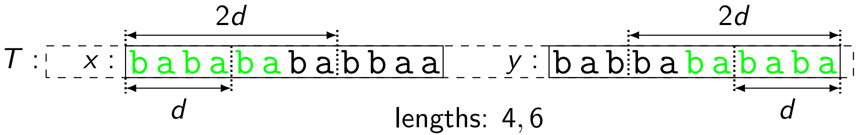

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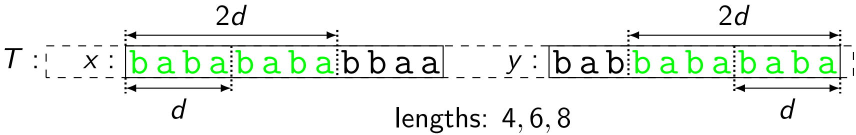

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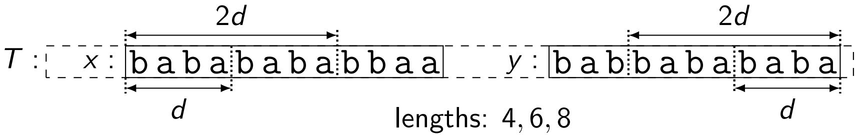

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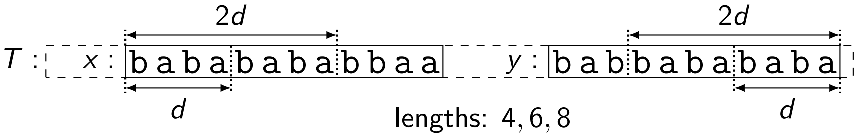
- $\mathcal{O}(1)$ time using IPM Queries.



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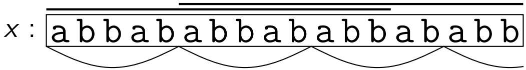

Problem (PERIOD QUERIES)

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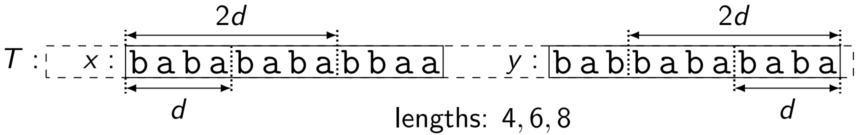

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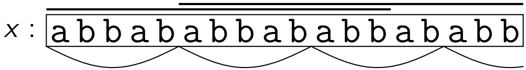


Problem (Period Queries)

Given a subword x of T, report all periods of x.

- $\mathcal{O}(\log |x|)$ time using IPM Queries.

Applications in Substring Compression

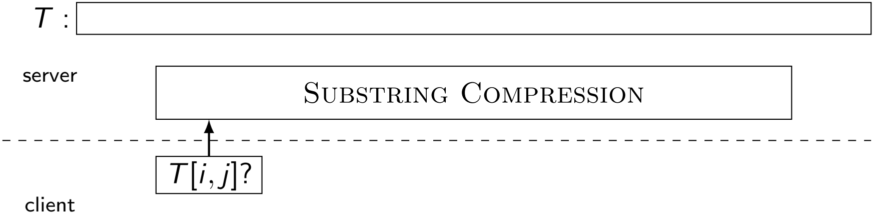
T :

server

SUBSTRING COMPRESSION

client

Applications in Substring Compression



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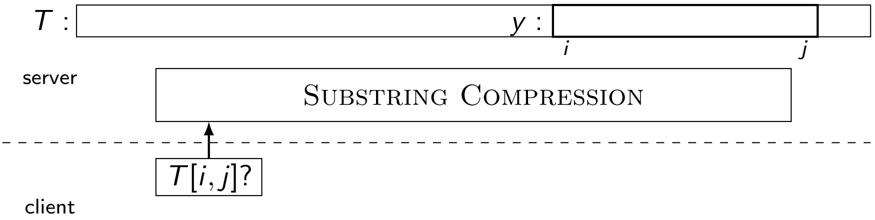

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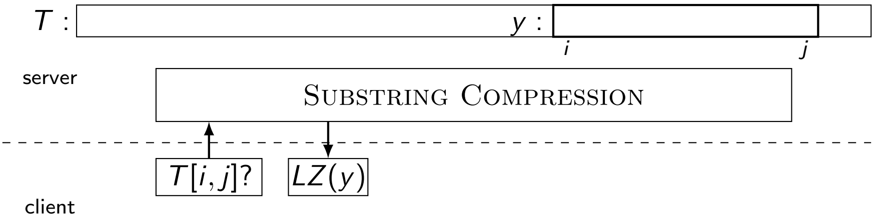

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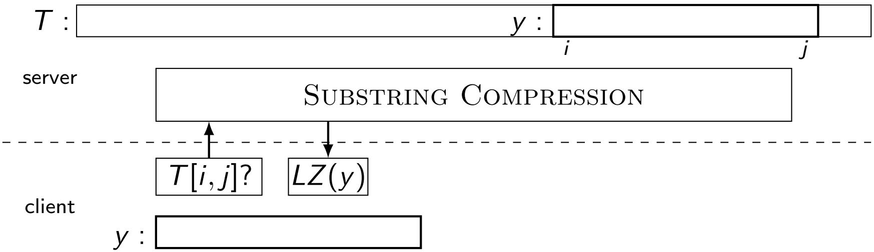

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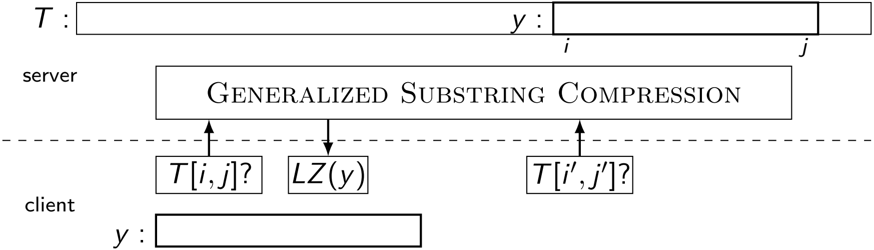

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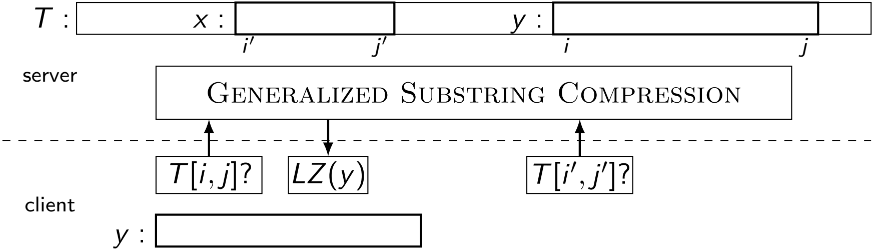

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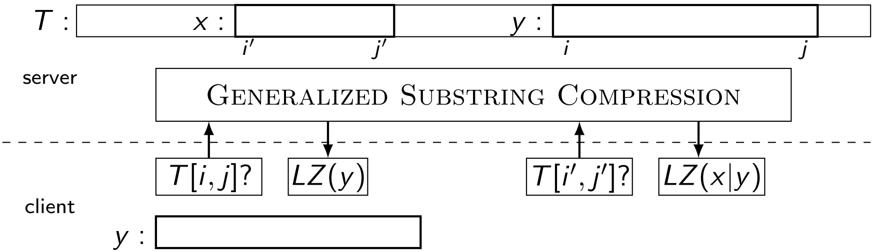

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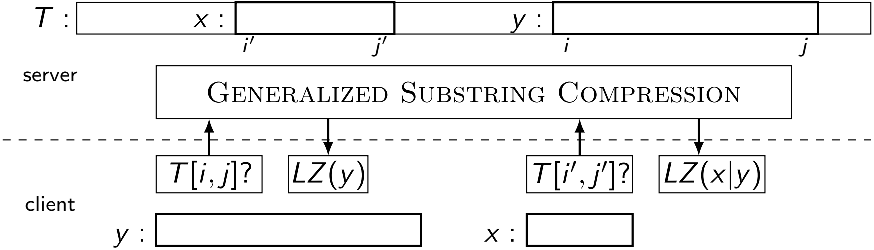

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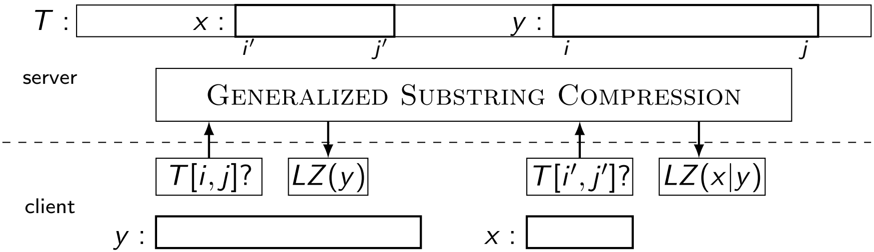

Problem (Cormode & Muthukrishnan; SODA 2005)

Given subwords x and y, compute $LZ(x|y)$, i.e., the section of the Lempel-Ziv (LZ77) factorization LZ($y\$x$) that corresponds to x.

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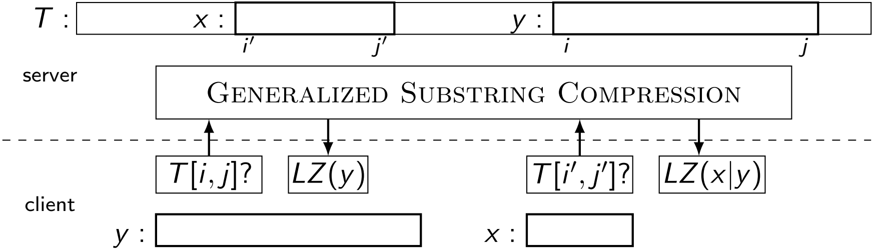

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- data structure of $\mathcal{O}(n)$ size,
- query time improved from $\mathcal{O}(C \log \frac{|x|}{C} \log^{\varepsilon} n)$ to $\mathcal{O}(C \log\log \frac{|x|}{C} \log^{\varepsilon} n)$ where C is the output size,
- combines orthogonal range queries with IPM QUERIES.

Other Applications

Problem (CYCLIC EQUIVALENCE QUERIES)

Decide whether given subwords x and y are cyclic shifts of each other ($x = uv$ and $y = vu$ for some strings u and v).

- $\mathcal{O}(1)$ query time using IPM QUERIES.

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Definition (δ-subrepetition)

A δ-*subrepetition* is a fragment x such that $\mathrm{per}(x) \leq \frac{|x|}{1+\delta}$ and x cannot be extended (to the left or right) preserving $\mathrm{per}(x)$.

- Improved algorithm for finding δ-subrepetitions in a word:
  - first (expected) linear-time algorithm for $\delta = \Theta(1)$.

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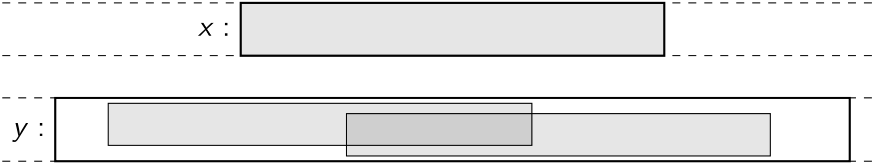
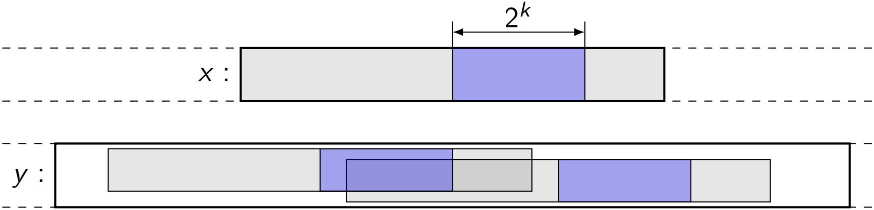
- More applications through wavelet suffix trees:
  - substring suffix rank & selection,
  - substring compression using Burrows-Wheeler transform.

High-level Ideas

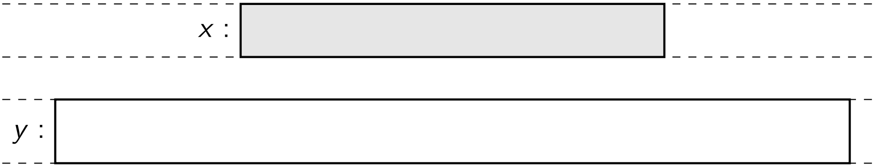
IPM Queries: Outline





Main idea:

- Design (locally consistent) representative assignment
  - representatives of length 2^k with $\frac{|x|}{4} < 2^k \le \frac{|x|}{2}$,
  - store information about location of representative only.



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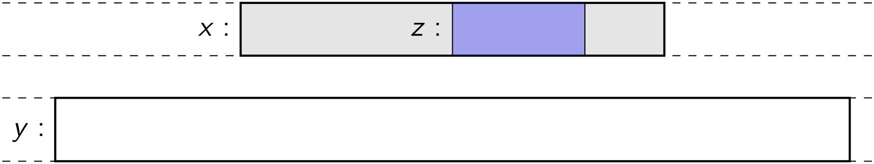

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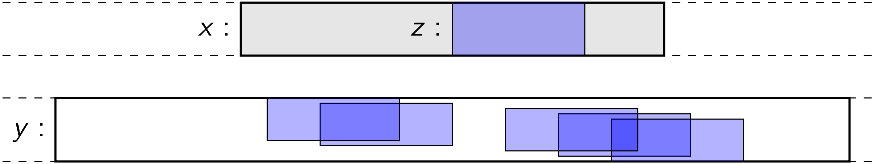

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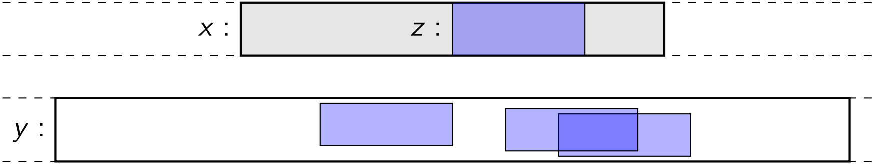

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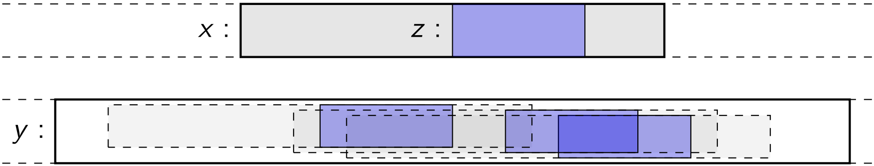

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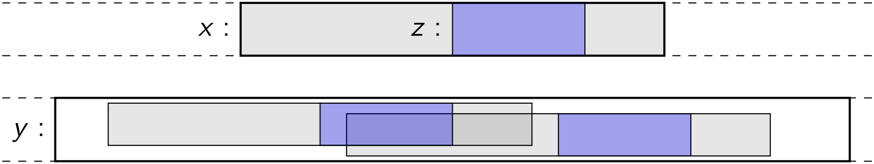
  - representatives of length 2^k with $\frac{|x|}{4} < 2^k \le \frac{|x|}{2}$,
  - store information about location of representative only.

Query algorithm:

1. Compute the representative z assigned to x.
2. Find occurrences of z (as representatives) within y.
3. Check which occurrences of z extend to occurrences of x.



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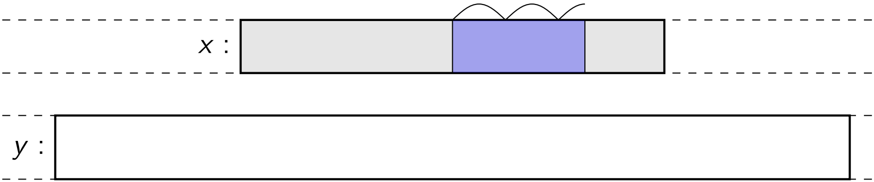
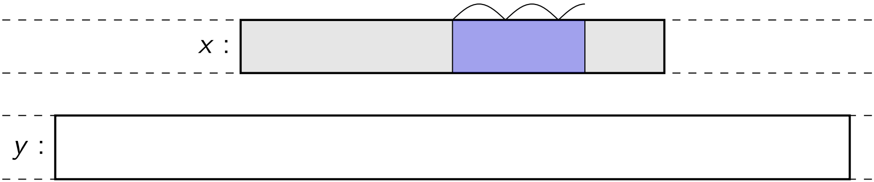


Representative: any periodic subword of appropriate length.

- use combinatorial and algorithmic tools for repetitions.

Periodic Representatives



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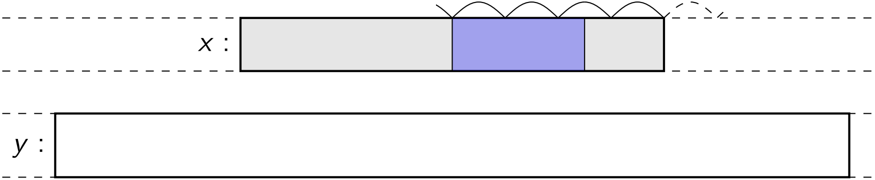

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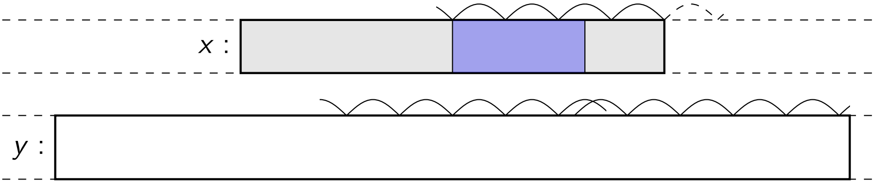

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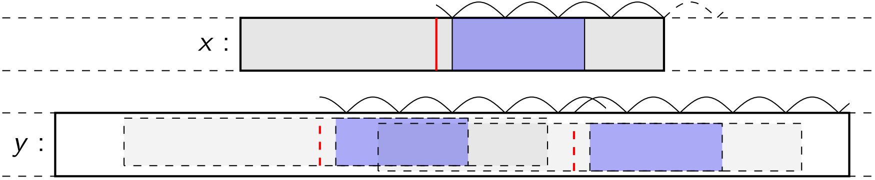

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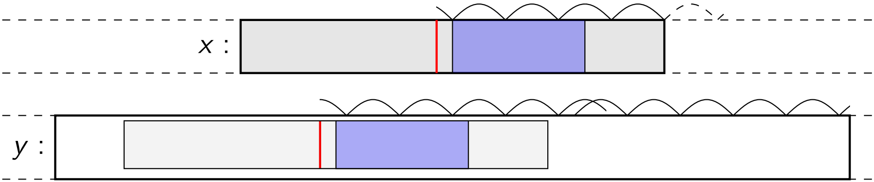

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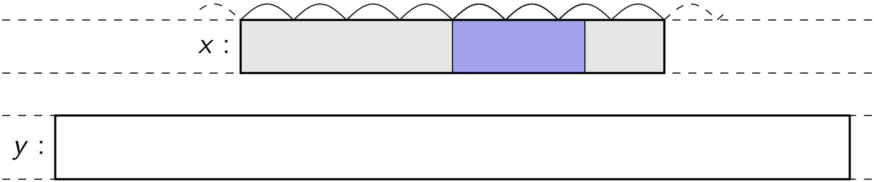

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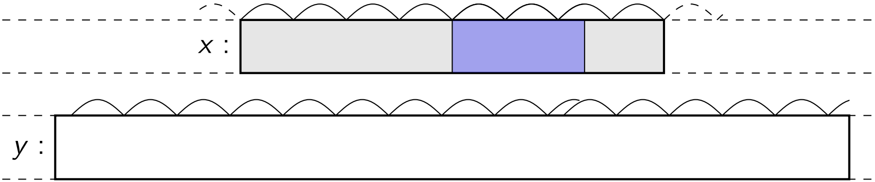

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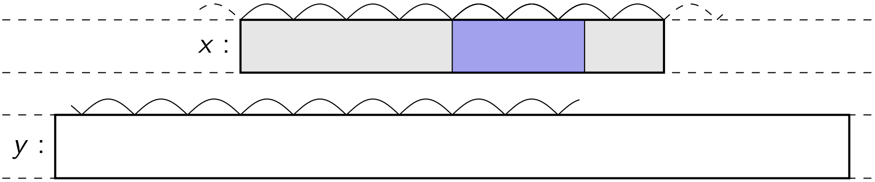

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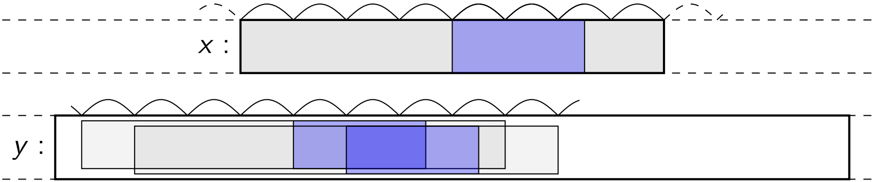

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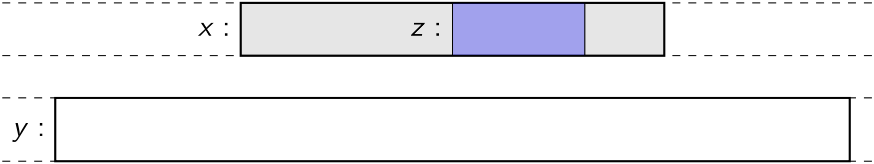
Non-periodic Representatives



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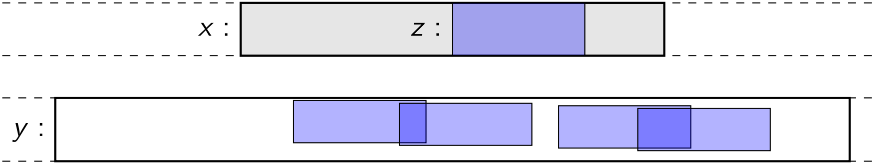

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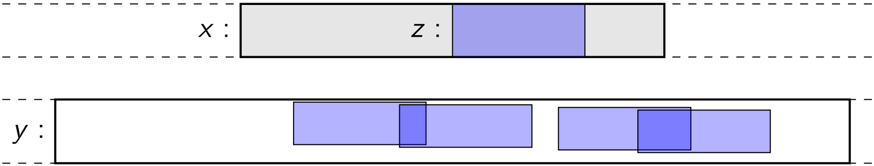

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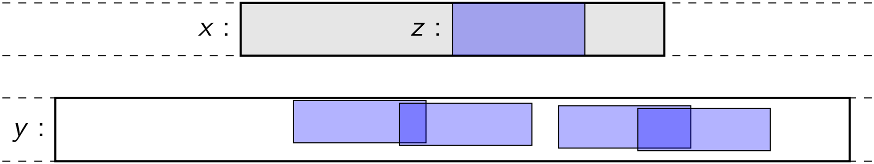

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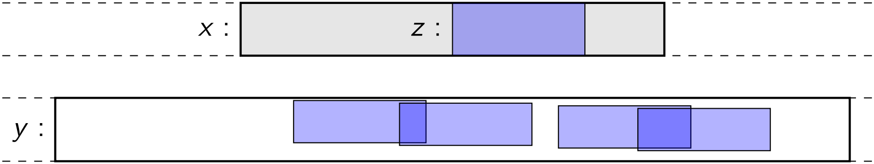

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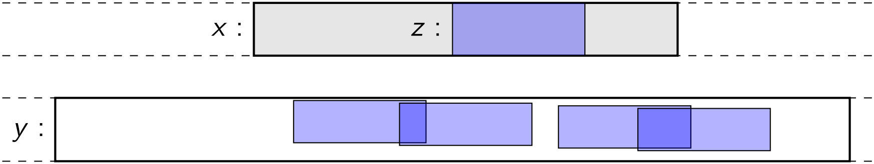

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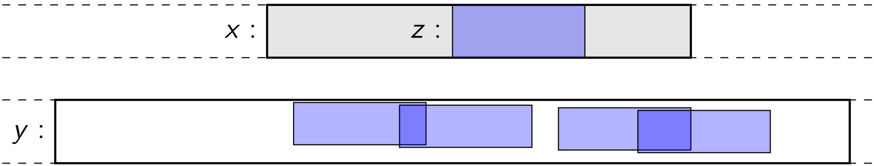

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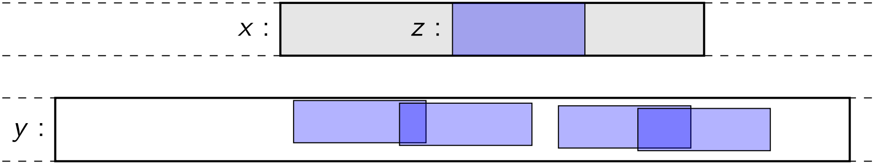

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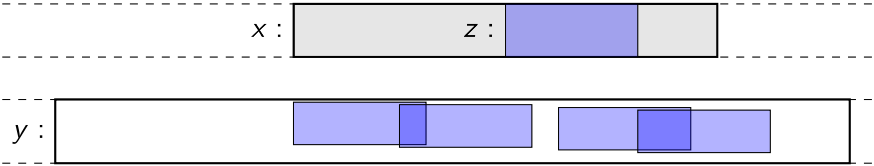

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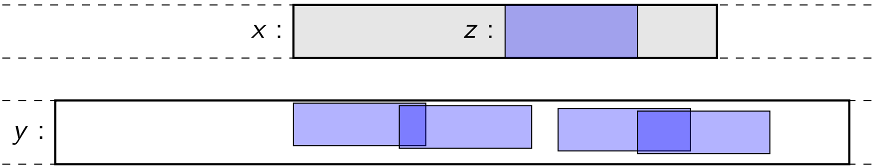

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  - in total $\mathcal{O}(n)$ representatives with $\mathcal{O}(n)$ occurrences as a representative.

Conclusions & Open Problems

Our results:

- a $\mathcal{O}(n)$-size data structure for IPM QUERIES with $\mathcal{O}(1)$-time queries and $\mathcal{O}(n)$-expected time construction,
- several applications of IPM QUERIES:
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- More applications of IPM QUERIES.
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- Can shortest periods be computed in $o(\log |x|)$ time?
  - Is there any lower bound for this problem?

Thank you for your attention!