Bisimulational Categoricity

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atomic propositions

$\exists \to p$ s.t. $p \models \varphi$

semantics

Bisimulational Categoricity

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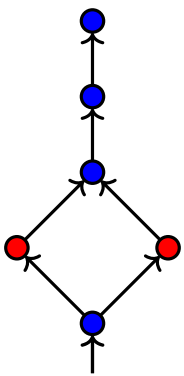

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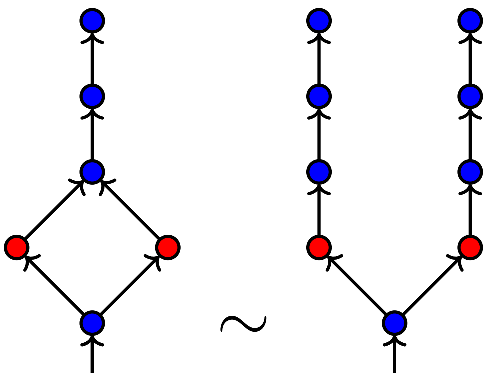

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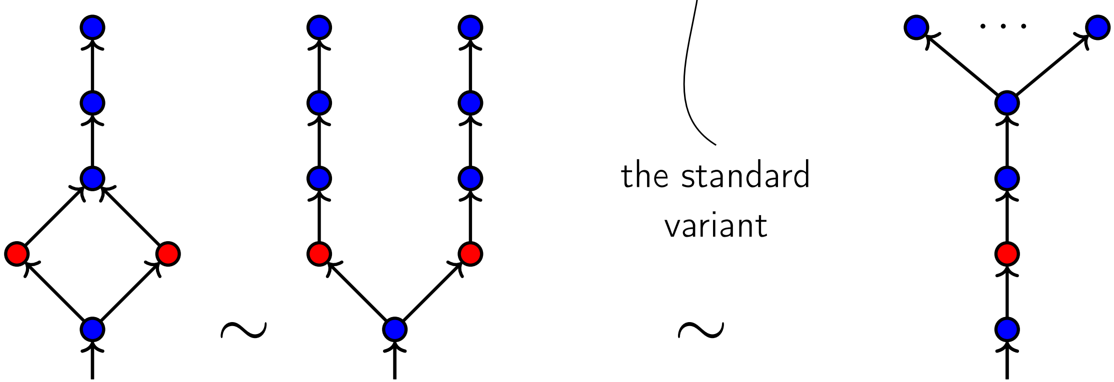

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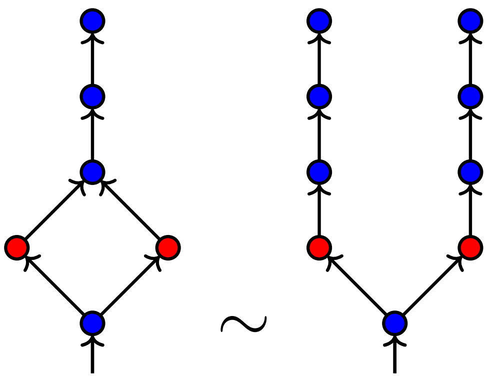

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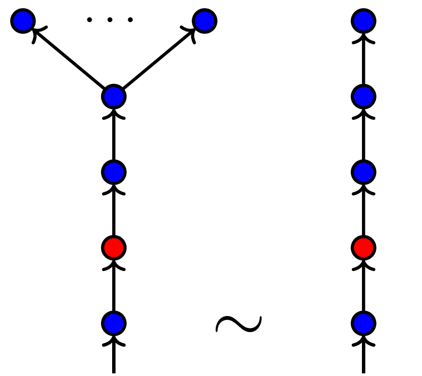

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$$\{\Box^{n}(\text{blue} \wedge \Diamond\top) \mid n \in \{0, 1, \ldots\}\}$$

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syntactic sugar for: $\overbrace{\Box \ldots \Box}^{n \text{ times}}$

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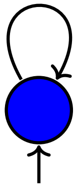

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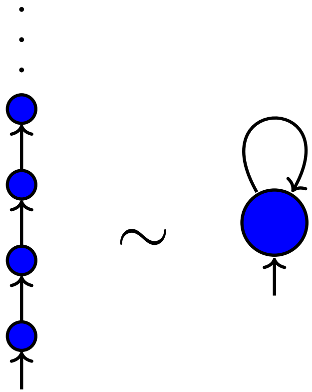

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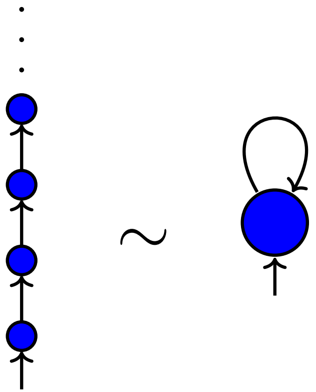

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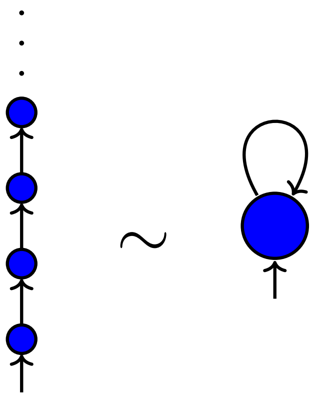

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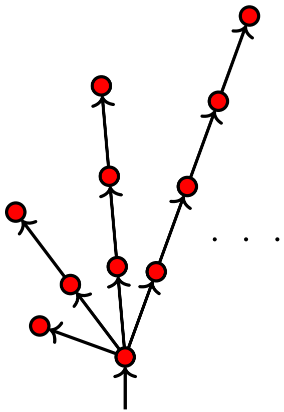

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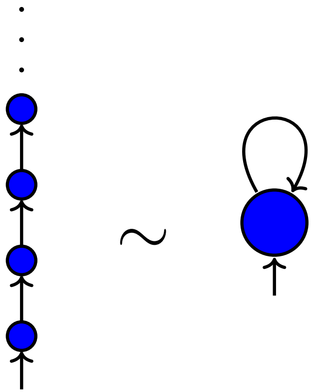

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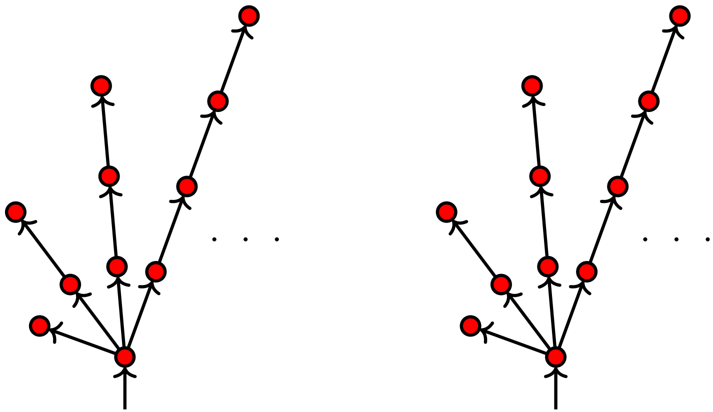

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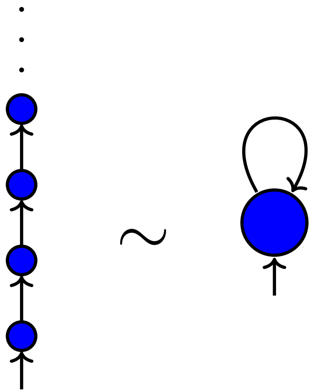

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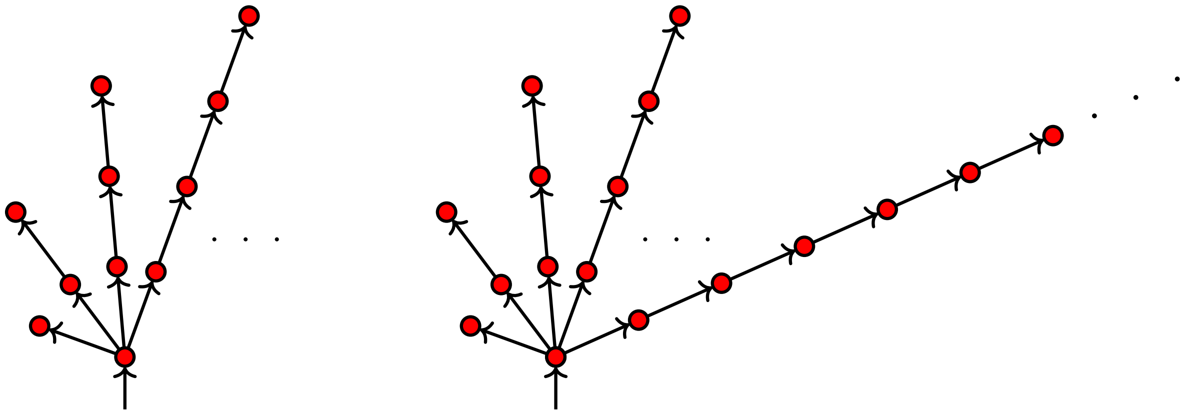

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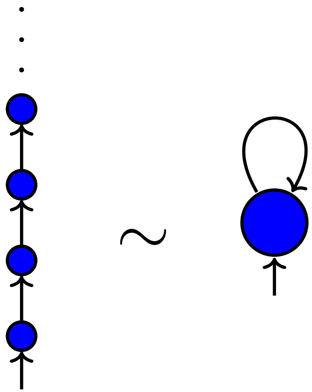

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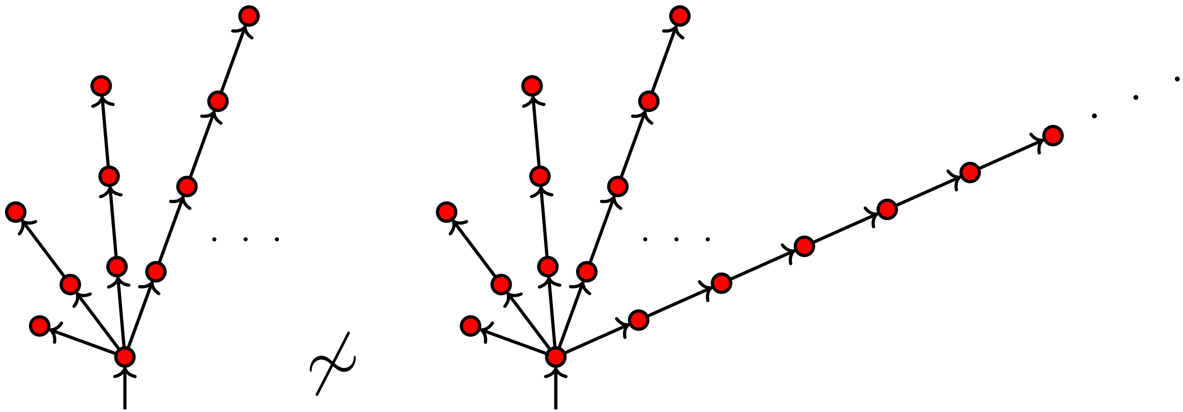

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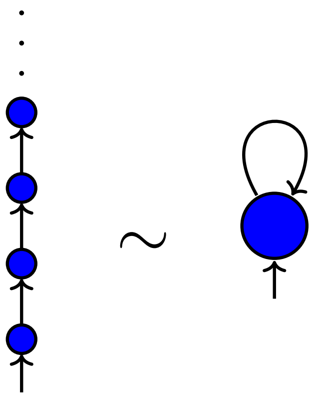

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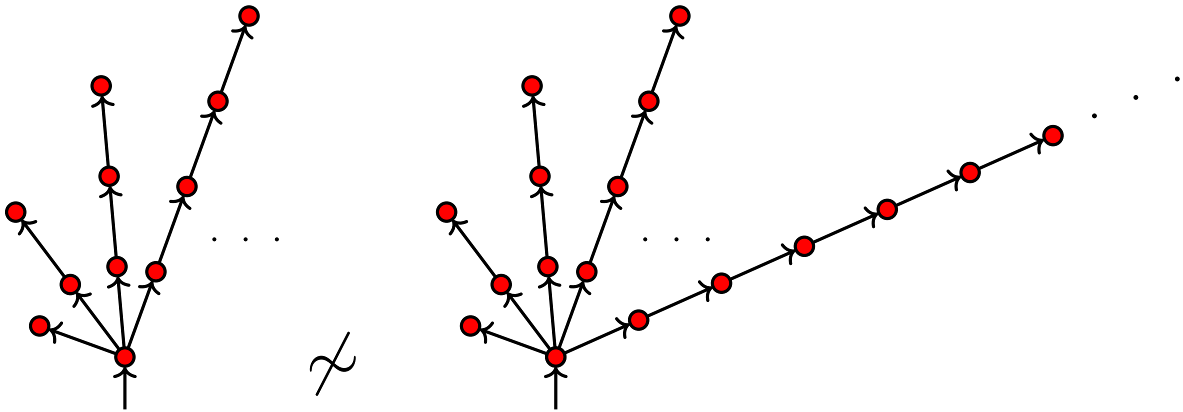

satisfy the same modal formulae, but are not bisimilar!

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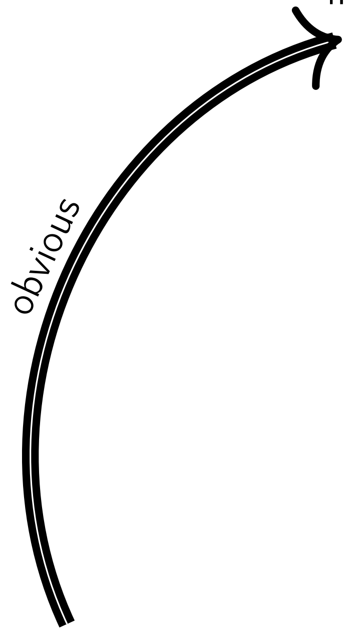

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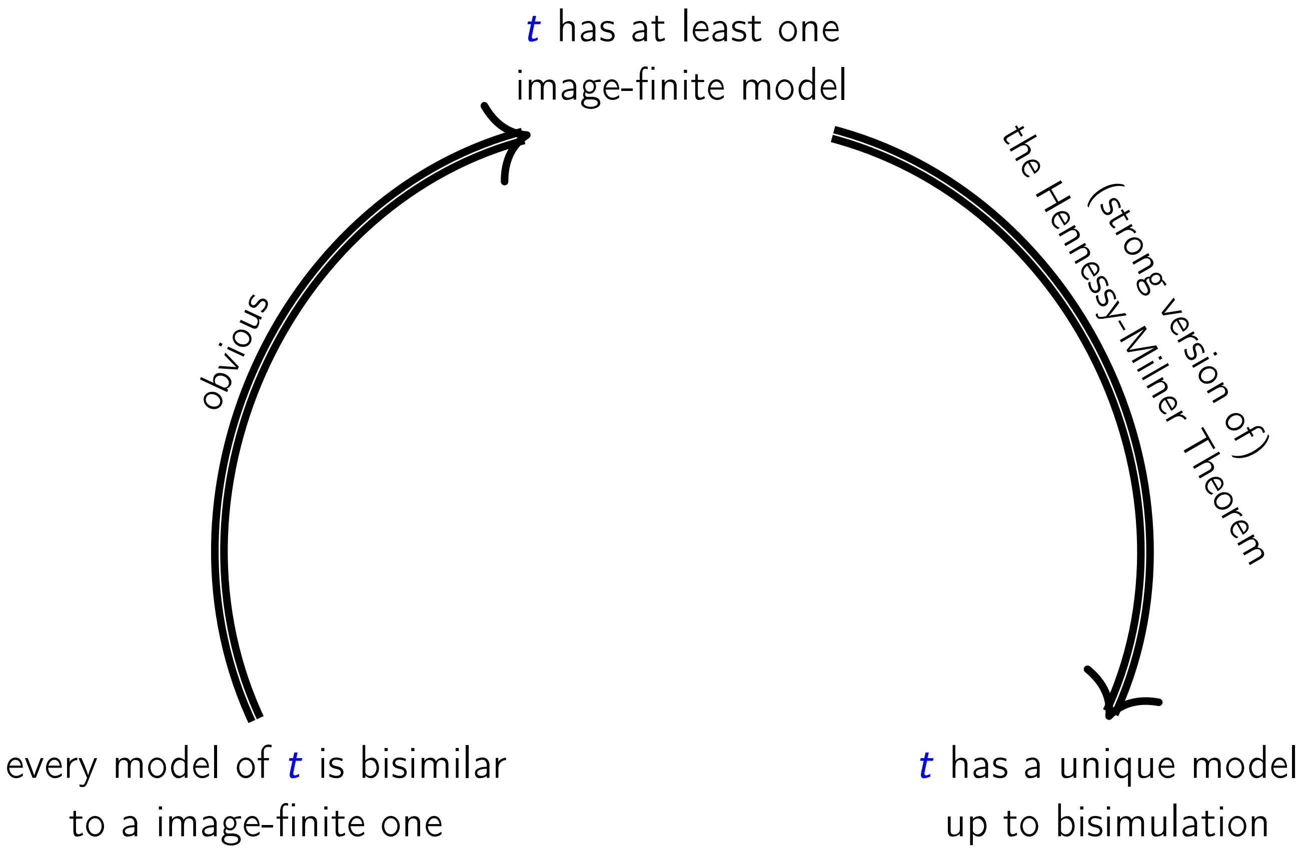

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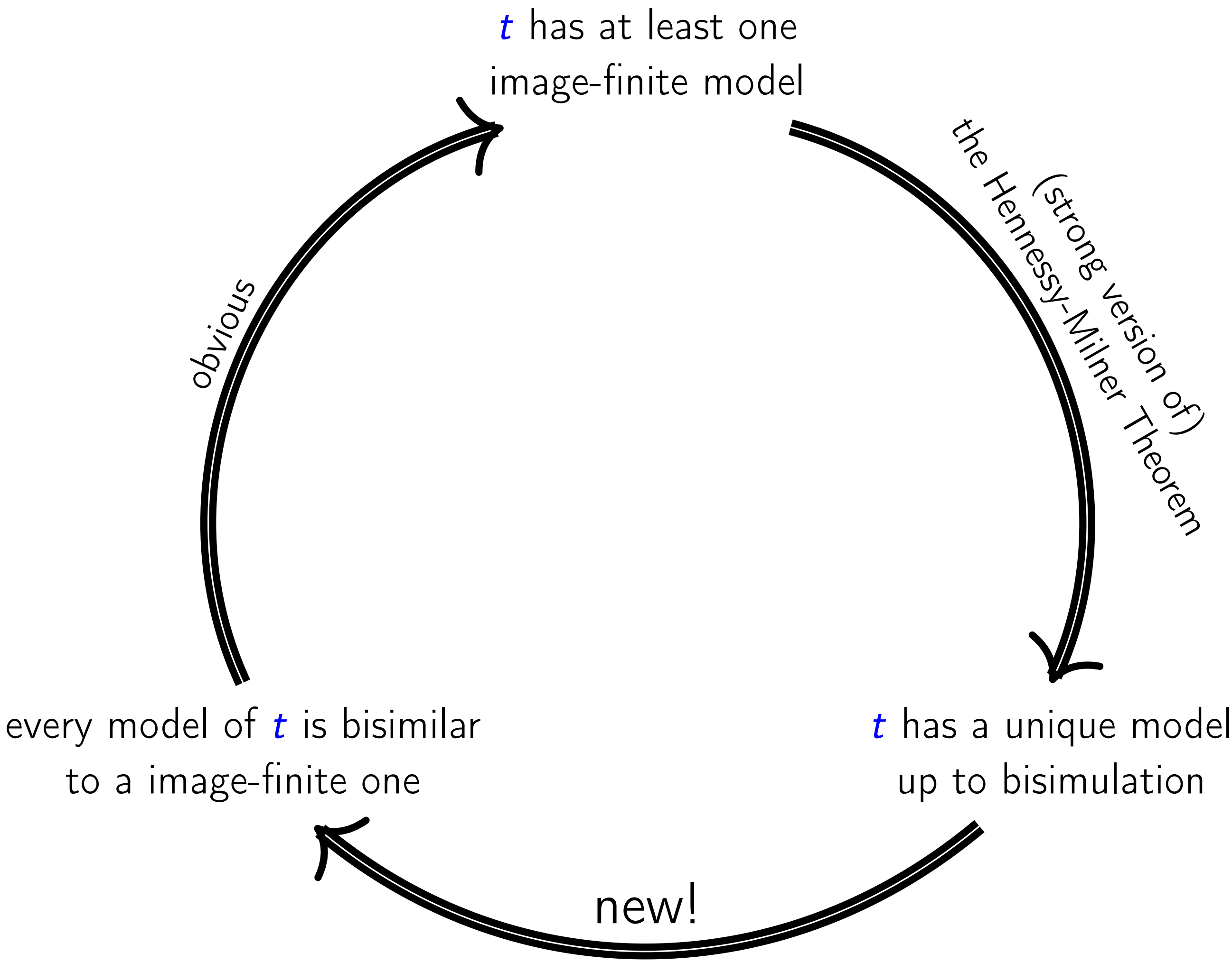

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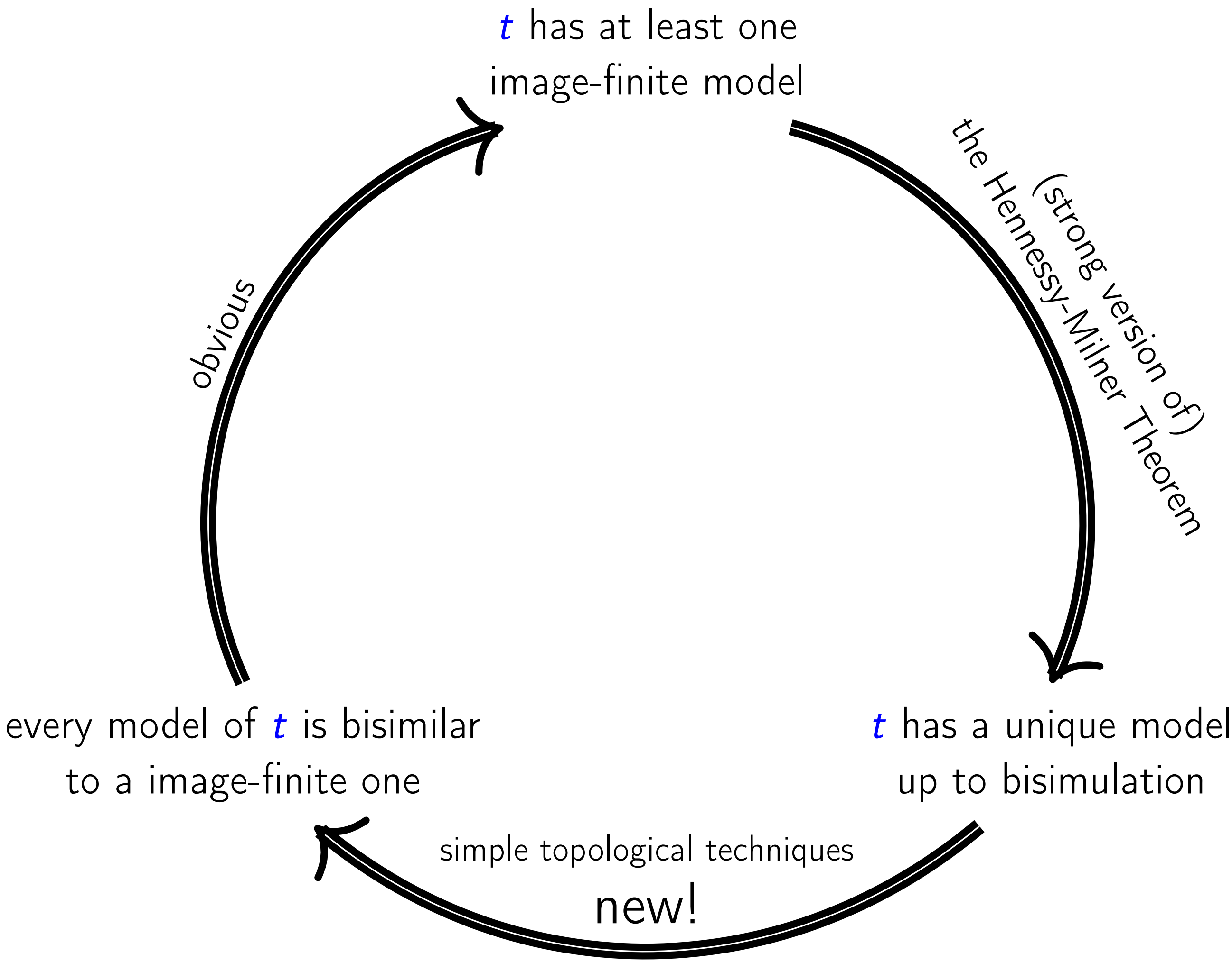

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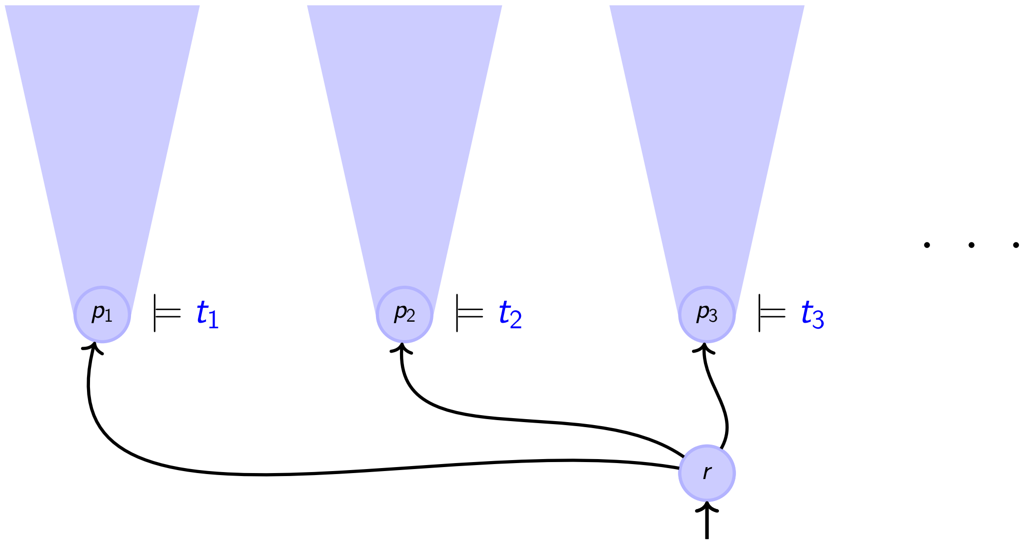

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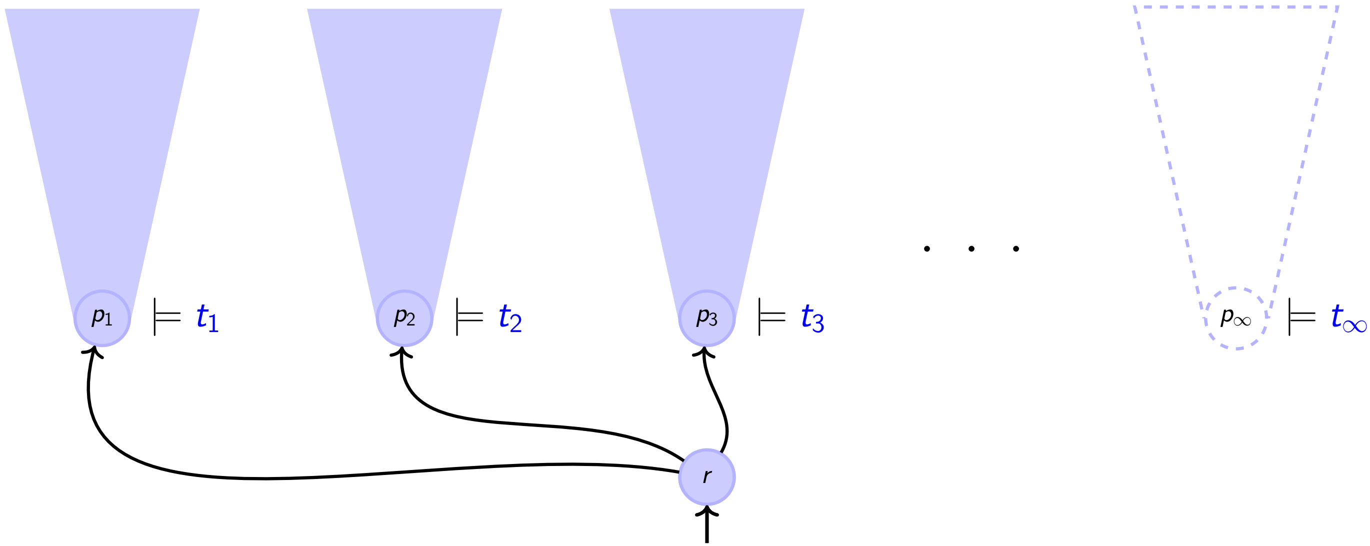

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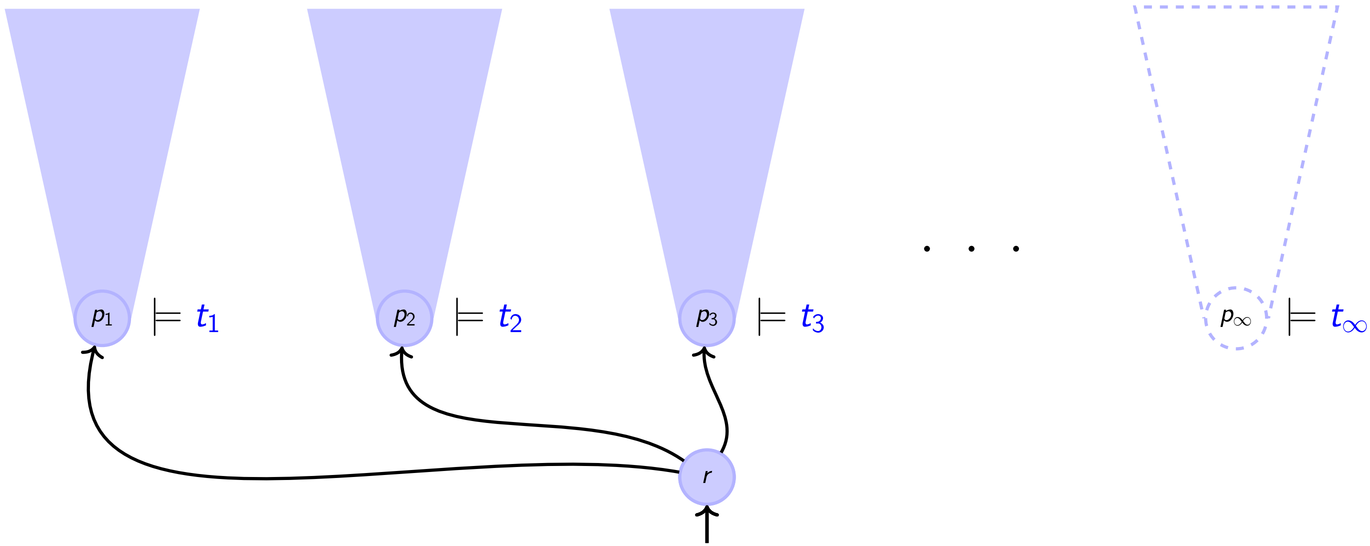

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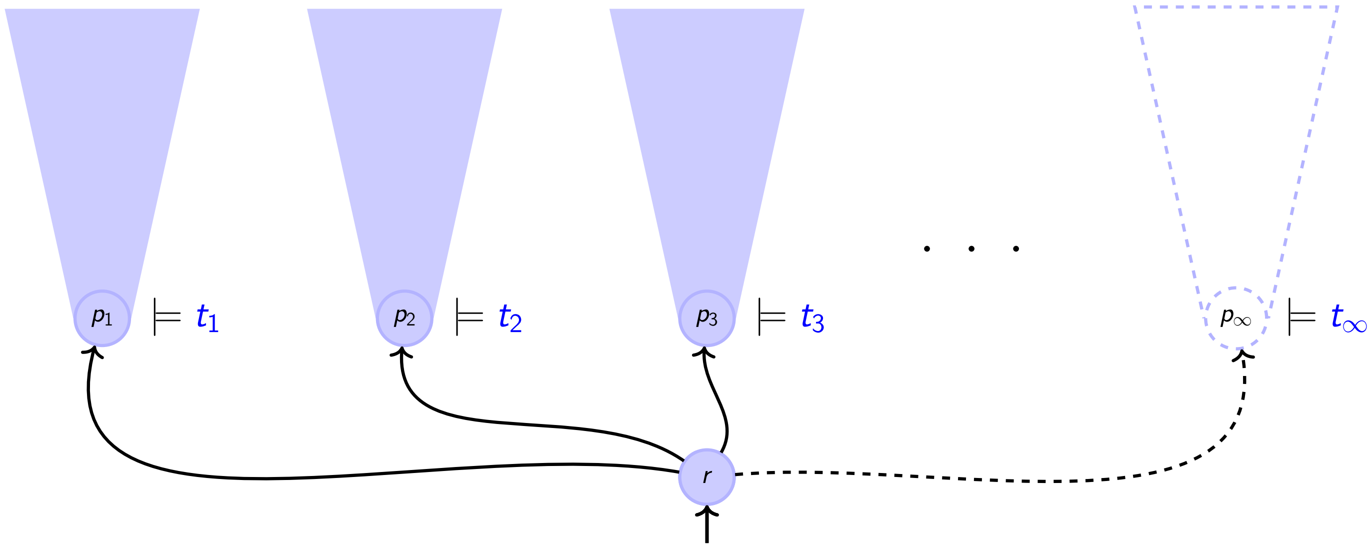

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- ...and include/exclude it, obtaining equivalent, but non-bisimilar models

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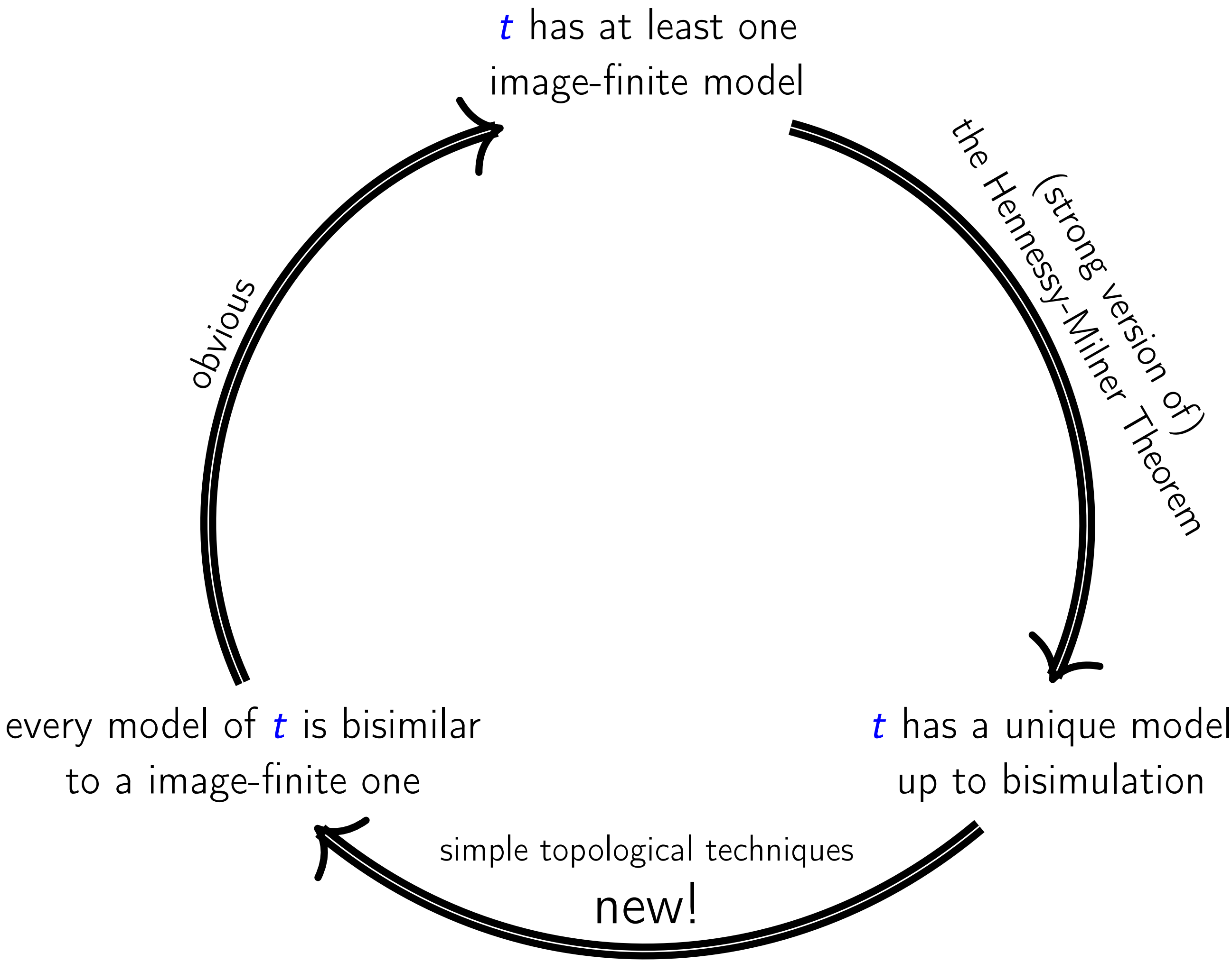

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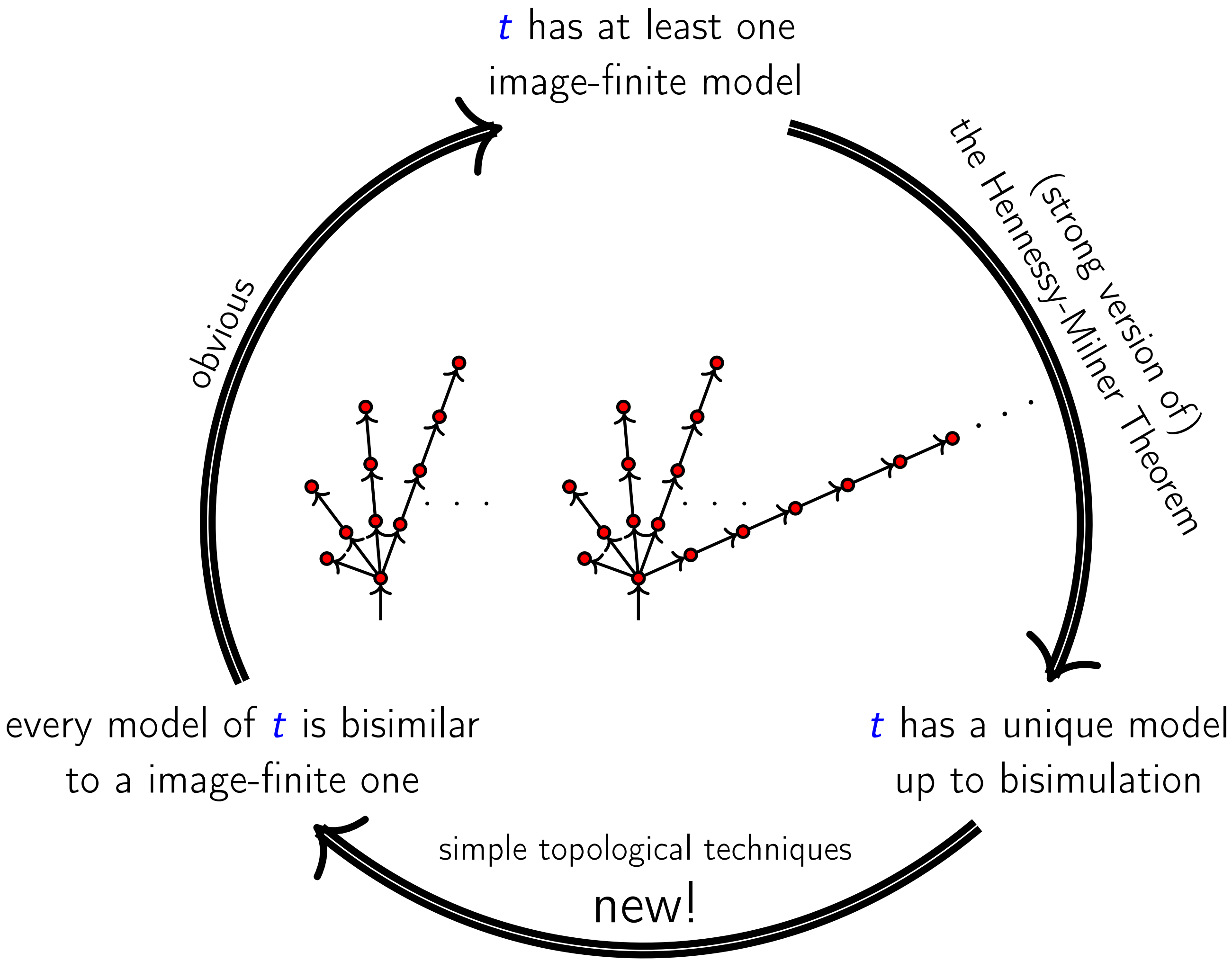

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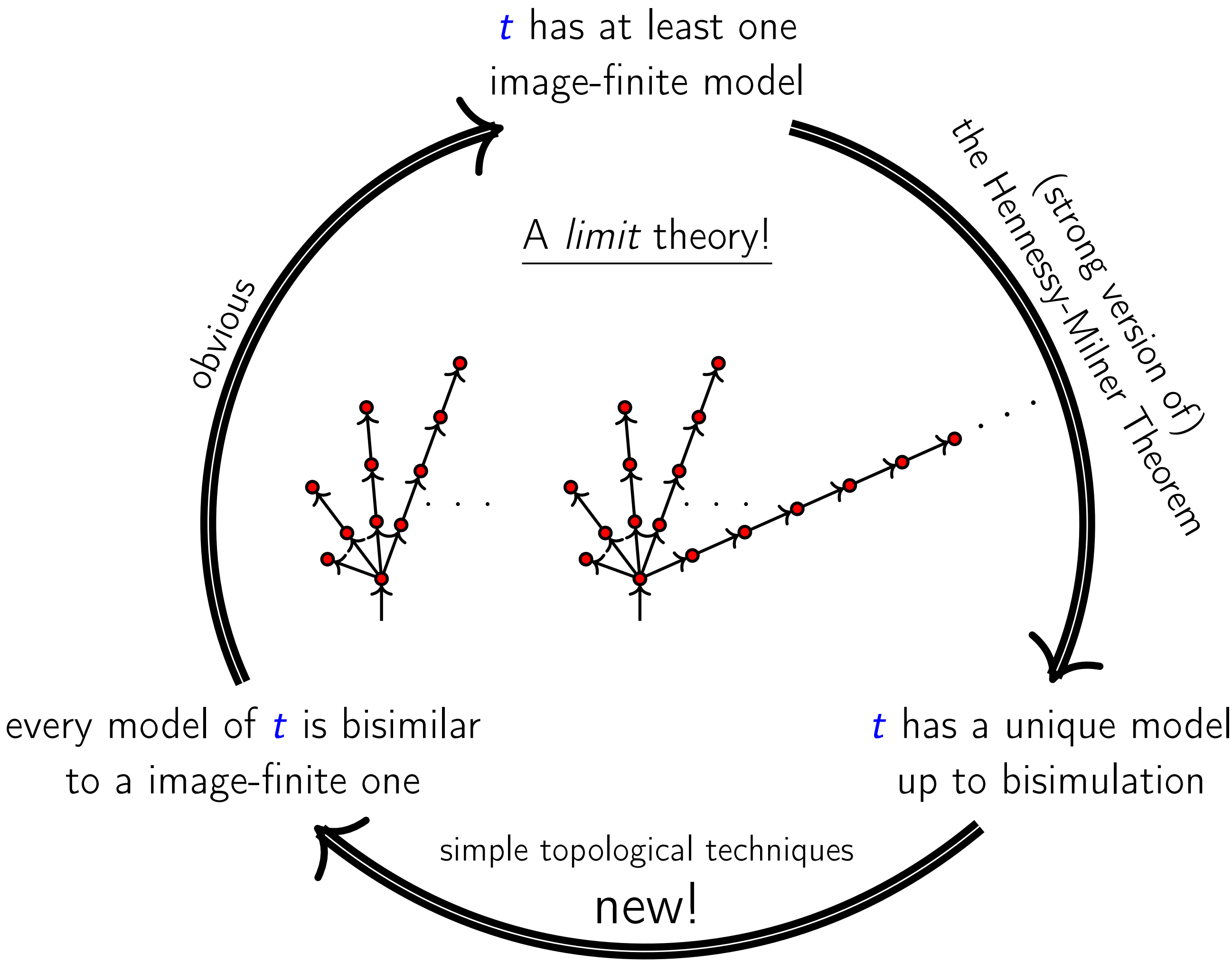

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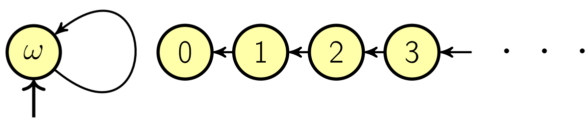

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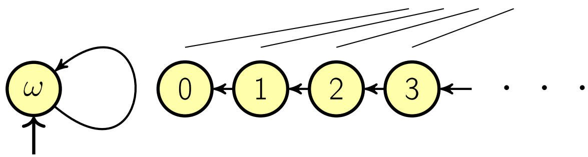

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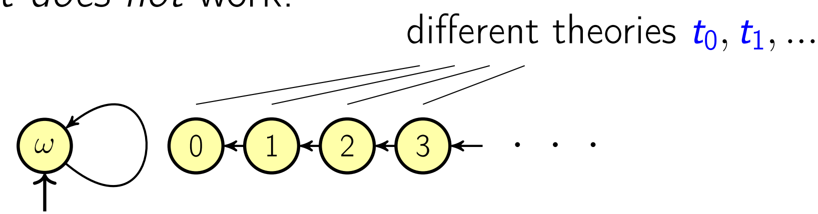

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ω has infinitely many $\langle\exists\rangle$-children with different theories $t_1, t_2, \ldots$, but their only limit is t_ω – the theory of ω which cannot be omitted!

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