Bisimulation-Invariant Logics: Beyond Finite (and Infinite)

Jędrzej Kołodziejski

24 V 2024
Warszawa

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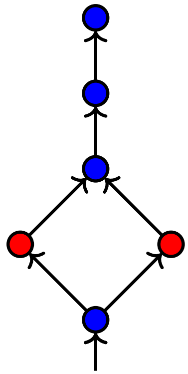

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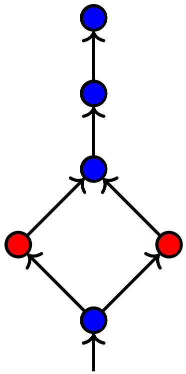

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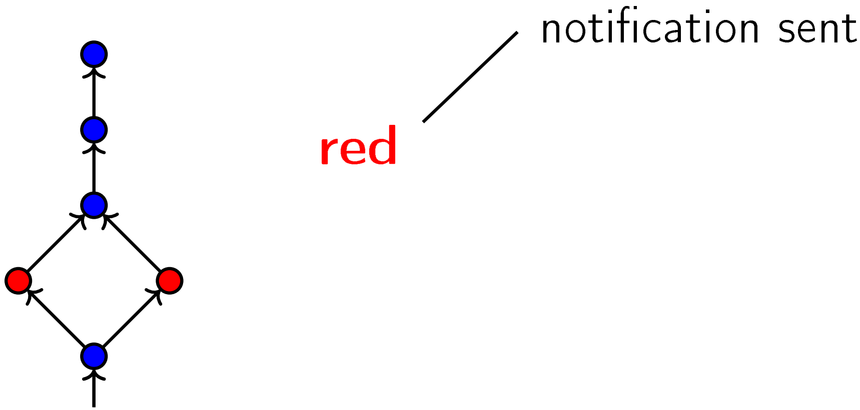

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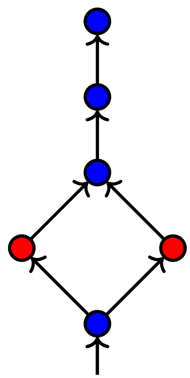

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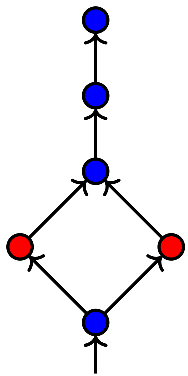

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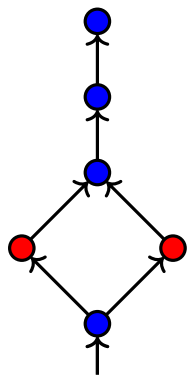

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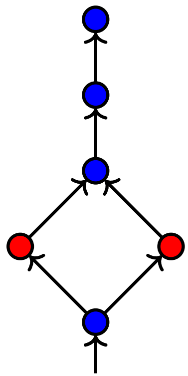

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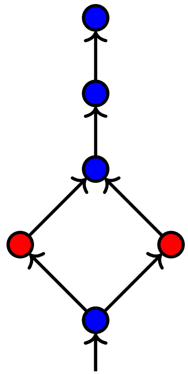

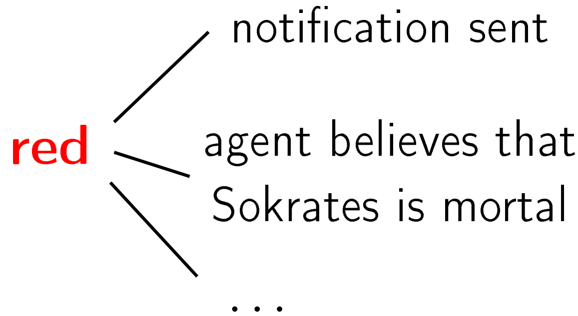

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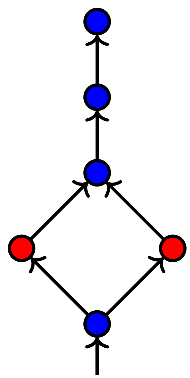

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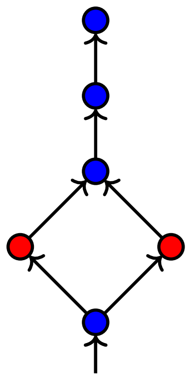

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finer: more information

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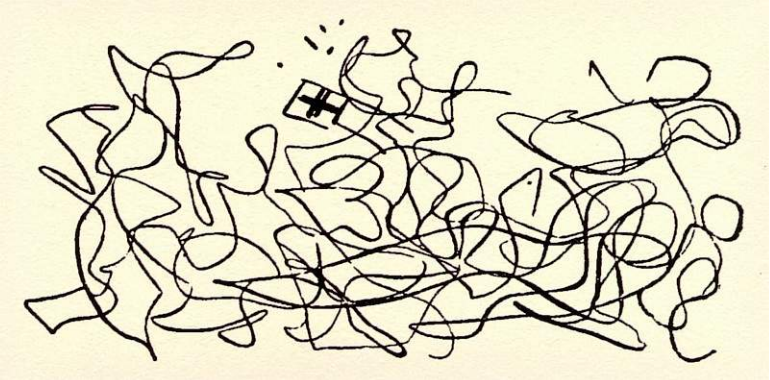
coarser: easier to understand



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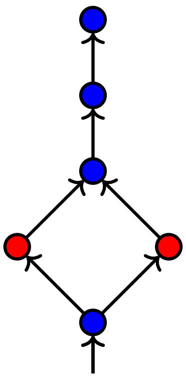

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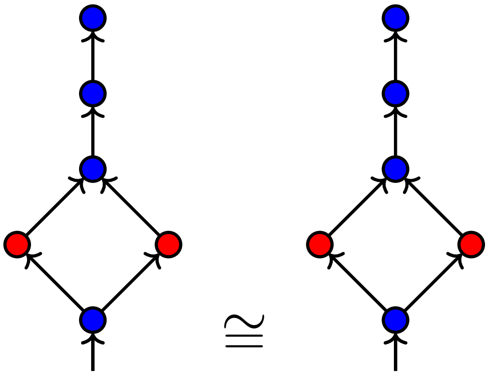

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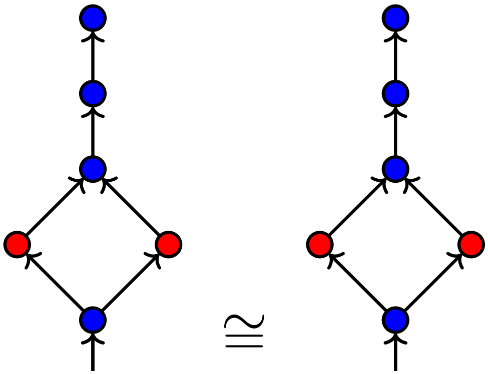

bisimulation: the same behavior

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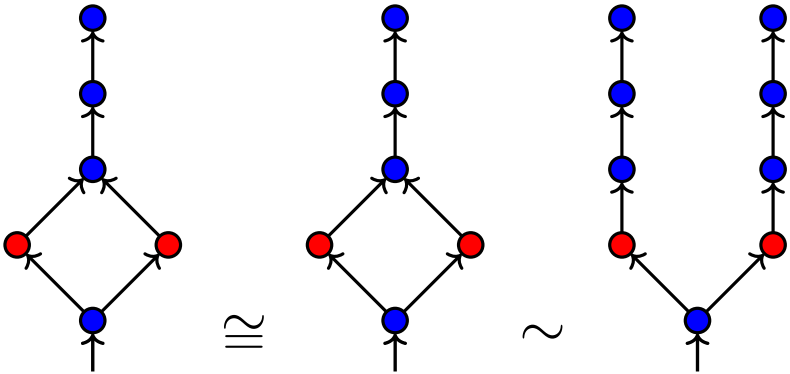

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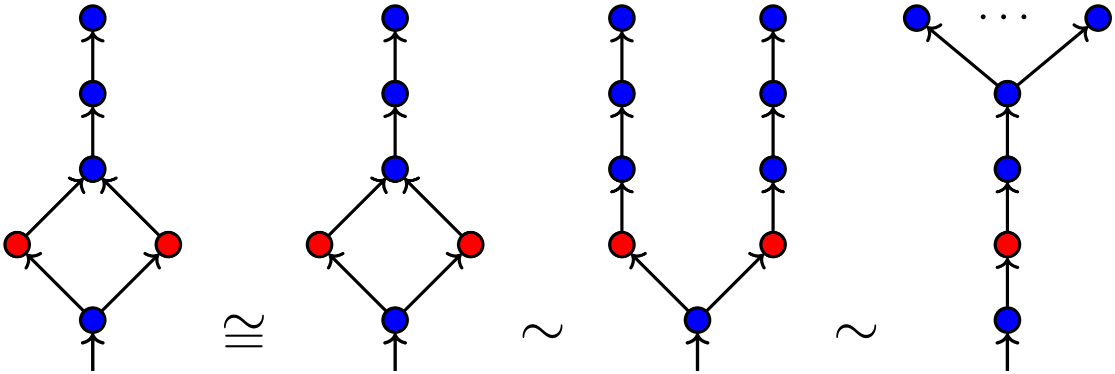

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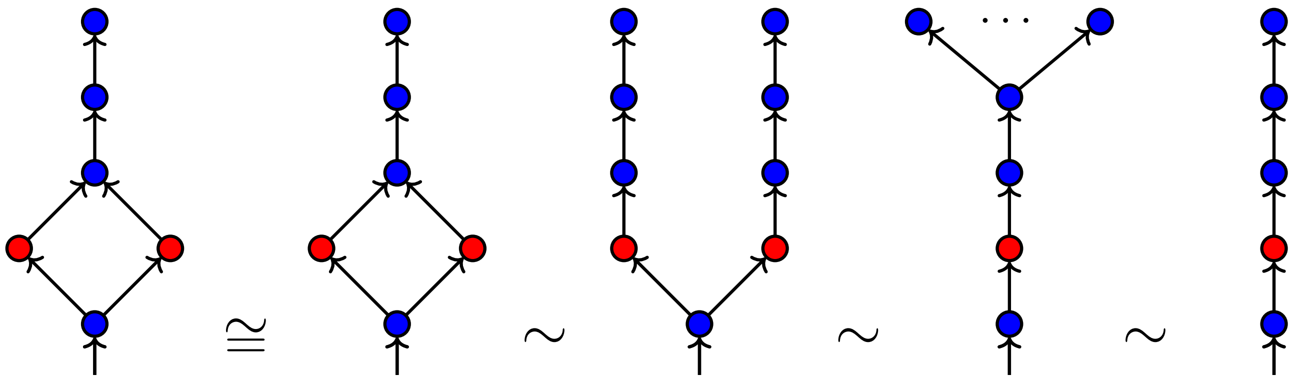

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Model theory for (sets of!) **modal** formulae

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- *and*
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"there is an outgoing infinite path" not local, so not expressible

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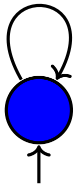

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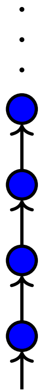

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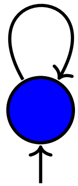

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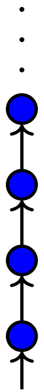

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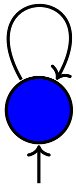

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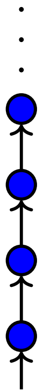
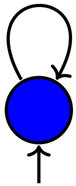
 $\sim$  $\models$ $\left\{ \begin{array}{l} \text{all points are } \textbf{blue}, \\ \text{all points have children} \end{array} \right\}$

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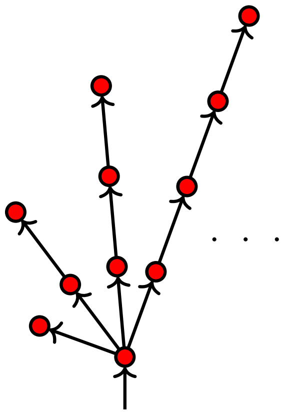

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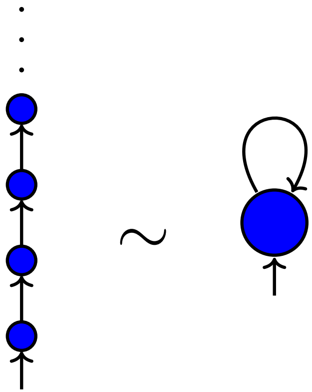

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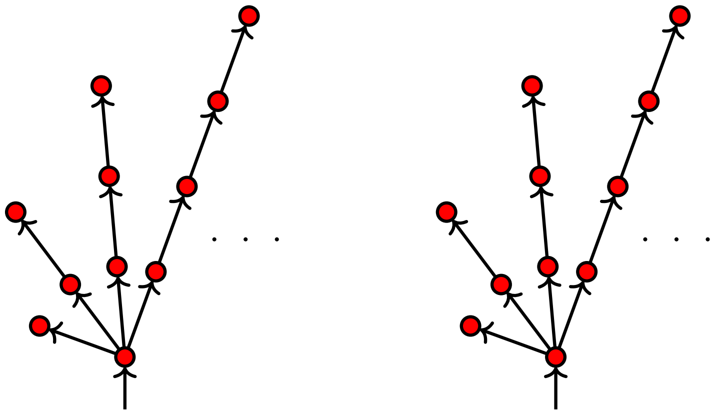

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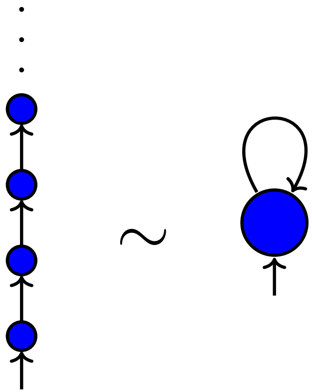

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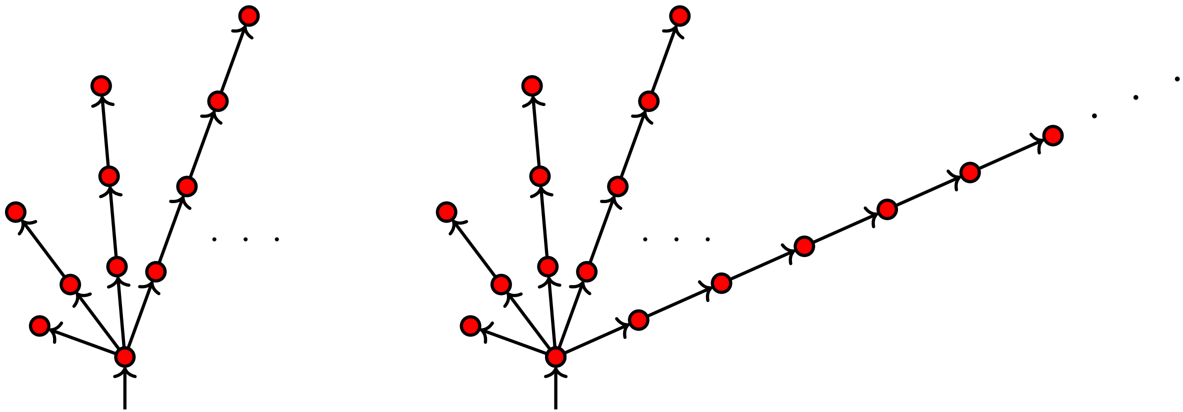

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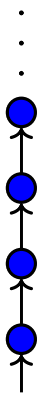

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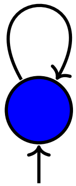

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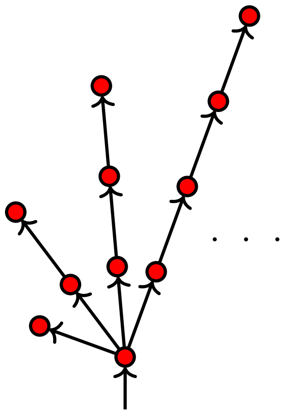

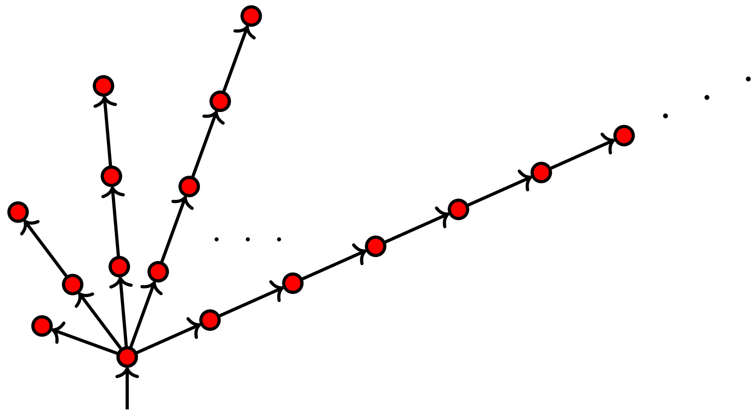

satisfy the same modal formulae, but are not bisimilar!

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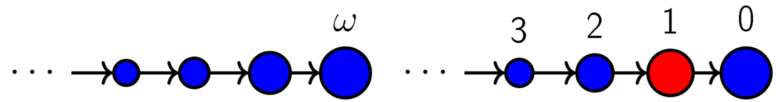

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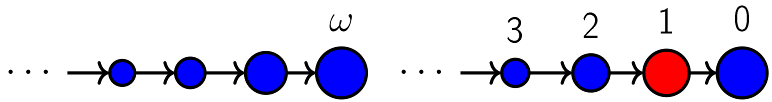

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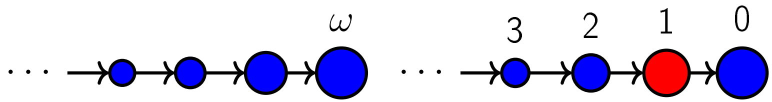

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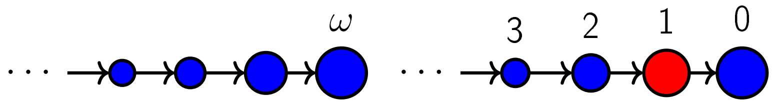

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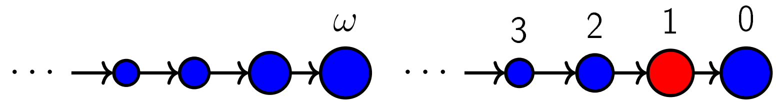

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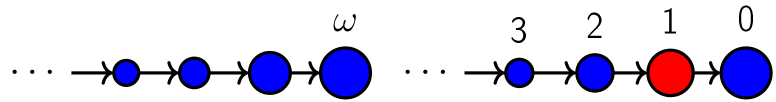

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Short Model Property:

If $t \subseteq \mathrm{ML}$ has an ordinal model then it has one of length $\leq \omega^{|\mathrm{colors}|} + 1$.

A classical approach to infinitary behavior: **fixpoints**

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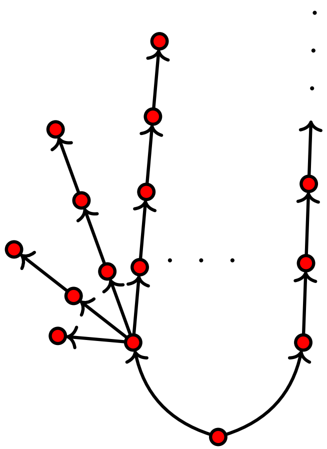

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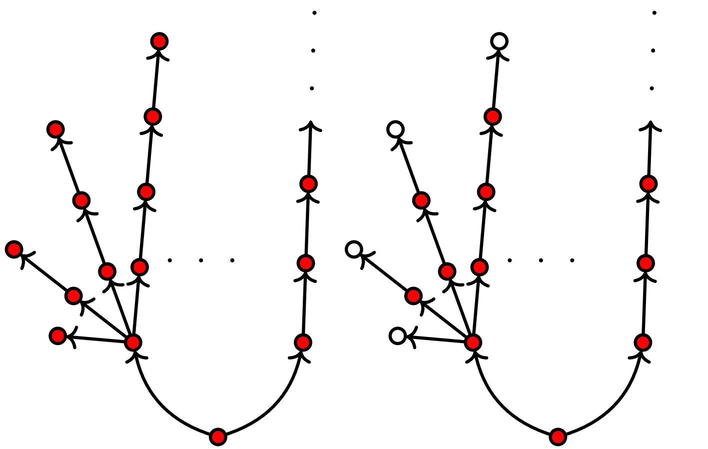

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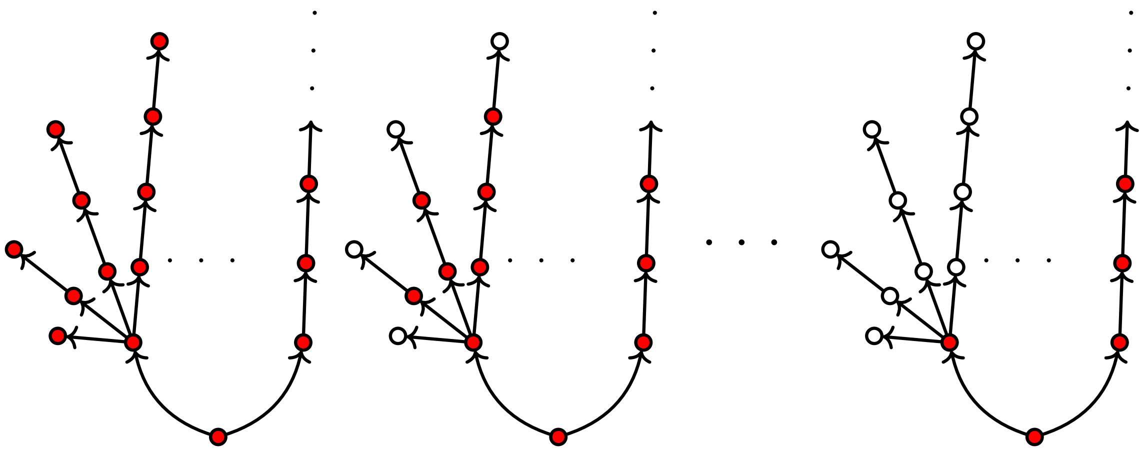

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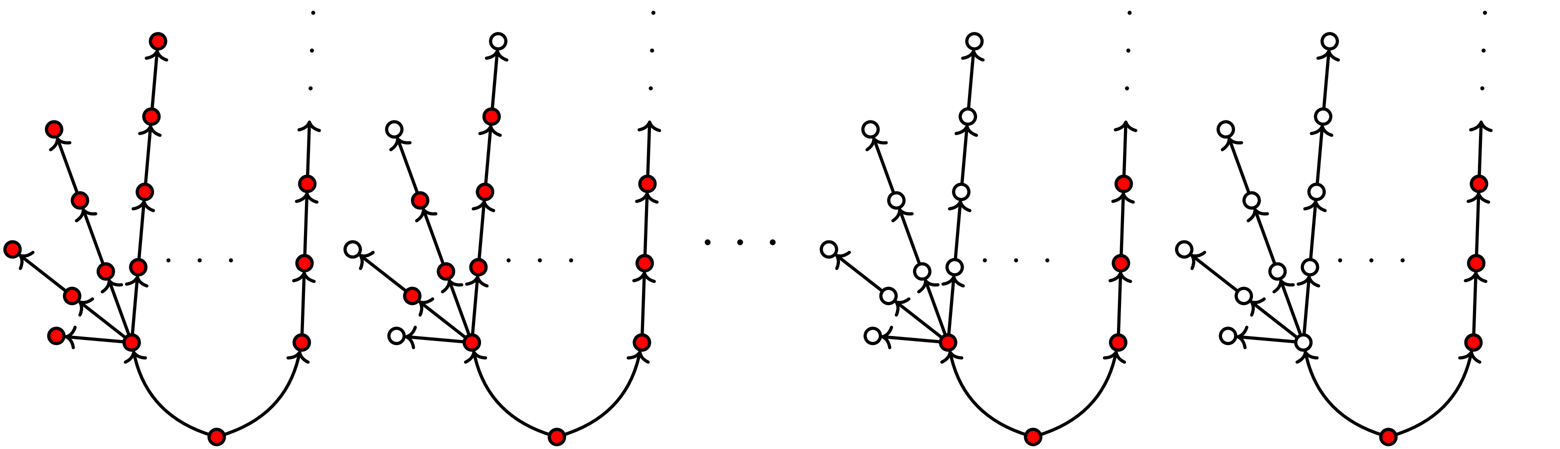

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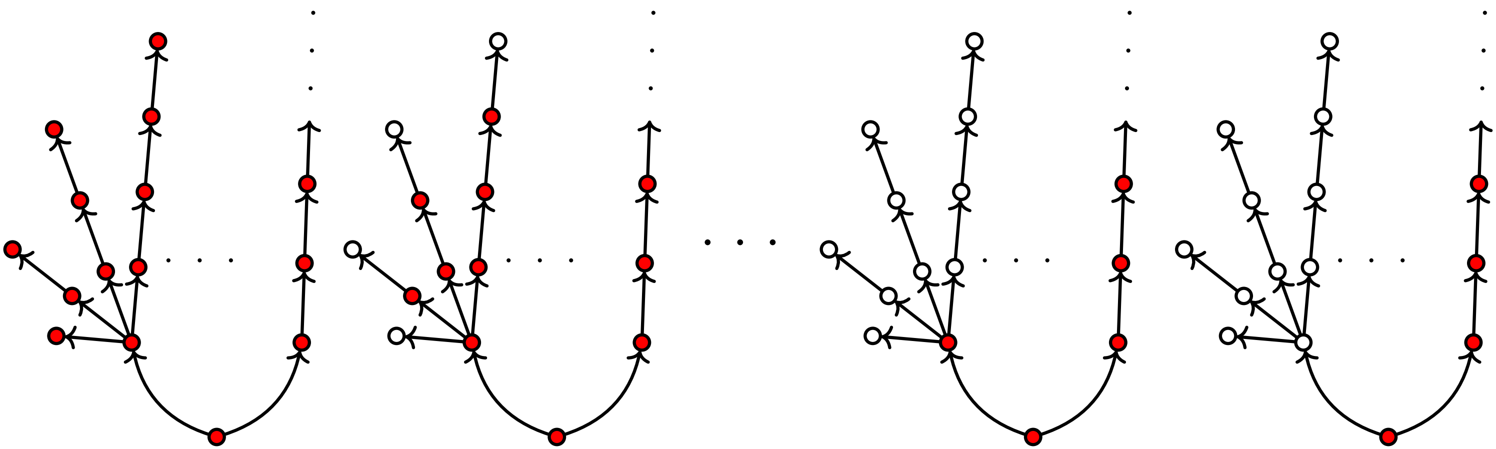

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  - "there are arbitrarily long finite paths" ✗

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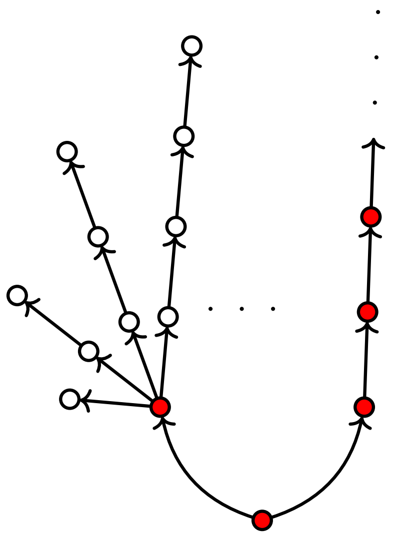

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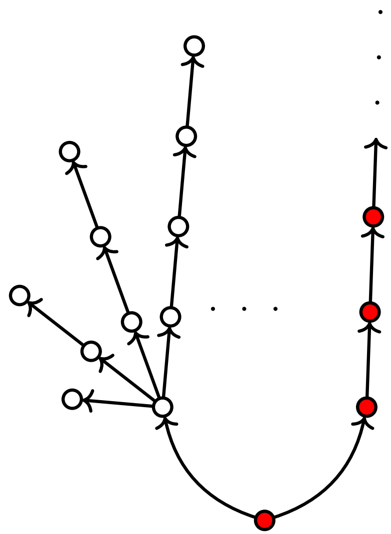

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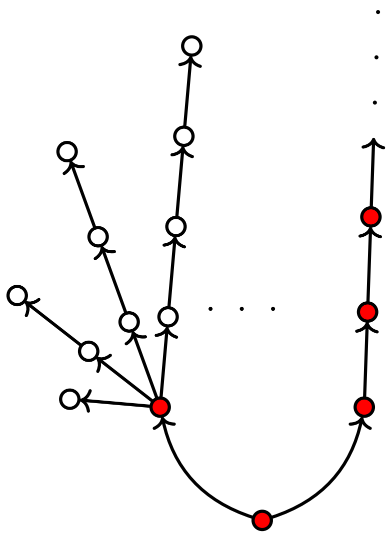
Extend the μ-calculus:

modal logic + **fixpoint approximations**

=

countdown μ-calculus



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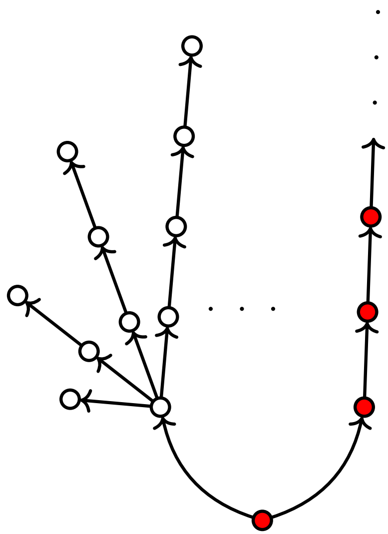

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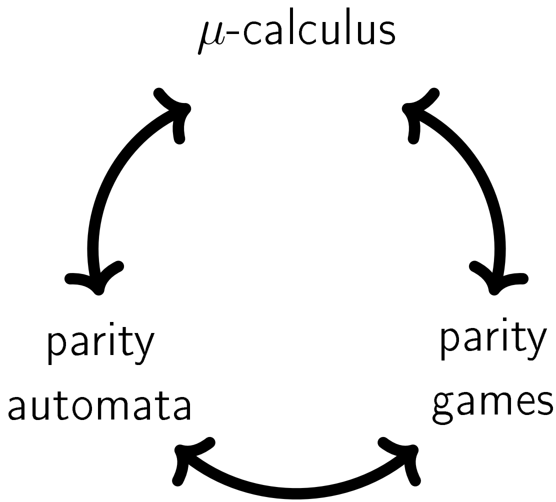

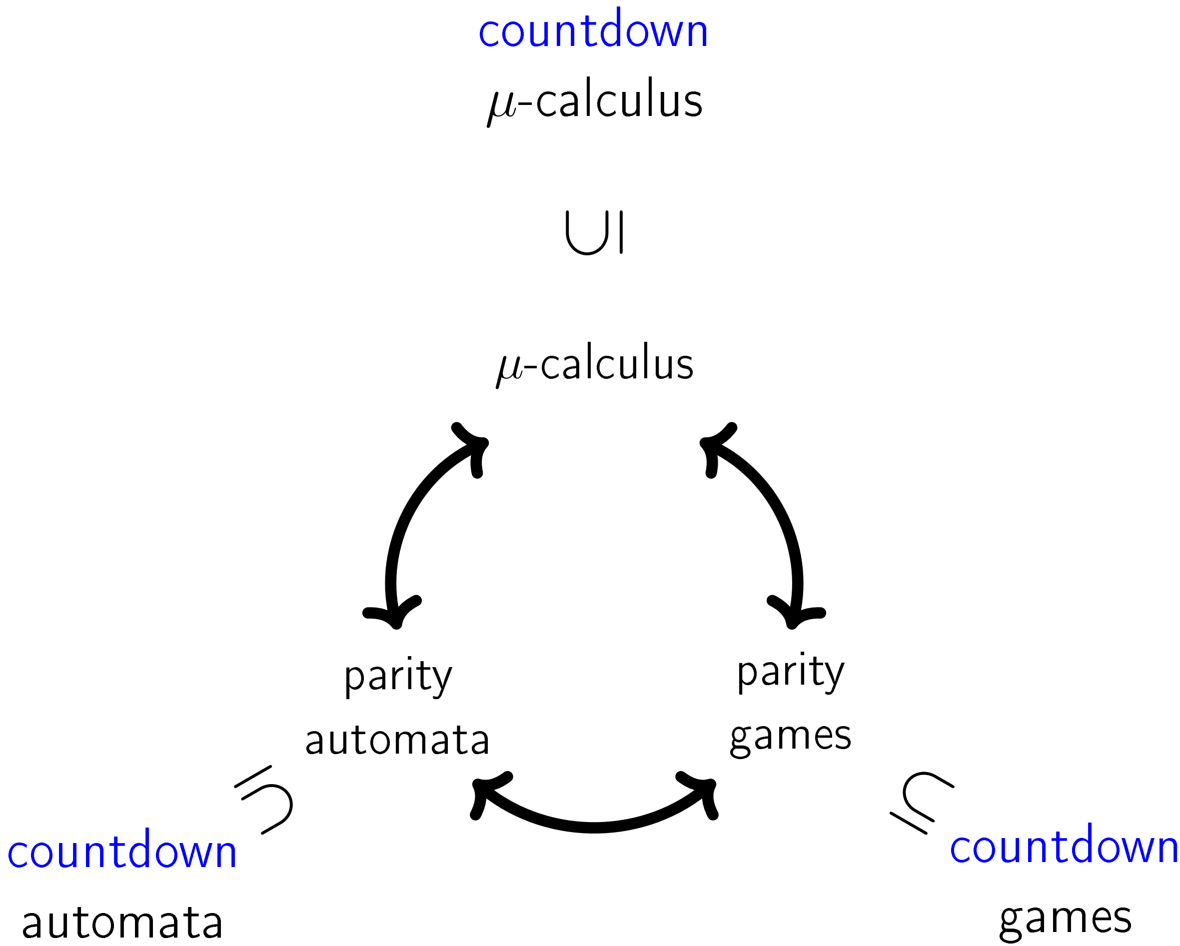
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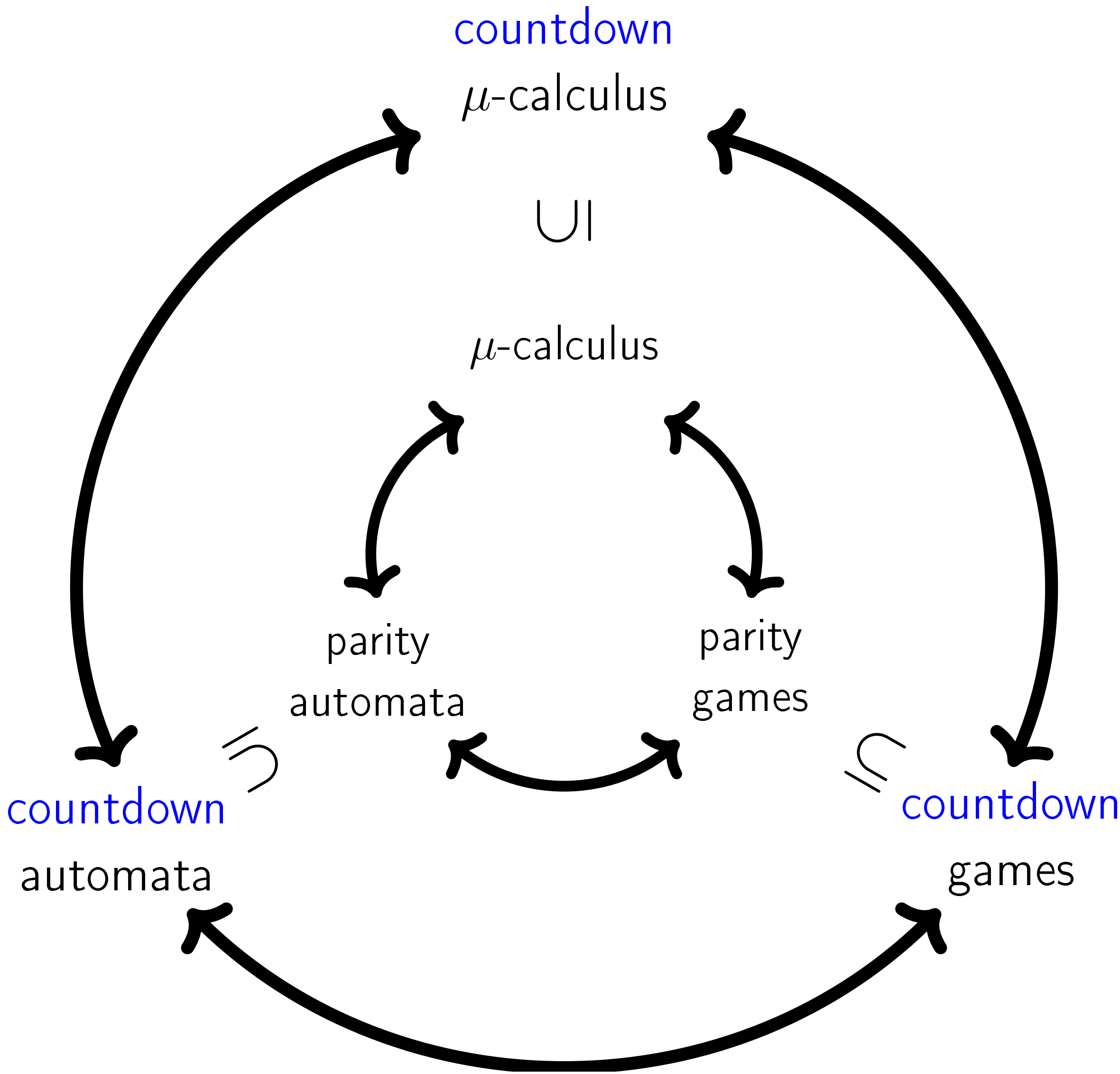
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COMPLICATIONS!!!

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Unlike with parity games:

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  scalar → correspond to a subclass of automata

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- players may need **unbounded memory** to win
- due to this, automata are more complicated:

alternating automata,
no **nondeterministic** model

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- works over models, words, trees, coalgebras...

Conclusion: (terribly) oversimplified slogan

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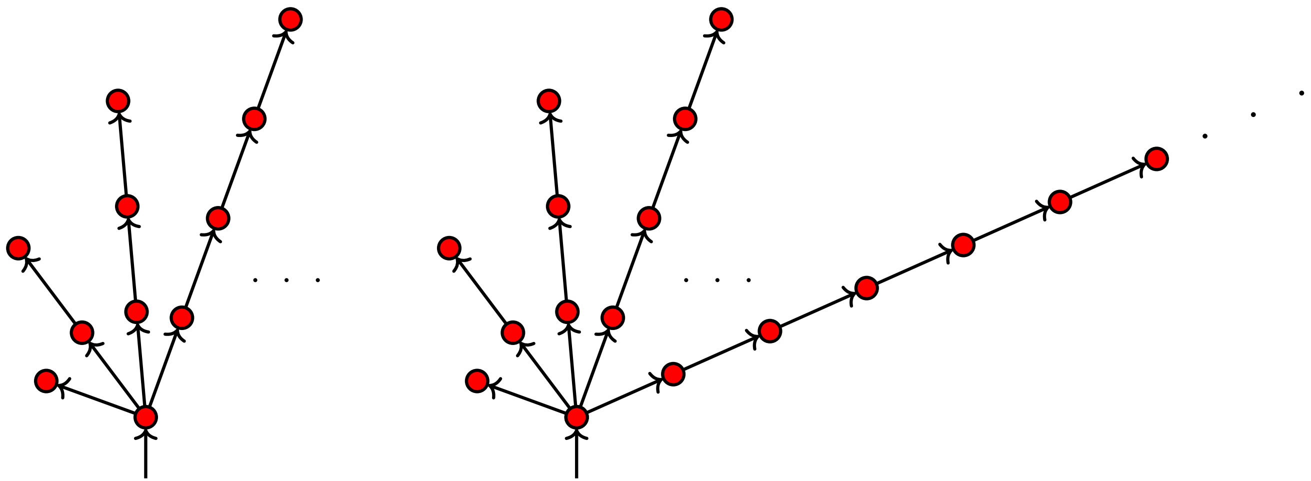

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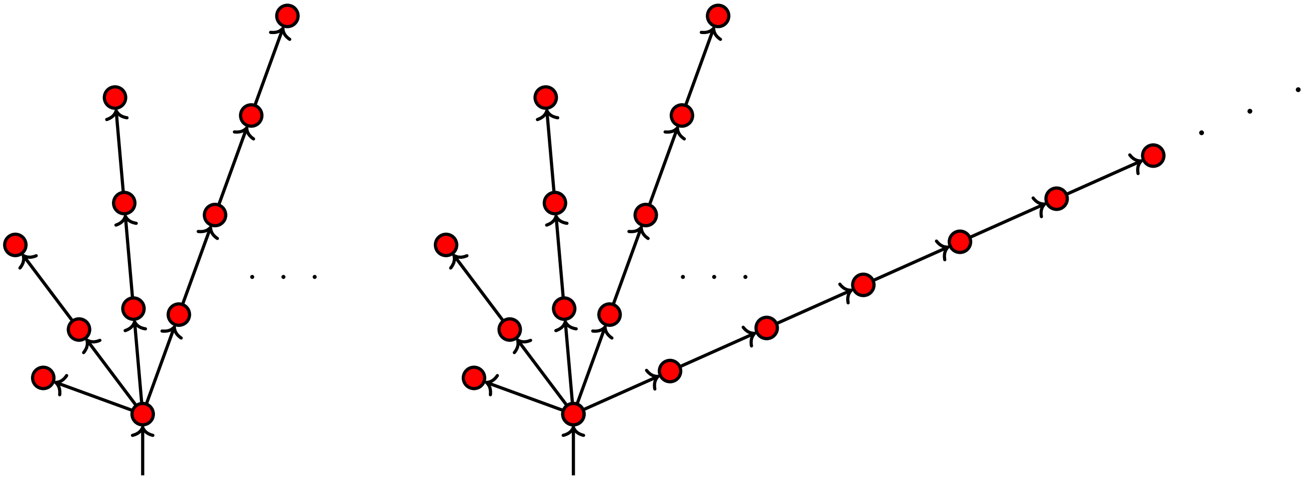

I studied the difference between these two pictures.

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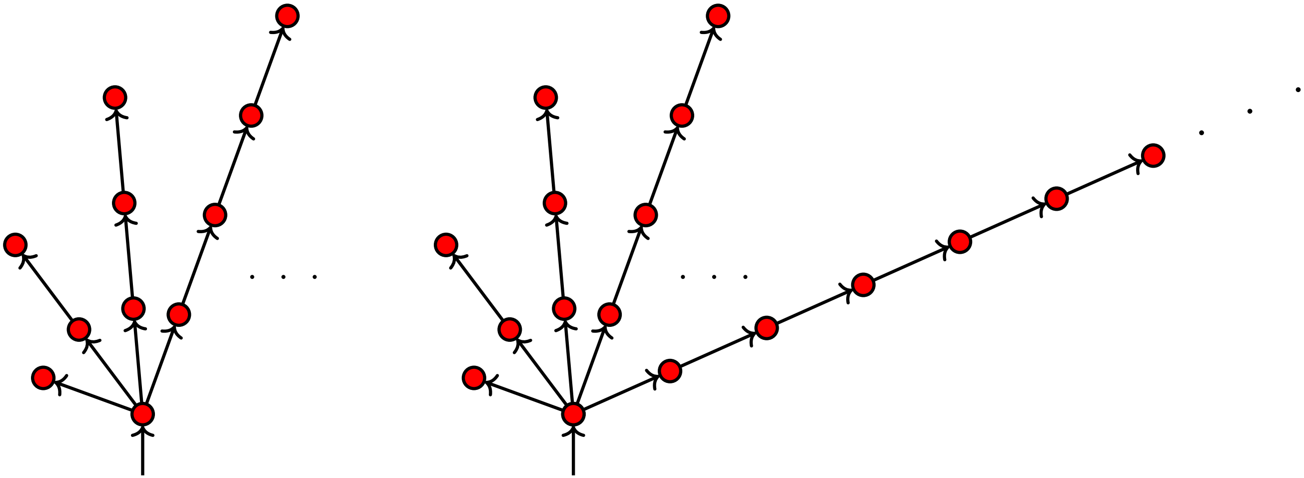

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Thank you!