String Powers in Trees

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Strings Powers

Definition (String Power)

The k-th power of a string u is the string $u^k = \underbrace{uu\ldots u}_{k \text{ times}}$.

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$k = 4$

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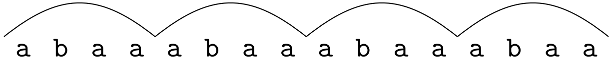

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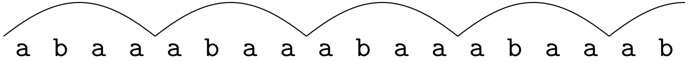

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Most commonly studied types of repetitions:

- squares ($k = 2$),
- cubes ($k = 3$).

Repetitions in Strings

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For a string s and an exponent α define $\text{powers}_\alpha(s)$ as the number of distinct substrings of s being powers of exponent α.

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Lower bounds

- $\text{powers}_\alpha(\mathrm{a}^m) = \Omega(m)$ for any fixed $\alpha \geq 1$:
  - a^{cx} for $1 \leq c \leq \lfloor \frac{m}{x} \rfloor$ where $\alpha = \frac{x}{y}$.

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  - a^{cx} for $1 \leq c \leq \lfloor \frac{m}{x} \rfloor$ where $\alpha = \frac{x}{y}$.
- $\text{powers}_\alpha(\text{a}^m\text{ba}^m) = \Omega(m^2)$ for any fixed $1 \leq \alpha < 2$:
  - $\text{a}^i\text{ba}^{cy-1-i}\text{a}^{cx}$ for $1 \leq c \leq \lfloor \frac{m}{y} \rfloor$, $cx \leq i \leq cy$ where $\alpha = 1 + \frac{x}{y}$.

Repetitions in Strings: Upper Bounds

Theorem (Fraenkel and Simpson, 1998)

For every strings s we have $powers_2(s) = O(|s|)$.

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String powers in strings:

$\alpha \in (1,2)$	$\Theta(n^2)$
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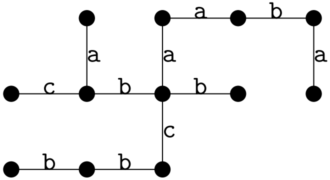
Unrooted, unoriented trees with edges labeled by single letters.

T :



Repetitions in Trees

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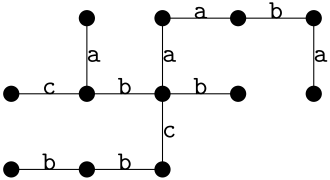

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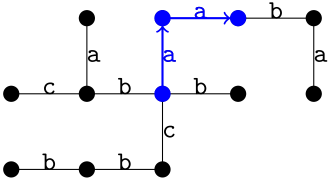

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Squares in T: aa

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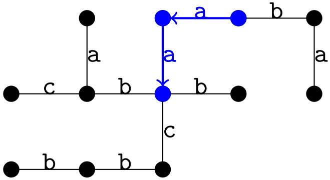

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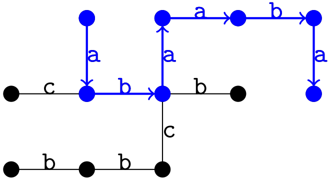

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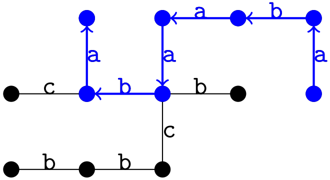

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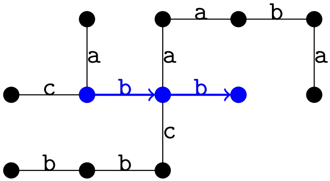

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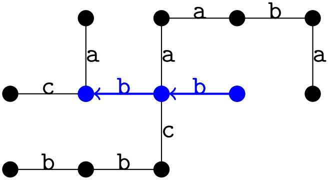

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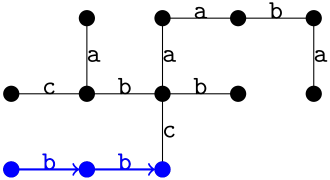

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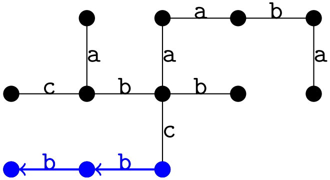

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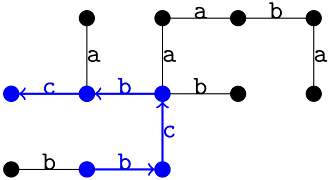

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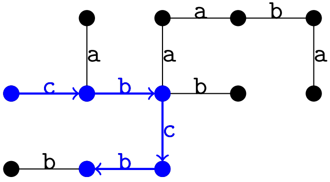

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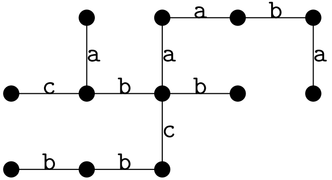

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$$\text{powers}_2(T) = 5$$

Origins:

- avoidability problems
- square-free strings (Thue, 1906)
  - semigroup theory (Burnside's problem for semigroups),
  - number theory (Prouhet-Tarry-Escott problem).

Related Work

Origins:

- avoidability problems
- square-free strings (Thue, 1906)
  - semigroup theory (Burnside's problem for semigroups),
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- non-repetitive colorings of trees
  (Brešar et al., 2007; Grytczuk, 2008)

We denote $\text{powers}_\alpha(T)$ as the number of distinct substrings of T which are α-powers and $\text{powers}_\alpha(n)$ as the maximum $\text{powers}_\alpha(T)$ over trees T with n edges.

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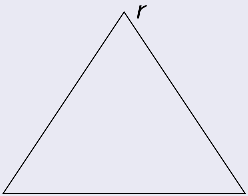
Proof.

- Fix a node r.



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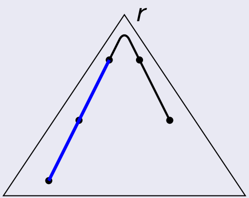

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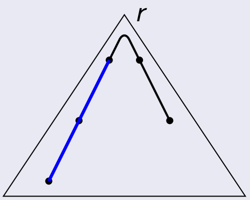
- At most 2 topmost occurrences of such squares at any position.



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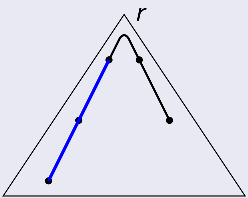
- $\text{powers}_4(n) \le 4n$.



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$\alpha \in (1,2)$	$\Theta(n^2)$
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$\alpha \in (1,2)$	$\Theta(n^2)$
$\alpha = 2$	$\Theta(n^{4/3})$
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Theorem (KRRW, CPM 2012)

$powers_2(n) = \Theta(n^{4/3})$.

Squares in Trees

String powers in trees:

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$\alpha \in (2,4)$	??
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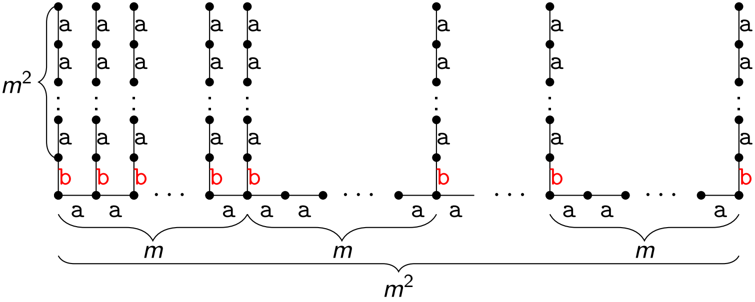
Extending the Lower Bound



Branches start at positions $\{0, 1, 2, \ldots, m-1, m, 2m, 3m, \ldots, m^2\}$, which form a difference cover (for distances $1, \ldots, m^2$).

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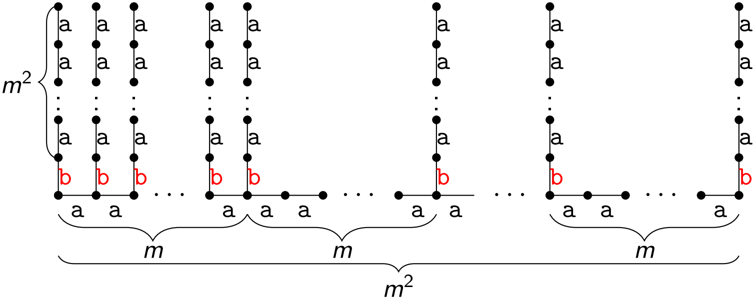

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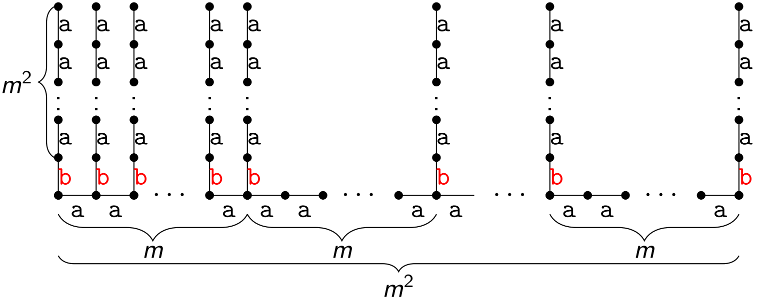
There are $\Theta(m^3)$ edges and $\Theta(m^4)$ squares:

$$\{\mathtt{a}^i\mathtt{b}\mathtt{a}^{i+j}\mathtt{b}\mathtt{a}^j = (\mathtt{a}^i\mathtt{b}\mathtt{a}^j)^2 : 1 \le i + j \le m^2\}.$$



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For fixed $\alpha = 2 + \frac{x}{y}$ there are $\Theta(m^4)$ α-powers:

$$\{(\mathtt{a}^i\mathtt{b}\mathtt{a}^{cy-1-i})^2 a^{cx} : 1 \le c \le \lfloor \tfrac{m^2}{y} \rfloor, cx \le i \le cy\}.$$

Cubes in Trees

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$\alpha \in [2,3)$	$\Theta(n^{4/3})$
$\alpha \geq 4$	$\Theta(n)$

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Theorem (this work)

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Theorem (this work)

$powers_3(n) = O(n \log n)$.

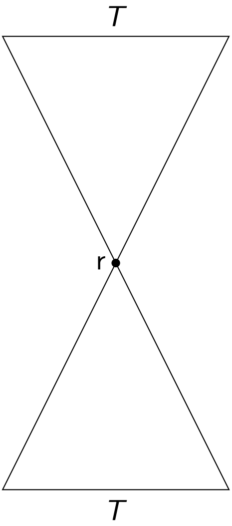
A substring is anchored at a node r if it is a label of a simple path containing a node r.

Theorem

A tree T with n edges and a fixed node r contains $O(n)$ distinct cubes anchored at r.

Core of the Problem

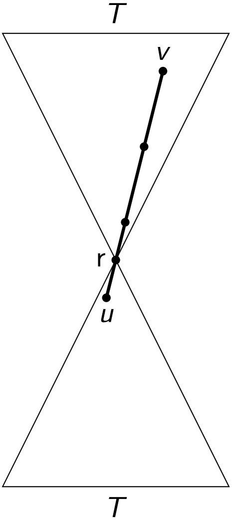
Consider a fixed anchor node r.



Consider a fixed anchor node r.

Definition

We say that (U, V) is a decomposition a cube X^3 if $\mathrm{val}(u, r) = U$, $\mathrm{val}(r, v) = V$ and $\mathrm{val}(u, v) = X^3$.



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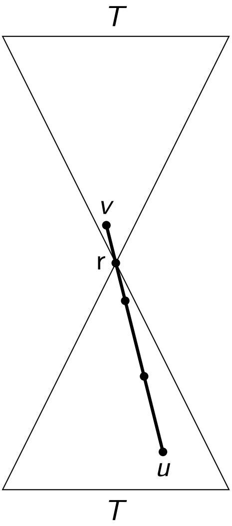
We limit to essential cube decompositions:

- leftist (with $|U| > |V|$),



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We limit to <u>essential</u> cube decompositions:

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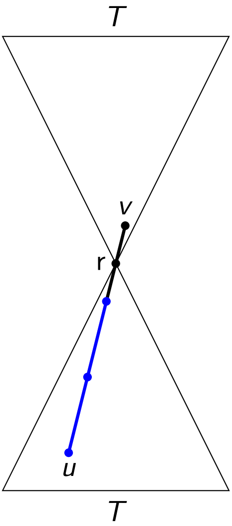

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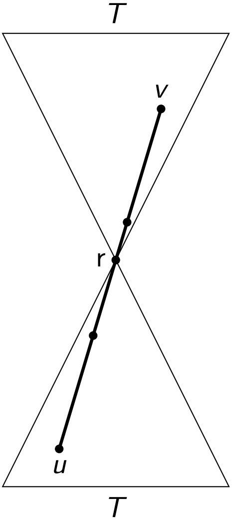

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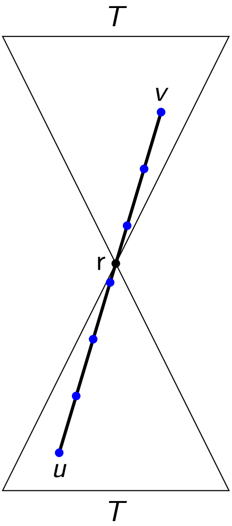

Core of the Problem

Consider a fixed anchor node r.

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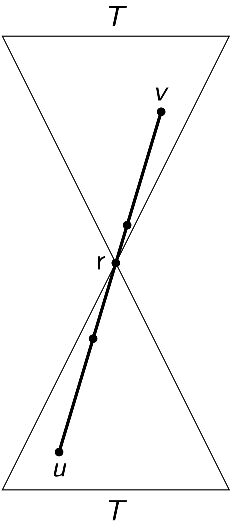
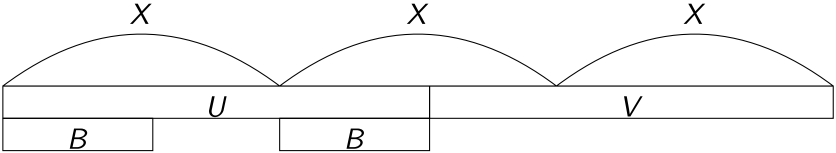
We limit to essential cube decompositions:

- leftist (with $|U| > |V|$),
- balanced (with $|U|, |V| > |X|$),
- with primitive base.



Type-1 Cube Decompositions



- Essential cube decomposition (U, V) induces a border B of U such that $U = XB$

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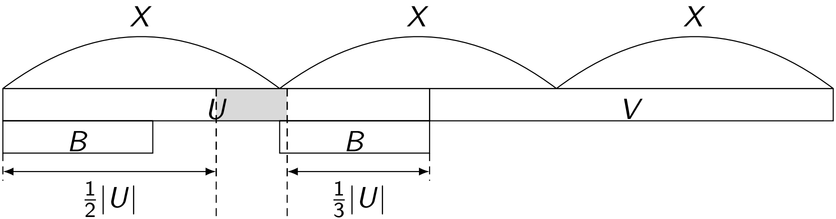

- Essential cube decomposition (U, V) induces a border B of U such that $U = XB$ and $\frac{1}{3}|U| \le |B| \le \frac{1}{2}|U|$.

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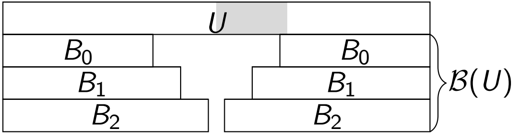

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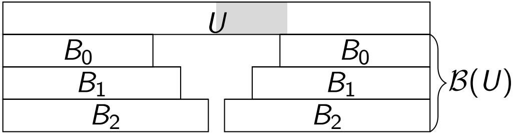

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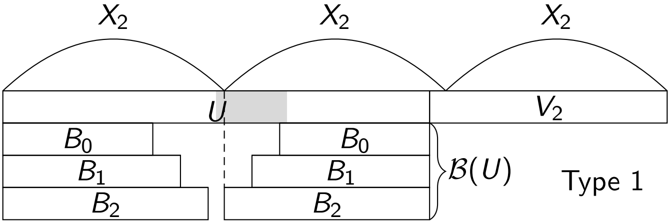

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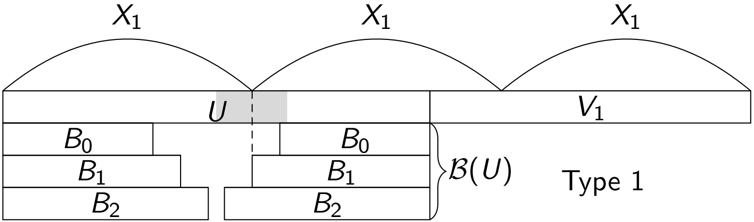

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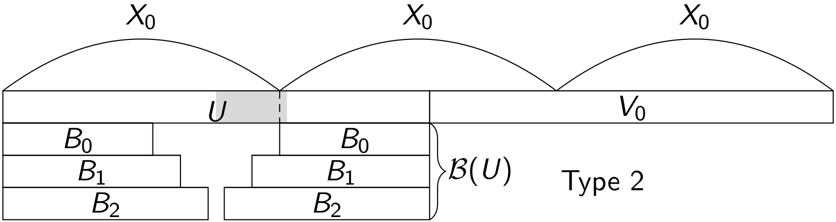

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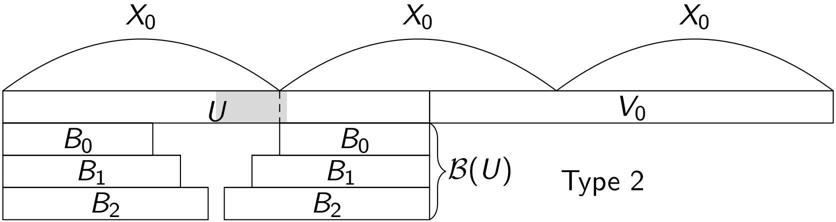

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- Classification of decompositions:
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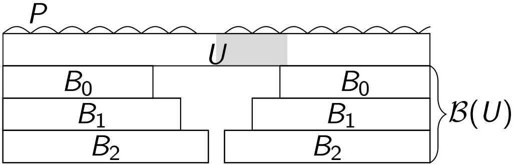
- For every string U there are at most two strings V such that (U, V) is an essential decomposition of Type 1.

Type-2 Cube Decompositions: Characterization



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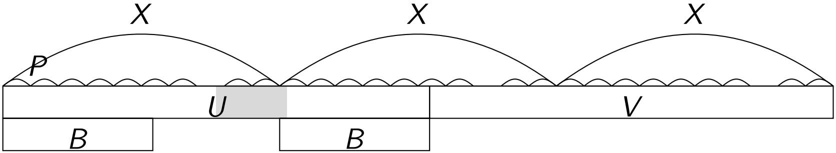

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- If (U, V) is a type-2 decomposition, then:

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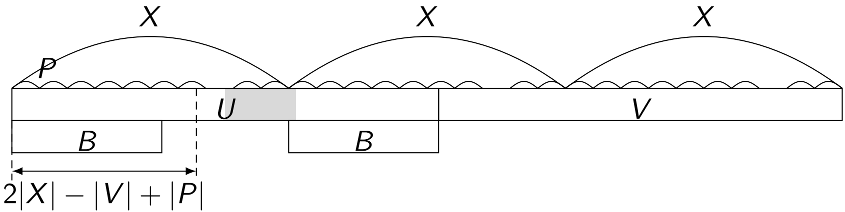

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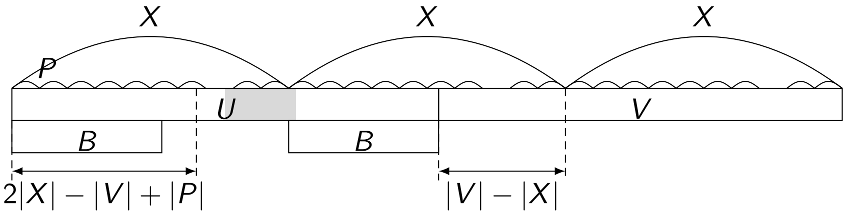

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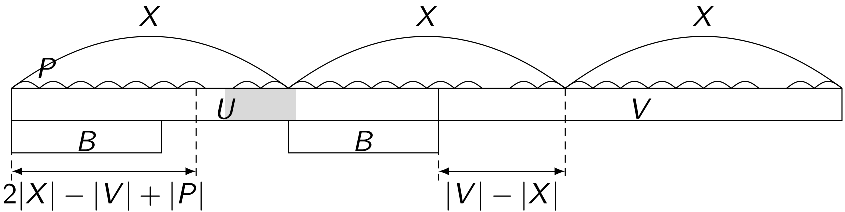

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For every string V there is at most one string U such that (U, V) is an essential decomposition of Type 2.

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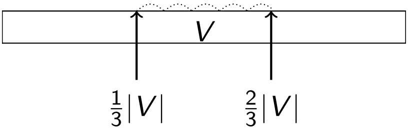

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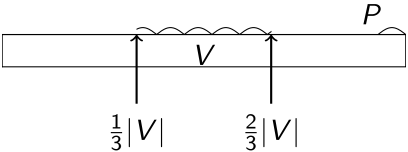

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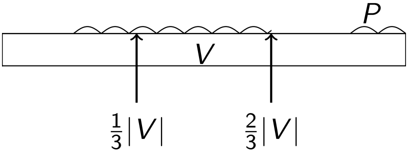

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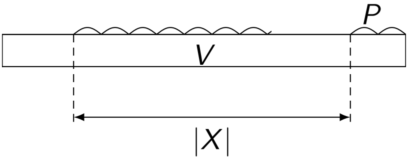

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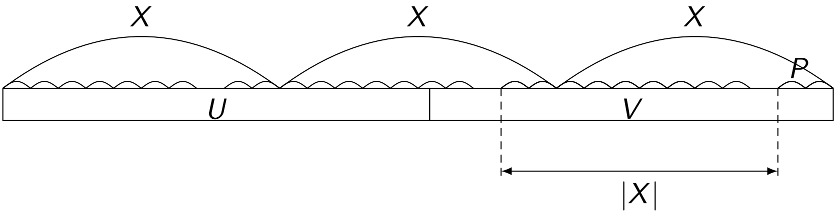

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Cubes in Trees: Summary

$O(n)$ cubes anchored at a fixed node r:

- $2n$ cubes with essential decomposition of type 1,
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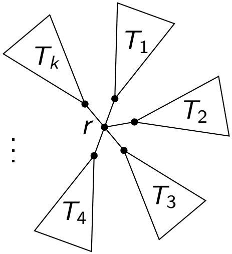

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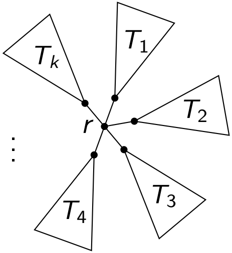

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Let $\text{powers}_3(T, r)$ be the number of cube substring of T anchored at r.

$$\text{powers}_3(T) \leq \text{powers}_3(T, r) + \sum_i \text{powers}_3(T_i)$$

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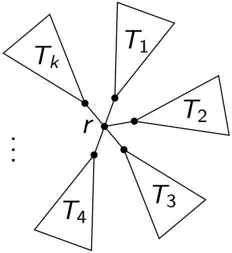

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Corollary

$\mathit{powers}_3(T) = O(|T| \log |T|)$.

Conclusions and Open Problems

String powers in trees:

$\alpha \in (1, 2)$	$\Theta(n^2)$
$\alpha \in [2, 3)$	$\Theta(n^{4/3})$
$\alpha \in [3, 4)$	$O(n \log n)$
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Related work: number of distinct palindromes

- $n + 1$ for strings (Droubay et al., 2001),
- $\Omega(n^{3/2})$ for trees (Brlek, Lafrenière and Provençal, DLT 2015),
- $O(n^{3/2})$ for trees (GKRRW, SPIRE 2015).

Thank you

Thank you for your attention!