Countdown μ-calculus

(with Automata and Games)

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MFCS 2022
Vienna

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- $\Diamond x$ induces an operation $F : \mathcal{P}(M) \to \mathcal{P}(M)$:

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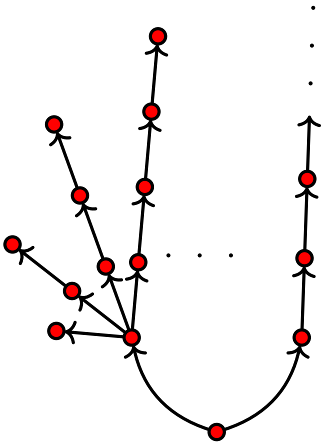

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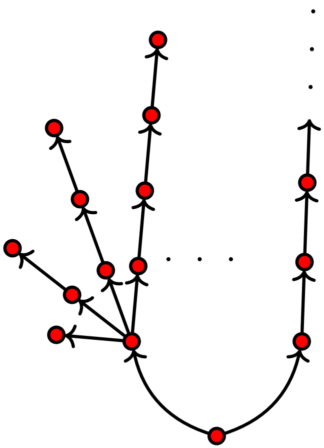

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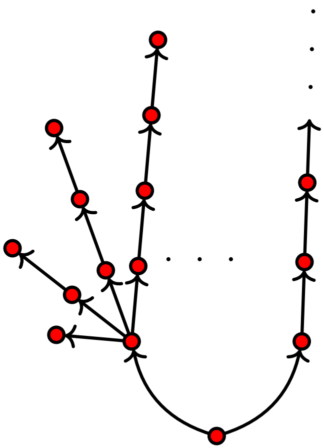

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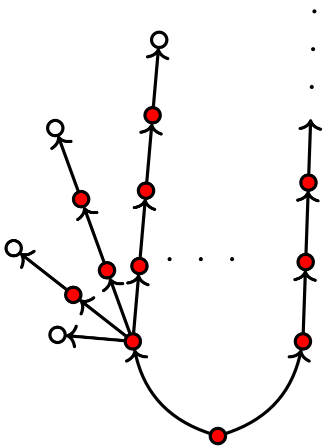

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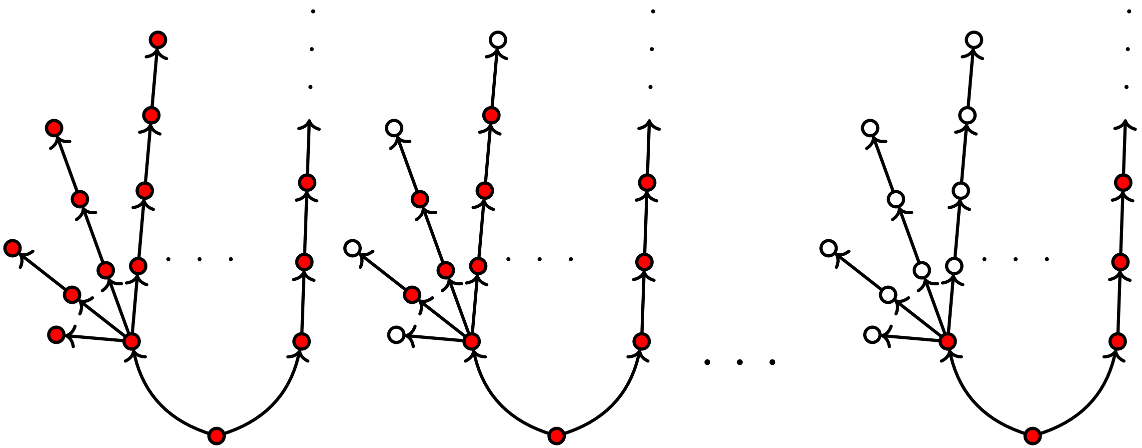

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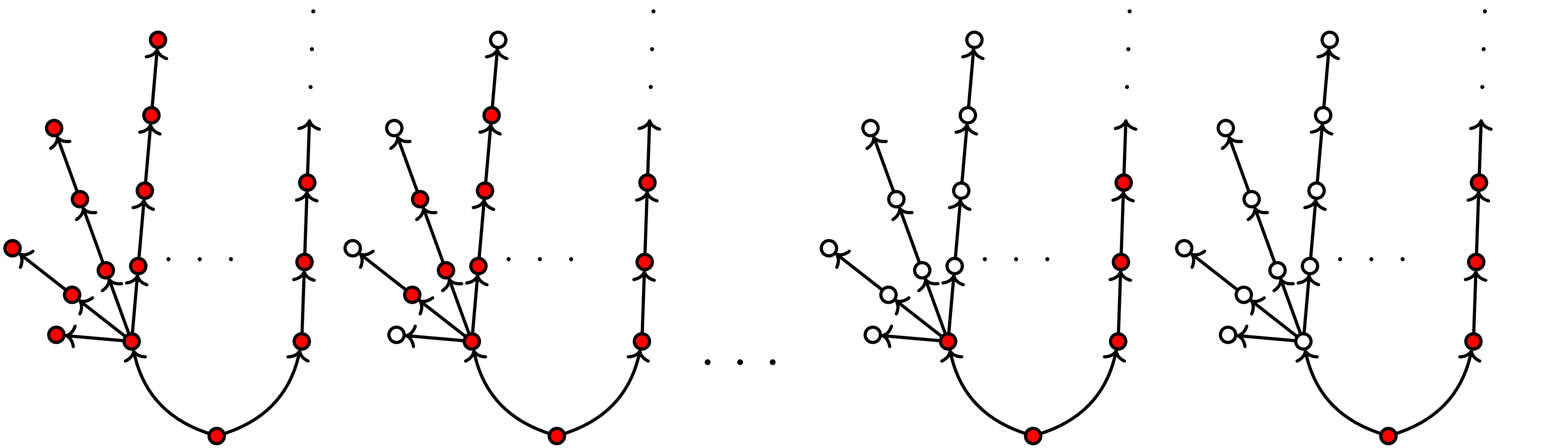

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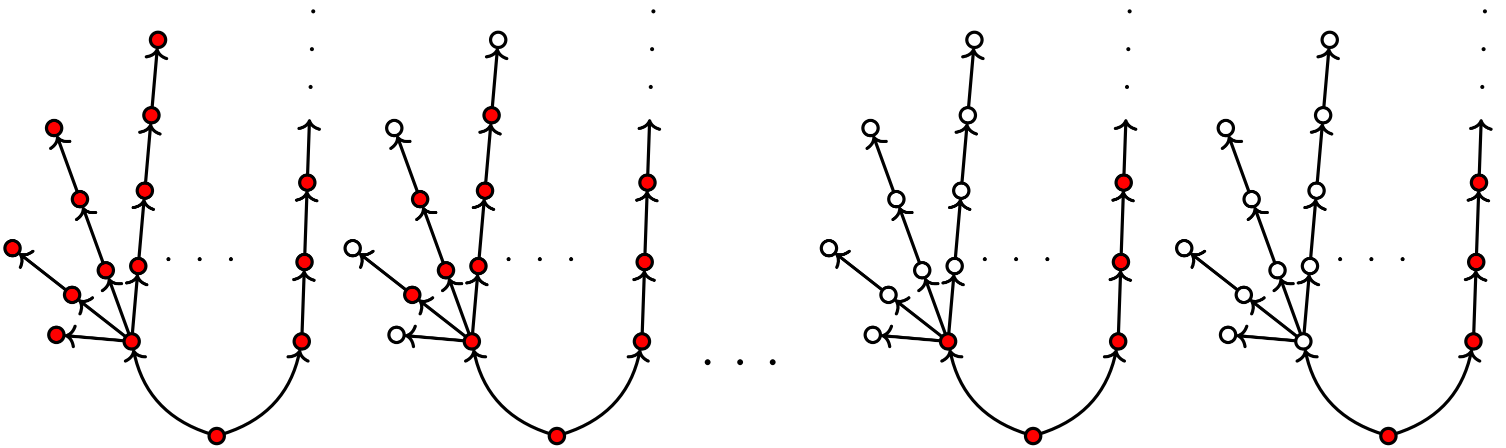

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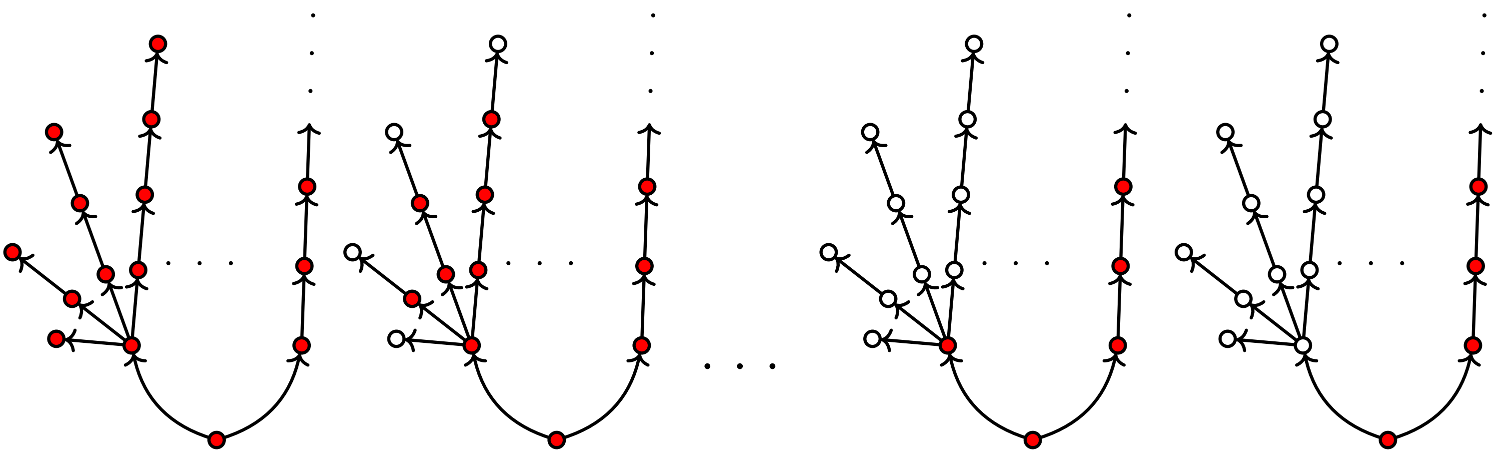

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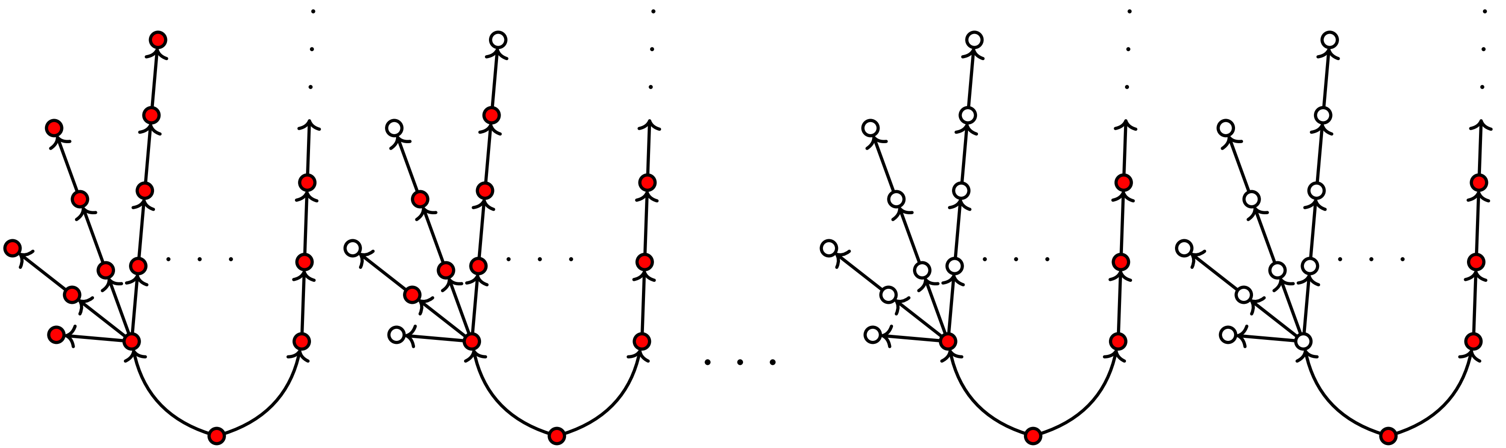

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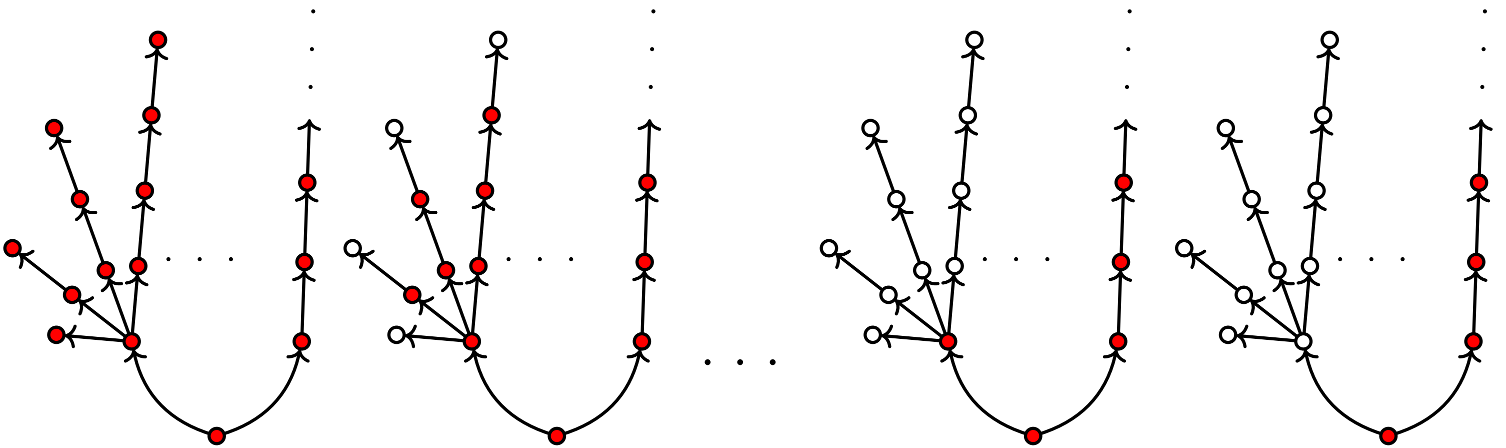
countdown calculus = μ-calculus + ν^{ω}



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countdown μ-calculus $\sim$ parity games

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countdown game = parity game + subset $\mathcal{D} \subseteq \{0,\ldots,d\}$

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  - and the game moves to $(w, \overline{C'_r})$.

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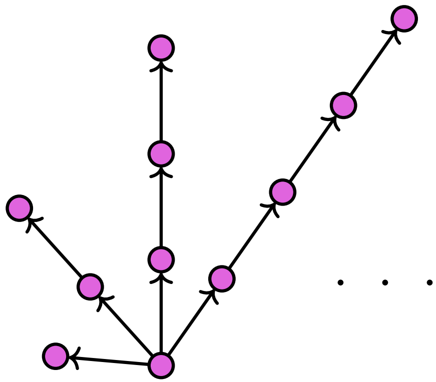

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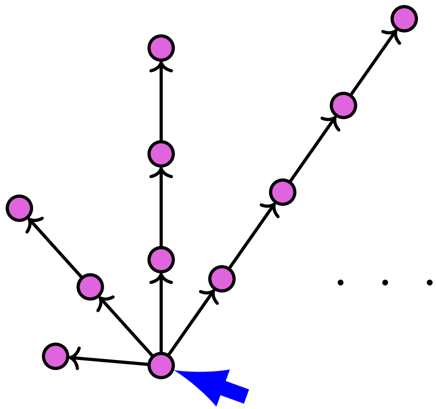

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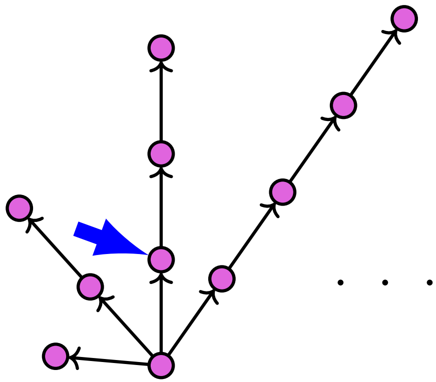

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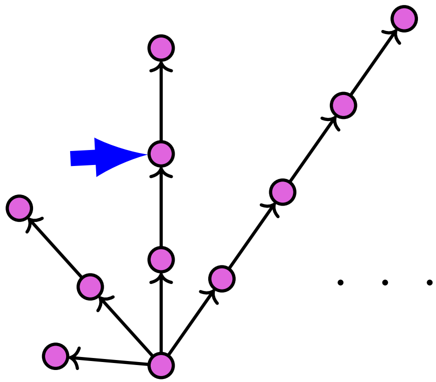

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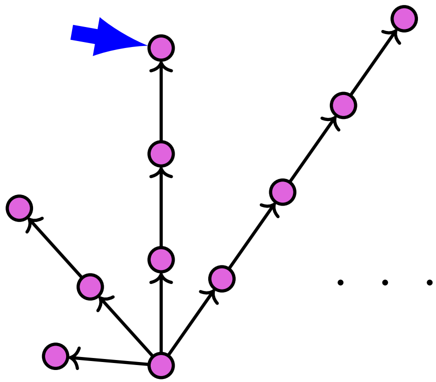

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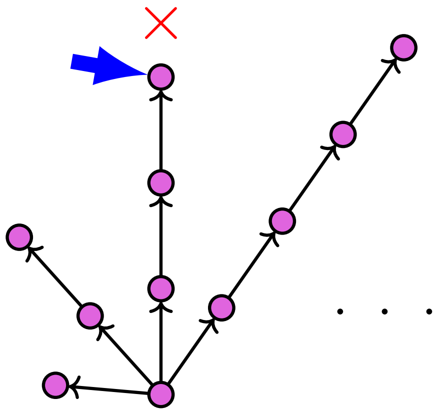

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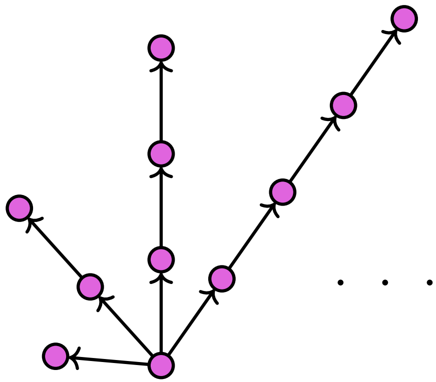

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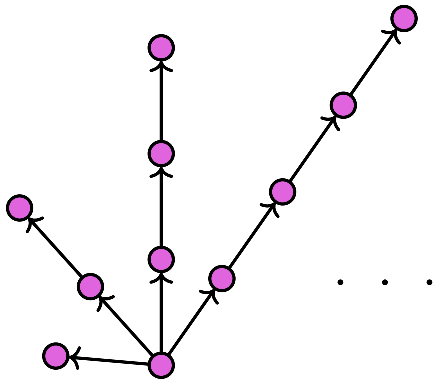

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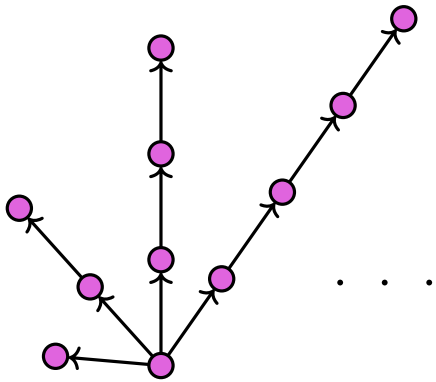

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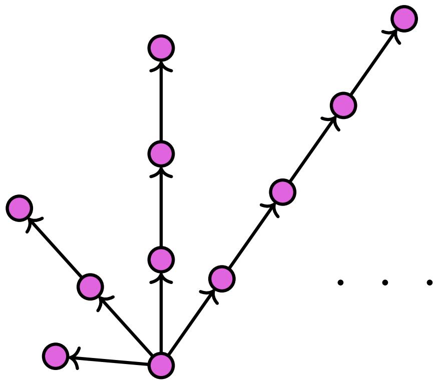

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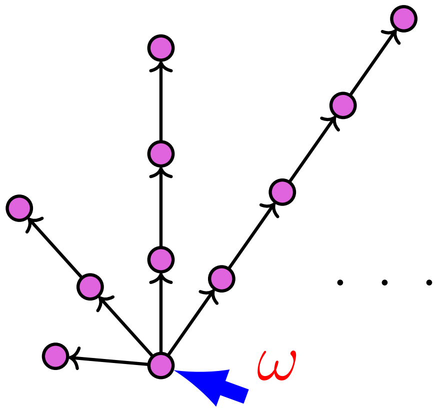

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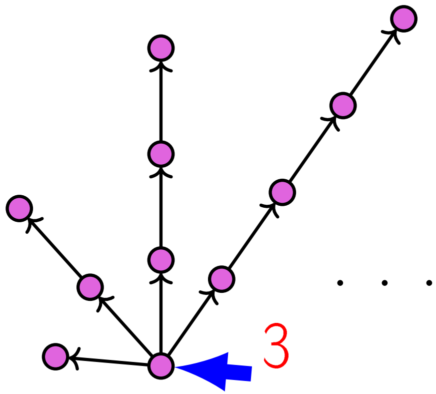

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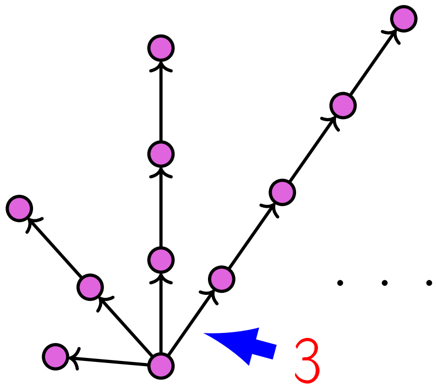

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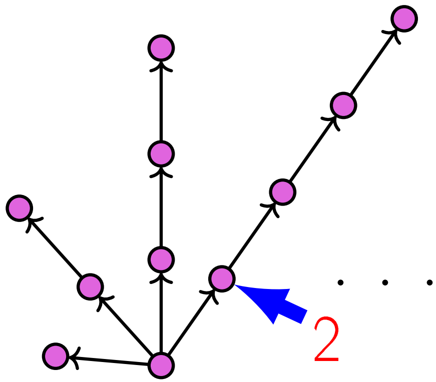

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- same as for $\nu x.\Diamond x$ except that rank r is nonstandard
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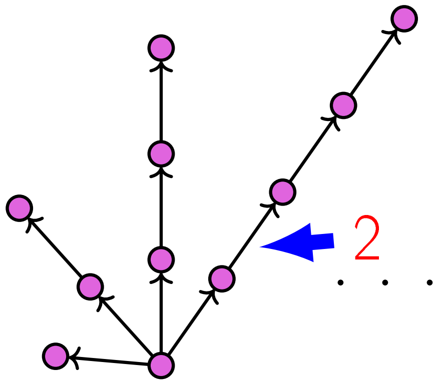

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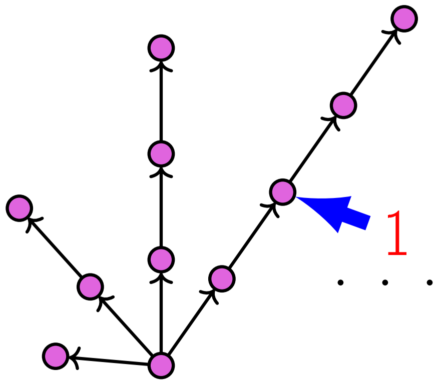

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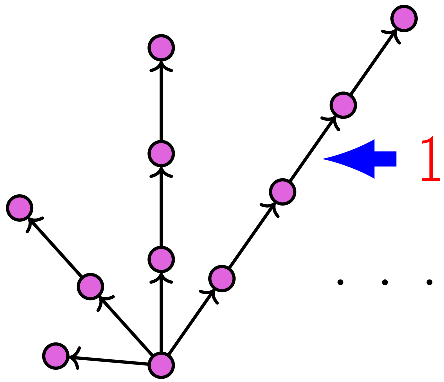

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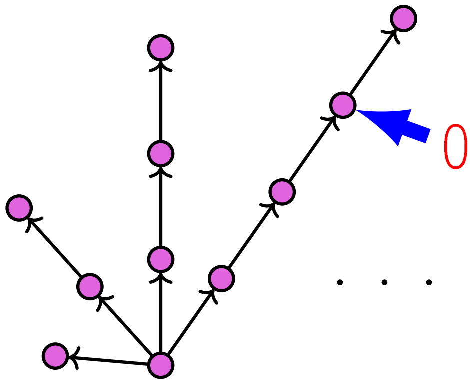

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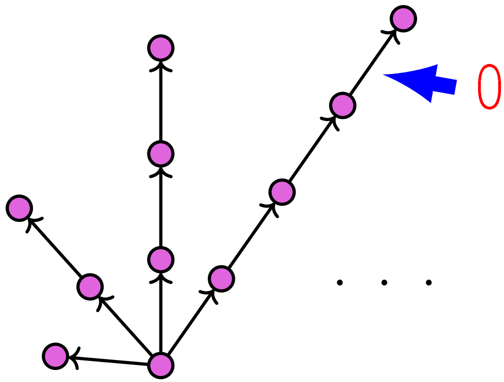

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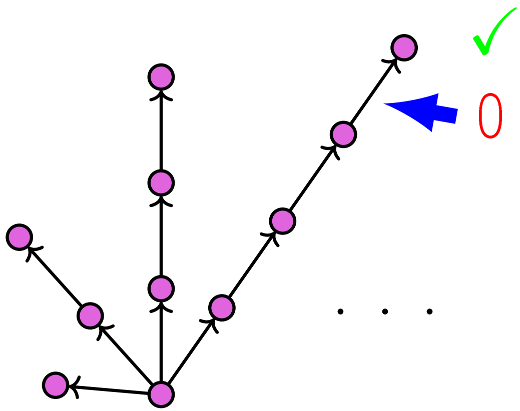

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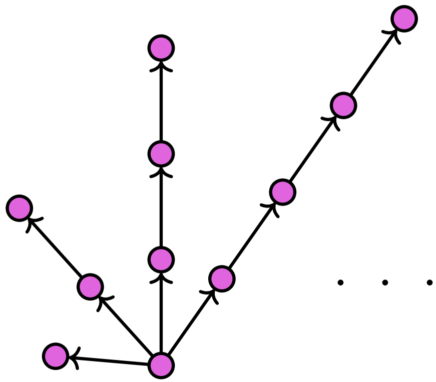

$\exists$ve wins $\mathcal{G}(\nu^\omega x.\Diamond x)$ $\iff$ $\exists$ arbitrarily long paths

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equivalent to alternating ~~parity~~ automata
countdown!

countdown μ-calculus $\sim$ ~~parity~~ games
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wining regions definable in logic

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equivalent for μ-ML

but not for μ^{ω}-ML!!!

wining regions definable in logic

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μ-ML, but with logical values from $[0,1]$ instead of just $\{0,1\}$

and the function $t \mapsto \frac{1}{2}t$ as an extra unary connective

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- more nesting of countdown operators $\implies$ more power
- nothing special about ω, take your favourite ordinal instead!

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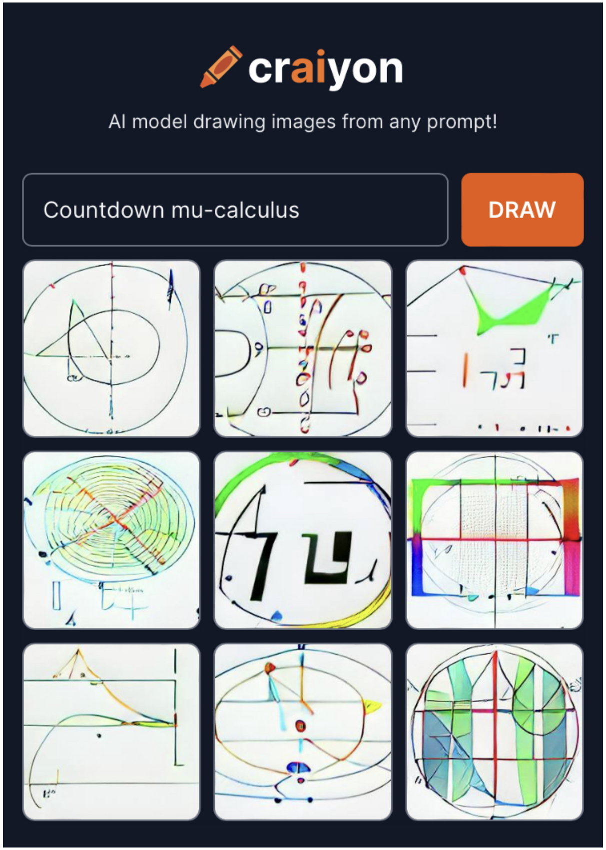
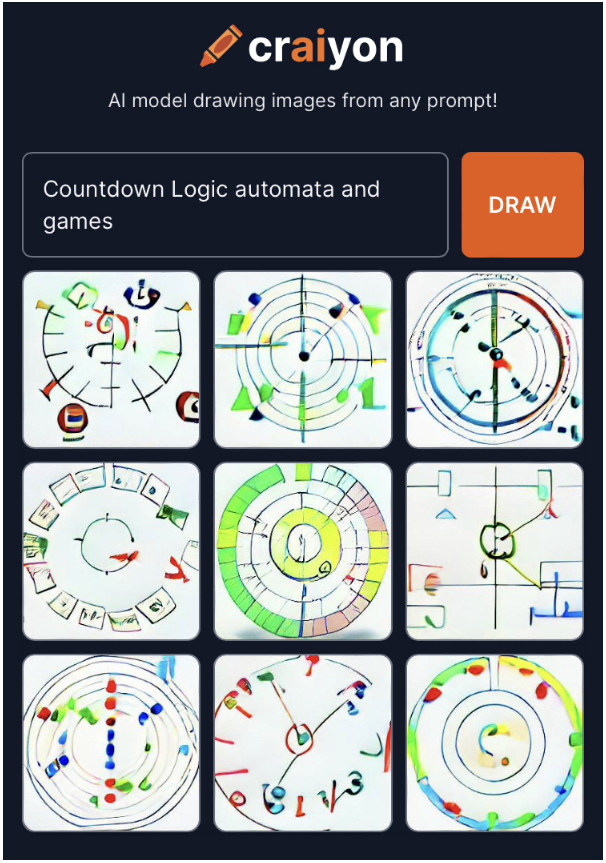
Thank you! :)


craiyon
AI model drawing images from any prompt!
Countdown mu-calculus
DRAW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