Efficient Algorithms for Shortest Partial Seeds in Words

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Moscow, June 16, 2014

Periodicity and quasiperiodicity

Periodicity: occurrences are aligned

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Covers and seeds

Definition (Apostolico, Farach, Iliopoulos; 1991)

A factor u is a *cover* of w if each position (letter) in w lies within an occurrence of u in w.

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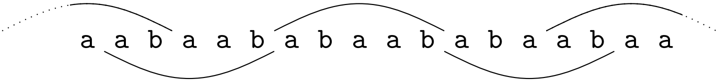
A factor u is a *seed* of w if u is a cover of a superstring of w.



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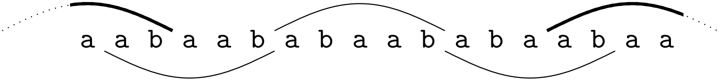

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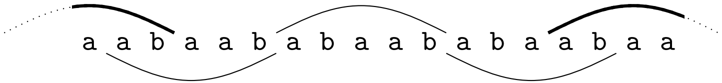

Observation

A factor u is a seed of w iff each position (letter) in w lies within a possibly overhanging occurrence of u in w.

Partial covers and partial seeds

Definition (KPRRW; CPM'13)

The *cover index* $\mathcal{C}(u)$ of u in w is the number of positions of w lying within an occurrence of u in w.

$$\texttt{a b a a b a a a b a a a a b a a a b a b a a a b}$$

$$\mathcal{C}(\texttt{a}) = 17$$

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$$\mathcal{C}(\mathtt{a}) = 17$$
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Definition

For a positive integer α an α-partial cover of w is a factor of w with cover index at least α.

Partial covers and partial seeds

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The *seed index* $\mathcal{S}(u)$ of u in w is the number of positions of w lying within a possibly overhanging occurrence of u in w.

a b a a b a a a b a a a a b a a a b a b a a a b

$$\mathcal{C}(\mathtt{a}) = 17 \qquad \mathcal{S}(\mathtt{a}) = 17$$

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Definition

For a positive integer α an α-partial seed of w is a factor of w with seed index at least α.

Other variants of covers and seeds

`b a b a a a b a b a b a a a a b`

- *k-covers* and *k-seeds* (Iliopoulos, Smyth; 1998) – each position lies within a (possibly overhanging) occurrence of at least one of the few factors of length k, together forming a k-cover (k-seed).

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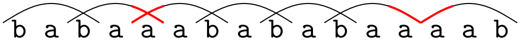

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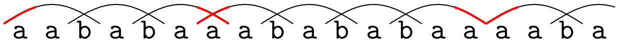

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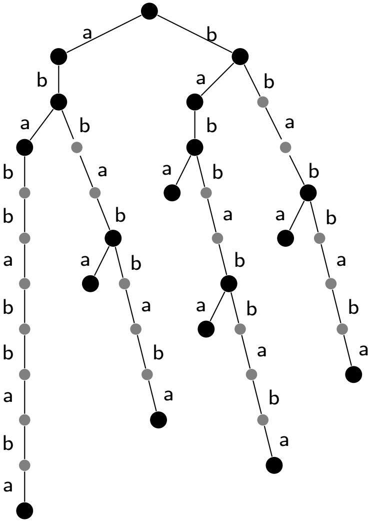
Main drawback: $\Omega(n^2)$ algorithms.

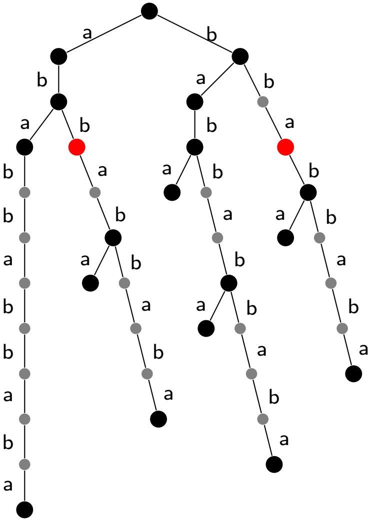
Cover Suffix Tree



CST(ababbabbaba)

The cover suffix tree of w (denoted $CST(w)$) is a suffix tree

Cover Suffix Tree



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The cover suffix tree of w (denoted $CST(w)$) is a suffix tree

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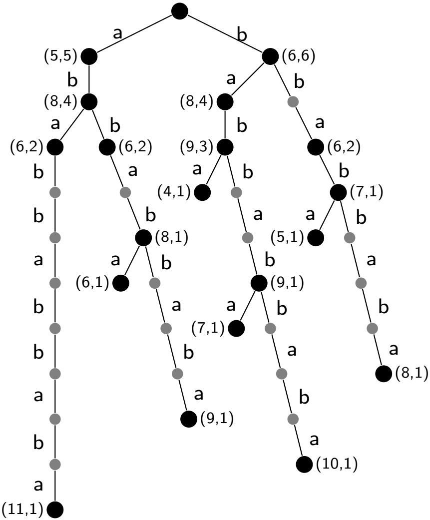

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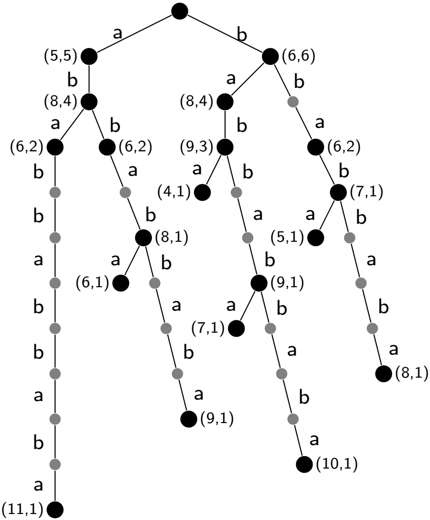

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Theorem (KPRRW; CPM'13)

The tree $CST(w)$ can be built in $\mathcal{O}(n \log n)$ time for any word w of length n.

Our results

Problem (PARTIAL SEEDS)

Given a word w of length n and a positive integer $\alpha \leq n$ find all shortest factors u of w such that $\mathcal{S}(u) \geq \alpha$.

Problem (LIMITED LENGTH PARTIAL SEEDS)

Given a word w of length n and an interval $[\ell, r]$ find a factor u of w maximizing $\mathcal{S}(u)$ among factors for which $|u| \in [\ell, r]$.

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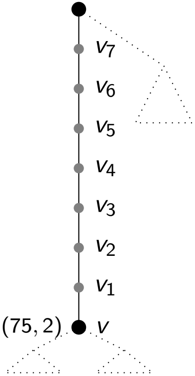
Theorem

Given $CST(w)$ both Partial Seeds *and* Limited Length Partial Seeds *can be solved in linear time.*

Determining the cover index



Lemma (CPM'13)

Let $v_0, v_1, \ldots, v_k$ be the nodes of an edge of $CST(w)$ with $v = v_0$ being the lowest (explicit) node. Then

$$\mathcal{C}(v_j) = \mathcal{C}(v) - j\Delta(v).$$

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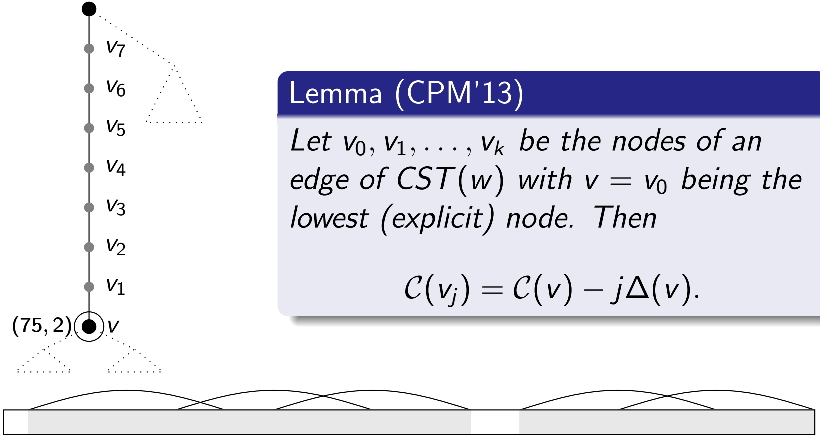

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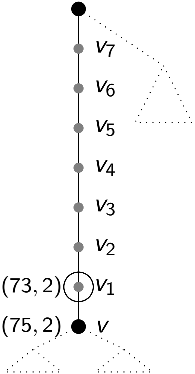

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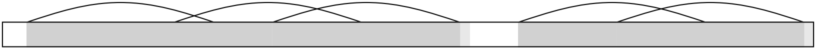

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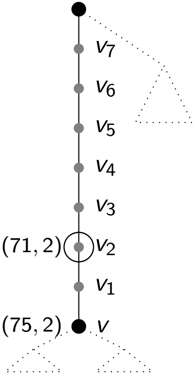

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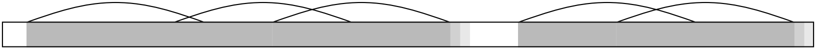

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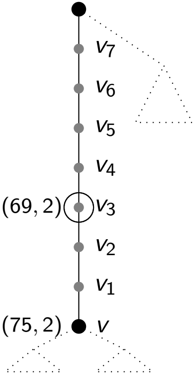

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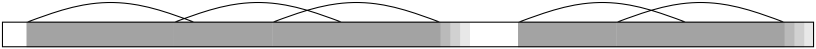

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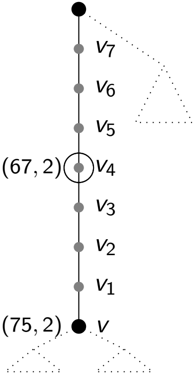

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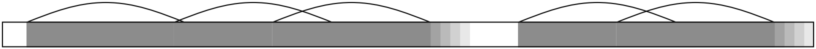

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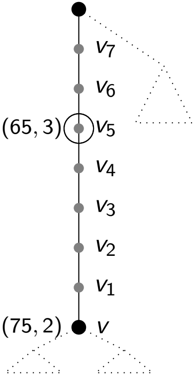

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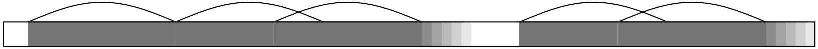

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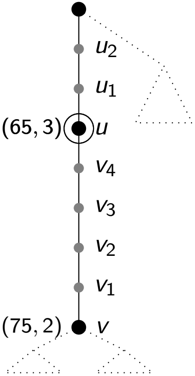

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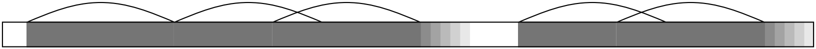

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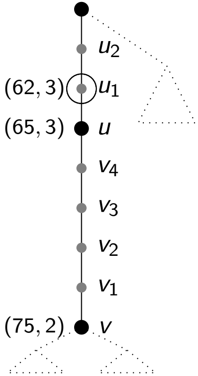

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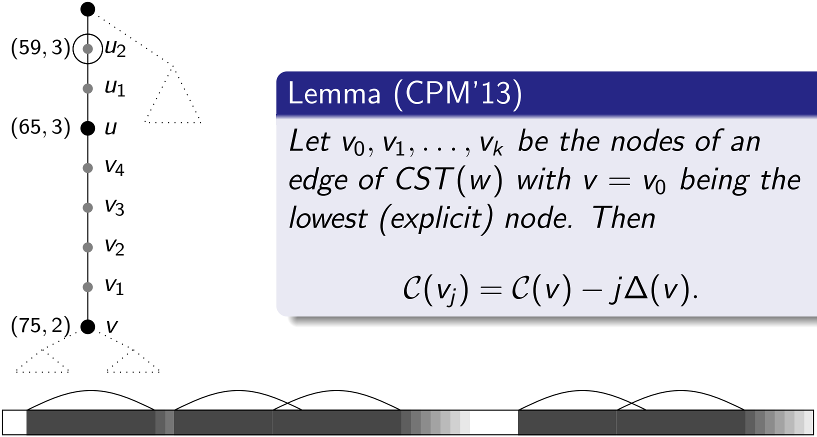

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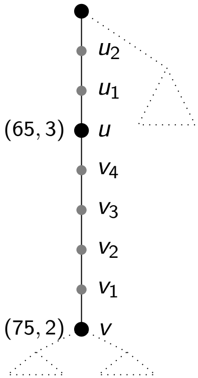
Determining the cover index



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Corollary

Given a locus of v in $CST(w)$, the cover index $\mathcal{C}(v)$ can be computed in $\mathcal{O}(1)$ time.

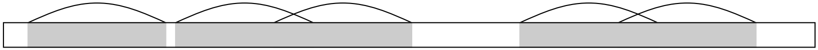
Seed index

$$\mathcal{S}(v) \quad = \qquad\qquad\qquad + \qquad\qquad + $$

Seed index

$$\mathcal{S}(v) \quad = \qquad\qquad\qquad + \quad \mathcal{C}(v) \quad +$$

Full occs

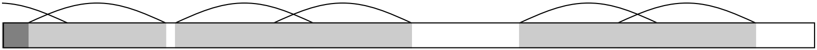


Seed index

$$\mathcal{S}(v) \quad = \quad \mathit{Left}\mathcal{S}(v) \quad + \quad \mathcal{C}(v) \quad +$$

Left-overhanging occs only

Full occs



Seed index

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Left-overhanging occs only

Full occs

Right-overhanging occs only

Seed index

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Left-overhanging occs only

Full occs

Right-overhanging occs only

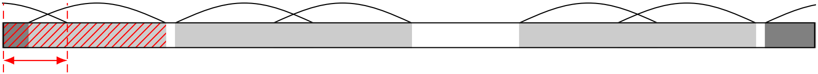
$$\mathit{Left}\mathcal{S}(v) = \min(B[\mathit{first}(v) + |v| - 1], \mathit{first}(v) - 1)$$

$\mathit{first}(v)$ start position of the first occurrence of v,

$B[i]$ largest border of $w[1..i]$.

Seed index

$$\mathcal{S}(v) \quad = \quad \underbrace{Left\mathcal{S}(v)}_{\text{Left-overhanging occs only}} \quad + \quad \underbrace{\mathcal{C}(v)}_{\text{Full occs}} \quad + \quad \underbrace{Right\mathcal{S}(v)}_{\text{Right-overhanging occs only}}$$



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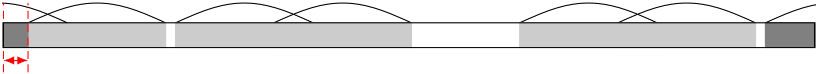
Left-overhanging occs only

Full occs

Right-overhanging occs only



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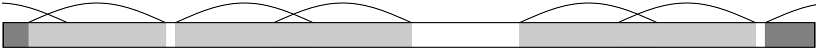


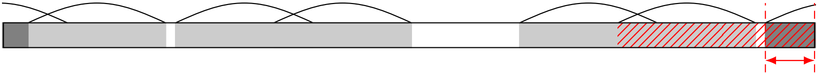
$$\mathit{Left}\mathcal{S}(v) = \min(B[\mathit{first}(v) + |v| - 1], \mathit{first}(v) - 1)$$
$$\mathit{Right}\mathcal{S}(v) = \min(B^R[\mathit{last}(v)], n - |v| + 1 - \mathit{last}(v))$$

$\mathit{last}(v)$ start position of the last occurrence of v,

$B^R[i]$ largest border of $w[i..n]$.

Seed index

$$\mathcal{S}(v) = \underbrace{\mathit{Left}\mathcal{S}(v)}_{\text{Left-overhanging occs only}} + \underbrace{\mathcal{C}(v)}_{\text{Full occs}} + \underbrace{\mathit{Right}\mathcal{S}(v)}_{\text{Right-overhanging occs only}}$$



$$\mathit{Left}\mathcal{S}(v) = \min(B[\mathit{first}(v) + |v| - 1], \mathit{first}(v) - 1)$$

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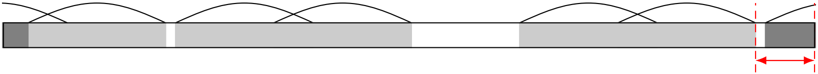

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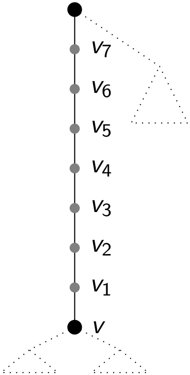

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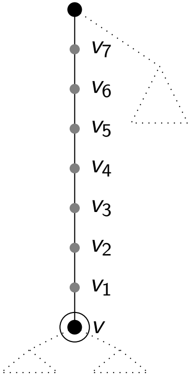

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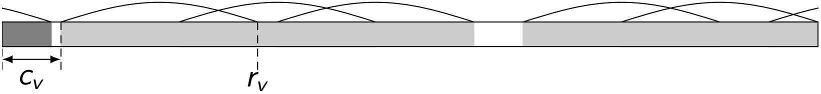

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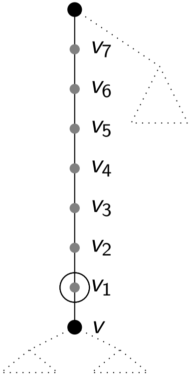

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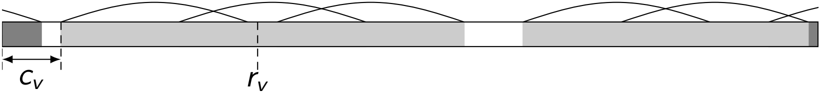

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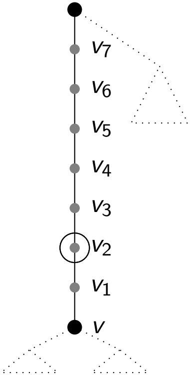

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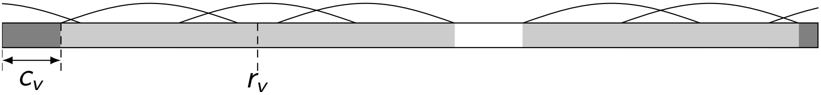

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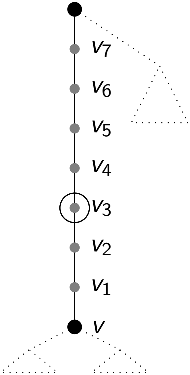

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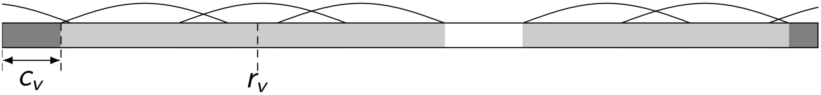

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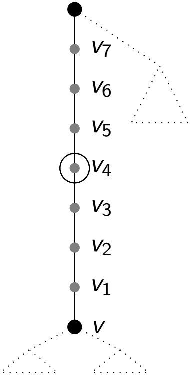

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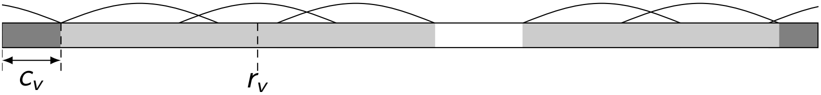

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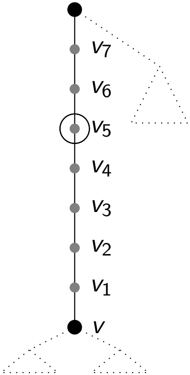

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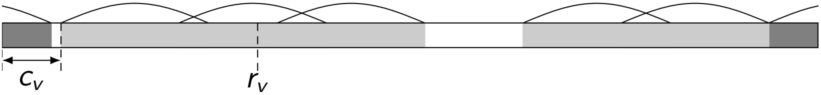

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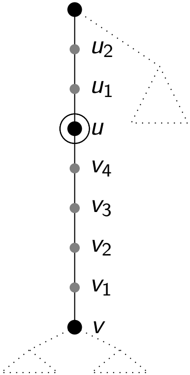

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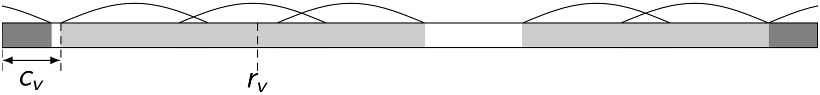

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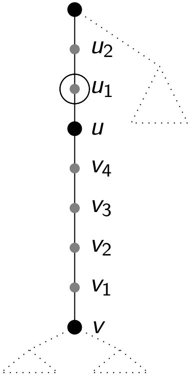

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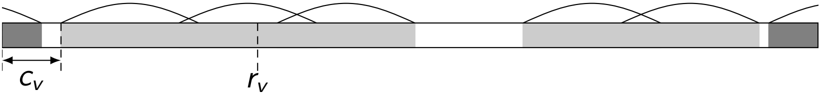

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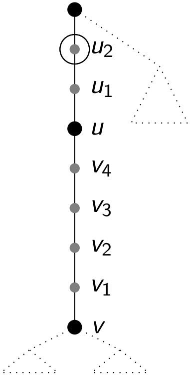

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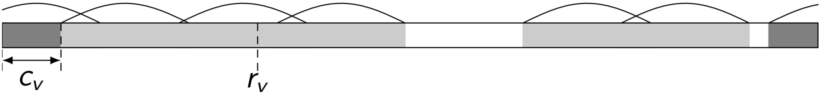

Seed Suffix Tree

$CST(w)$ can be further augmented in $\mathcal{O}(n)$ time to $SST(w)$ (*Seed Suffix Tree*) such that

- for each node v there exists a function

$$\phi_v(x) = a_v x + b_v + \min(c_v, B[x])$$

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Observation

Given a locus of v in $SST(w)$ and the border table B, the seed index $\mathcal{S}(v)$ can be computed in $\mathcal{O}(1)$ time.

Abstract problems

Problem

Input: *pairs* (ϕ_i, R_i), *where* $\phi_i(x) = a_i x + b_i + \min(c_i, B[x])$ *is a function and* $R_i = (\ell_i, r_i] \subseteq [1, n]$ *is a non-empty range*
Output:

(a) $\operatorname{argmax}\{\phi_i(x) : x \in R_i\}$ *for each pair,*

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Workaround for $\sum a_i = \mathcal{O}(n)$:

- use (a) queries to restrict the set of edges queried for (b).

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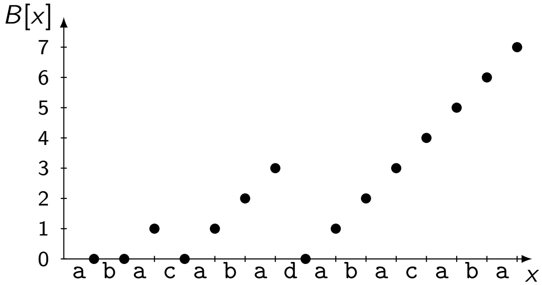
For the border array B to answer (off-line) the following queries: given a non-negative coefficient a_i and a range $R_i = (\ell_i, r_i]$ compute $x_i = \text{argmax}\{a_i x + B[x] : x \in R_i\}$.



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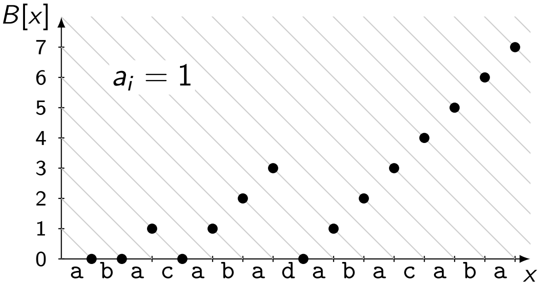

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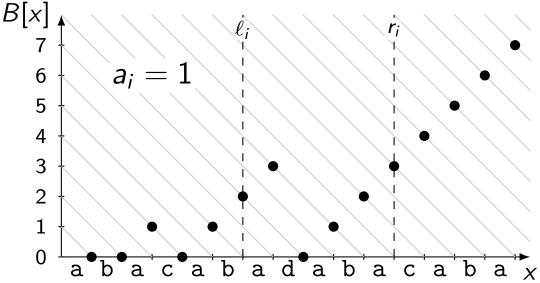

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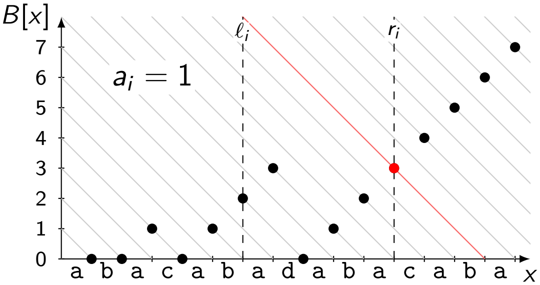

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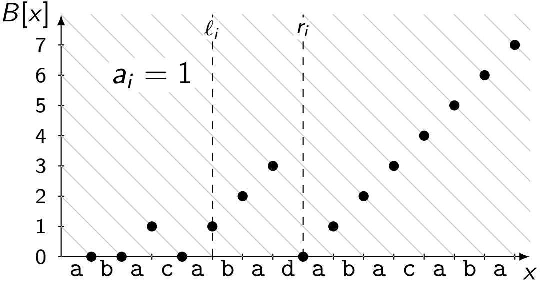

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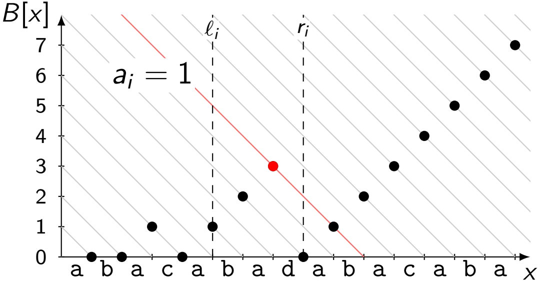

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Thank you for your attention!