FFT FAST FOURIER TRANSFORM - SZYBKIE PRZEKSZTAŁCENIE FOURIERA

$$c_q = \frac{1}{N} \sum_{j=0}^{N-1} f_j e^{-i\frac{2\pi qj}{N}}$$

$$N = 2^r$$

$$q = \sum_{k=1}^{r} q_k 2^{k-1} =$$
$$= q_1 2^0 + q_2 2^1 + \cdots + q_r 2^{r-1},$$

$$j = \sum_{m=1}^{r} j_{r-m+1} 2^{m-1} =$$
$$= j_r 2^0 + j_{r-1} 2^1 + \cdots + j_1 2^{r-1}.$$

.

$$\frac{qj}{N} = \frac{qj}{2^r} =$$

$$= \sum_{m=1}^{r} \sum_{k=1}^{r} q_k j_{r-m+1} 2^{m+k-r-2} =$$

$$= s + \sum_{m=1}^{r} j_{r-m+1} \sum_{k=1}^{r-m+1} 2^{m+k-r-2} q_k,$$

s- część całkowita.

Ponieważ $N = 2^r$ i $e^{-i2\pi s} = 1$

$$c_q = \frac{1}{N} \sum_{j=0}^{N-1} f_j.$$

$$.e^{-i2\pi \sum_{m=1}^{r} j_{r-m+1} \sum_{k=1}^{r-m+1} 2^{m+k-r-2} q_k} =$$

$$= \frac{1}{2}\frac{1}{2} \cdots \frac{1}{2} \sum_{j_r=0}^{1} \sum_{j_{r-1}=0}^{1} \cdots \sum_{j_1=0}^{1} f_{j_1 j_2 j_3 \cdots j_r}.$$

$$.e^{-2\pi i [j_r 2^{-r} \sum_{k=1}^{r} 2^{k-1} q_k]}.$$

$$.e^{-2\pi i [j_{r-1} 2^{1-r} \sum_{k=1}^{r-1} 2^{k-1} q_k]}.$$

$$\cdots\cdots$$

$$.e^{-2\pi i [j_1 2^{-1} q_1]}.$$

Po uporządkowaniu

$$c_q =$$

$$= \frac{1}{2} \sum_{j_r=0}^{1} e^{-2\pi i [j_r 2^{-r} \Sigma_{k=1}^{r} 2^{k-1} q_k]}.$$

$$\cdot \frac{1}{2} \sum_{j_{r-1}=0}^{1} e^{-2\pi i [j_{r-1} 2^{1-r} \Sigma_{k=1}^{r-1} 2^{k-1} q_k]}.$$

$$\cdots\cdots$$

$$\cdot \frac{1}{2} \sum_{j_2=0}^{1} e^{-2\pi i [j_2 2^{-2} \Sigma_{k=1}^{2} 2^{k-1} q_k]}.$$

$$\cdot \frac{1}{2} \sum_{j_1=0}^{1} e^{-2\pi i [j_1 2^{-1} q_1]} f_{j_1 j_2 \cdots j_r}$$

Określimy rekurencyjnie, $l = 0, 1, \cdots, q$

$$c^0(j_1 j_2 \cdots j_r) = f_{j_1 j_2 \cdots j_r},$$

$$c^1(q_1 j_2 \cdots j_r) =$$
$$= \frac{1}{2} \sum_{j_1=0}^{1} e^{-2\pi i [j_1 2^{-1} q_1]} c^0(j_1 j_2 \cdots j_r),$$

$$c^2(q_1 q_2 j_3 \cdots j_r) =$$
$$= \frac{1}{2} \sum_{j_2=0}^{1} e^{-2\pi i [j_2 2^{-2}(q_1 2^0 + q_2 2^1)]} c^1(q_1 j_2 \cdots j_r),$$
$$\cdots$$

$$c^l(q_1 q_2 \cdots q_l j_{l+1} \cdots j_r) =$$
$$= \frac{1}{2} \sum_{j_r=0}^{1} e^{-2\pi i [j_l 2^{-l}(q_1 2^0 + q_1 2^1 + \cdots q_l 2^{l-1})]}.$$
$$\cdot c^{l-1}(q_1 q_2 \cdots q_{l-1} j_l \cdots j_r).$$
$$\cdots$$

Zatem

$$c_q = c^r(q_1 q_2 \cdots q_r) =$$

$$= \frac{1}{2} \sum_{j_r=0}^{1} e^{-2\pi i[j_r 2^{-r}(q_1 2^0 + q_2 2^1 + \cdots q_r 2^{r-1})]}.$$

$$\cdot c^{r-1}(q_1 q_2 \cdots q_r {}_1 j_r).$$

Przykład
Niech $N = 8 = 2^3$, zatem $r = 3$. W tym przypadku algorytm **FFT** wykonuje $r = 3$ kroki. Oto poszczególne etapy wypisane dla obliczenia współczynnika

$$c_q = c_{q_3 q_2 q_1} :$$

$$\begin{array}{llll}
c^0(000) & & & \\
c^0(001) & & & \\
c^0(010) & c^1(q_1 00) & & \\
c^0(011) & c^1(q_1 01) & c^2(q_1 q_2 0) & \\
\cdots & \cdots & \cdots & c^3(q_1 q_2 q_3) = c_q \\
c^0(100) & c^1(q_1 10) & c^2(q_1 q_2 1) & \\
c^0(101) & c^1(q_1 11) & & \\
c^0(110) & & & \\
c^0(111) & & &
\end{array}$$