Modal Separability of Fixpoint Formulae

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&
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Bergen

Separators

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formulae $\varphi \models \neg\varphi'$

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simple ψ s.t.
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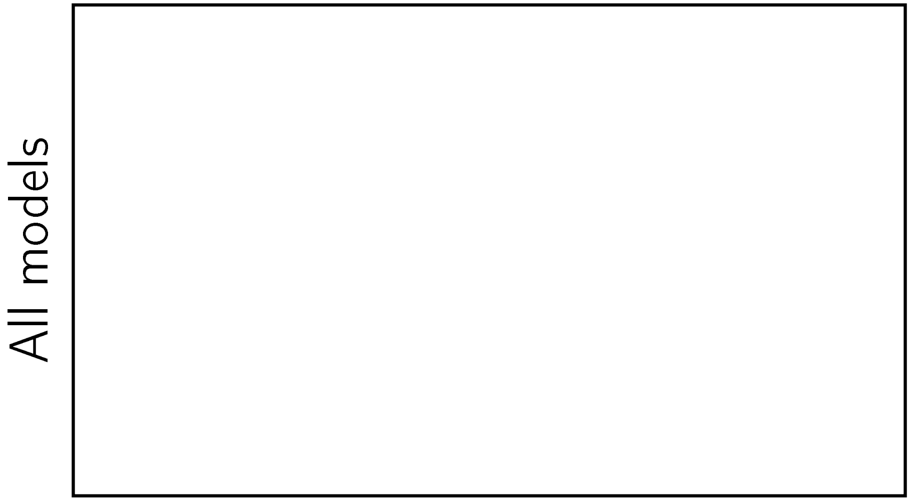

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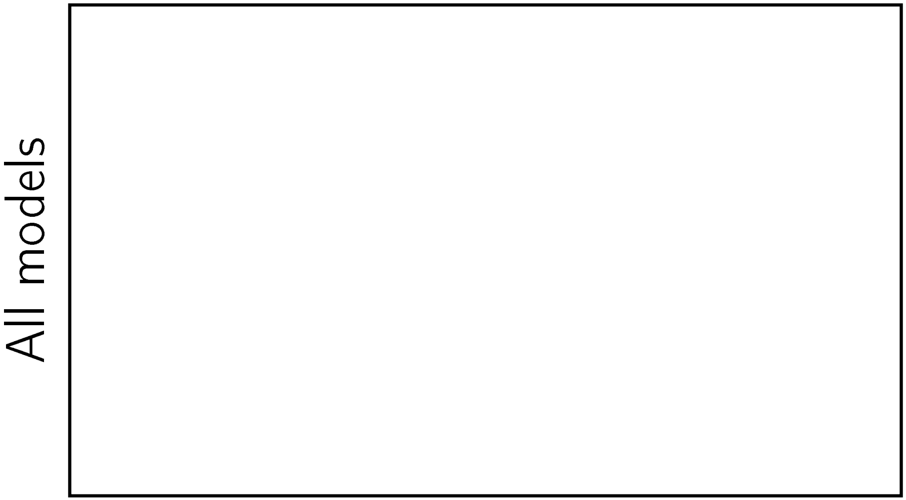
Colored directed graphs with chosen point (root)



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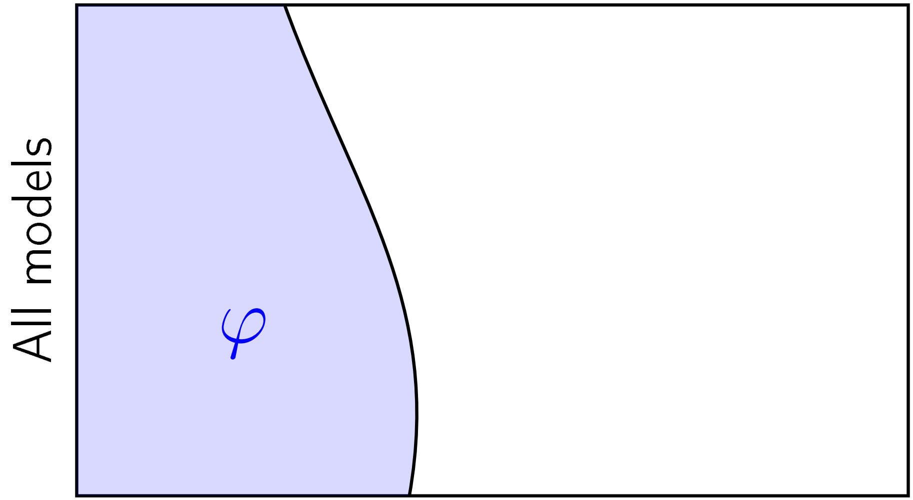

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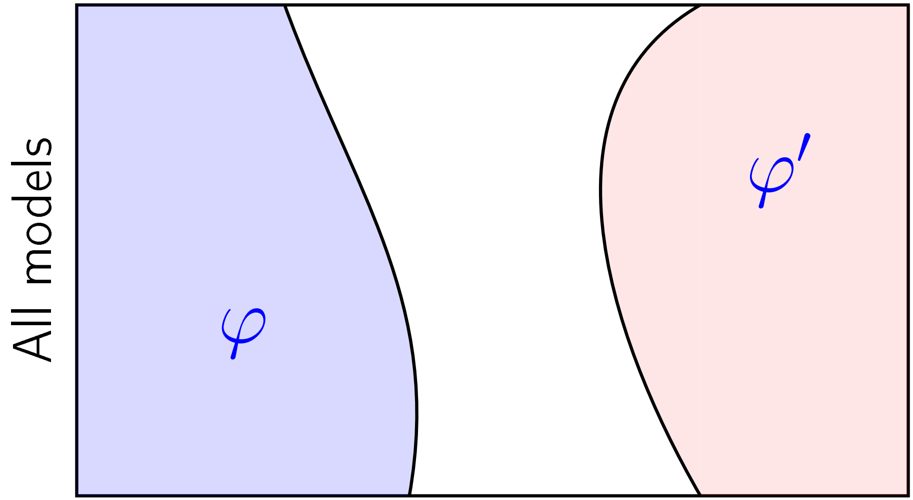

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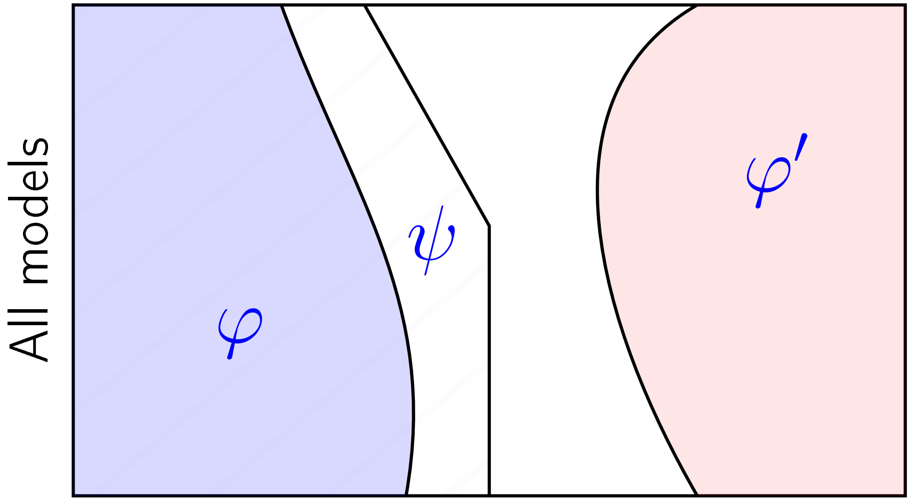

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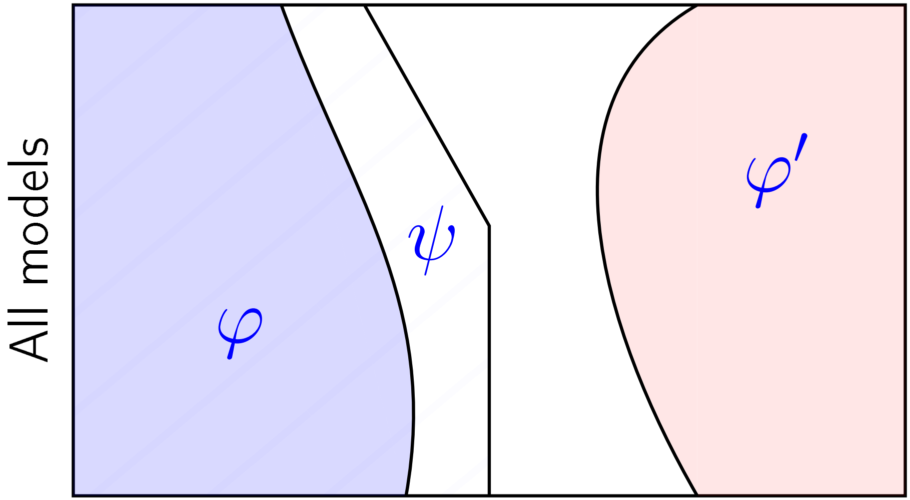

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simple explanation of contradiction

complicated formulae $\varphi \models \neg\varphi'$

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Example

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some path
starting in root
has labels in:

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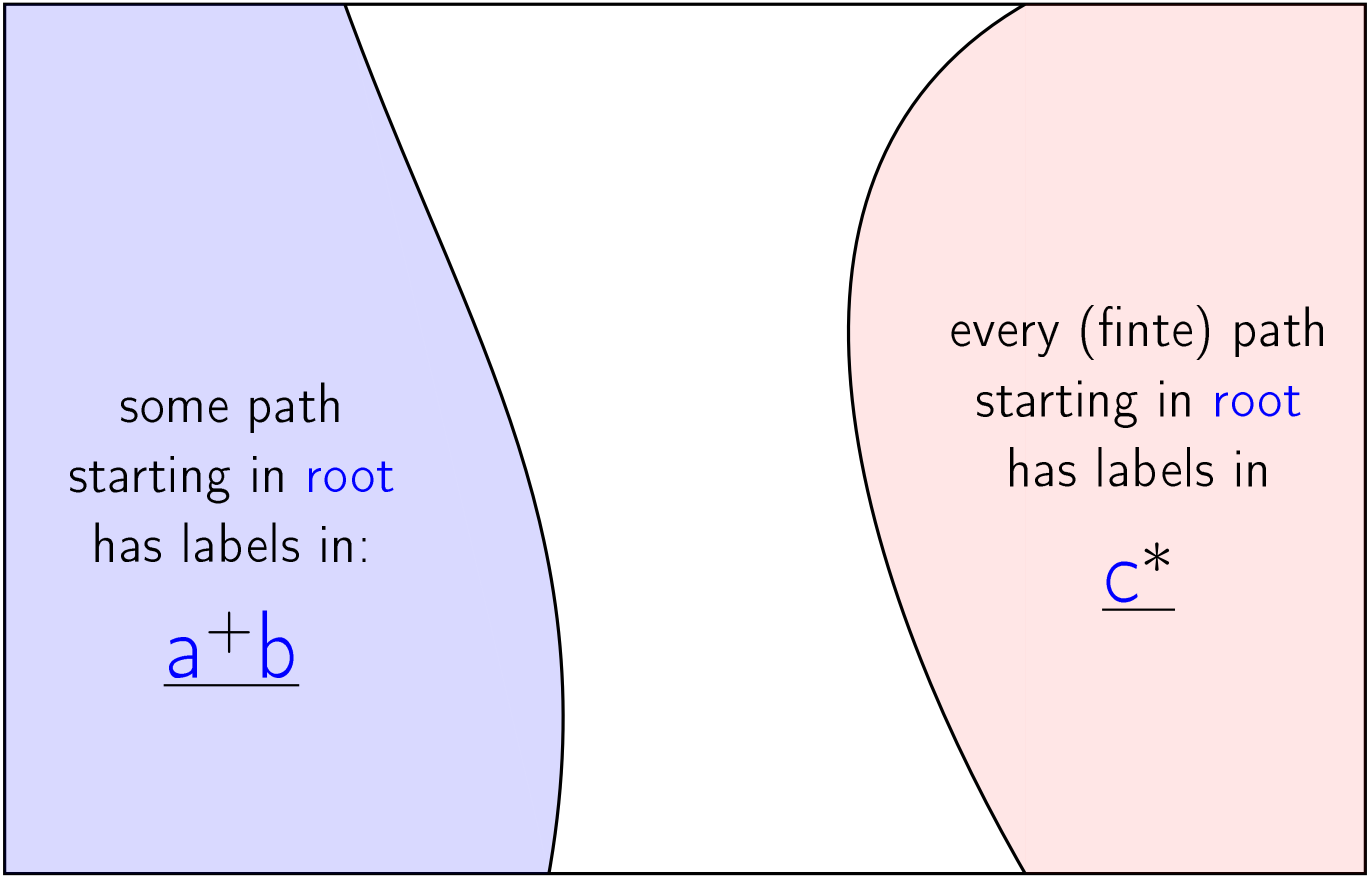
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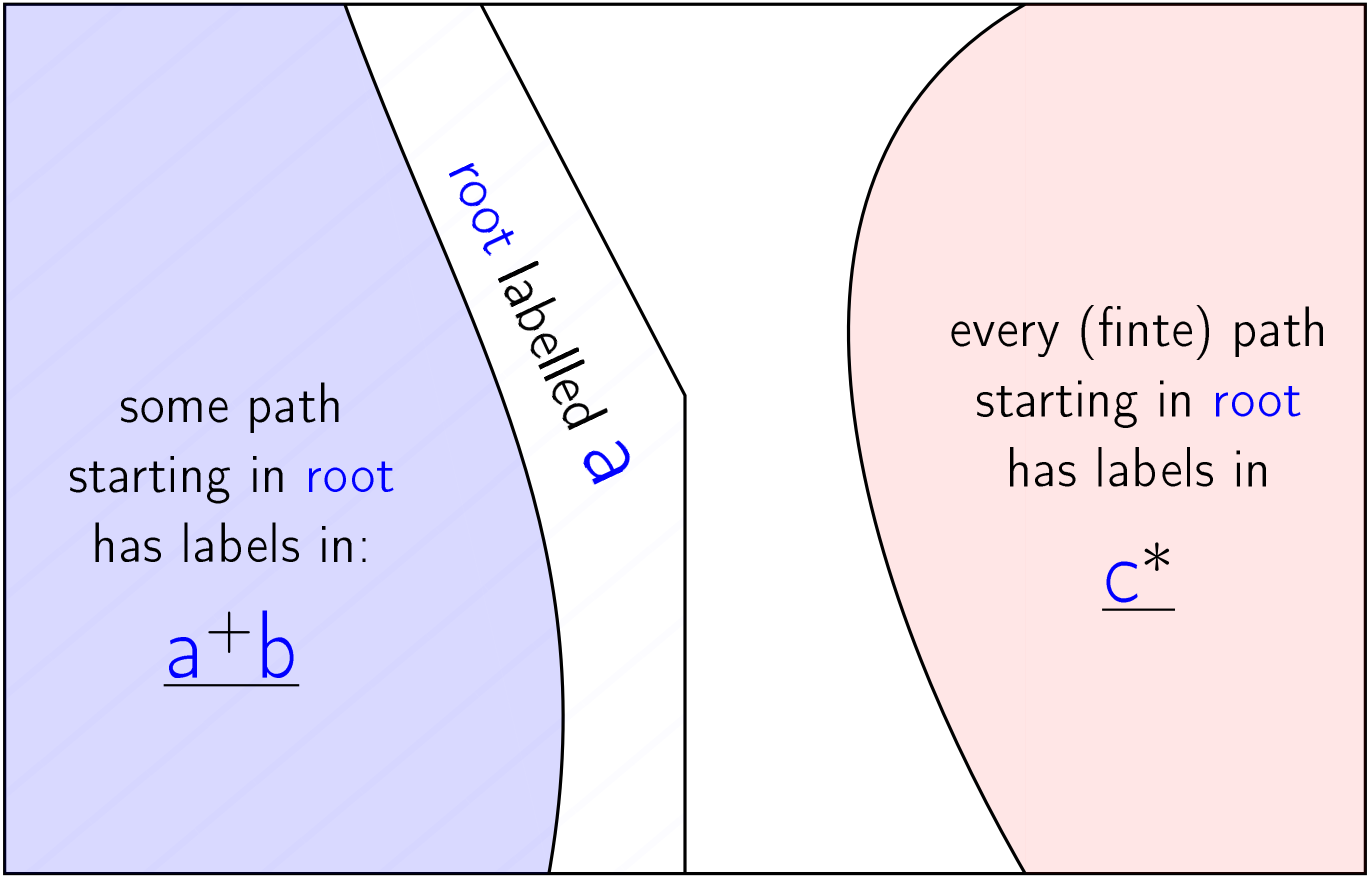

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$$\mathcal{L}^+ = \mu\text{-ML} = \text{ML} + \textbf{fixpoints}$$

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$\mathcal{L}^+$ = μ-ML = ML + **fixpoints**

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μ-ML

=

Automata

=

MSO

Translations

Translations

μ-ML formulae

parity automata

Translations

EXPONENTIAL

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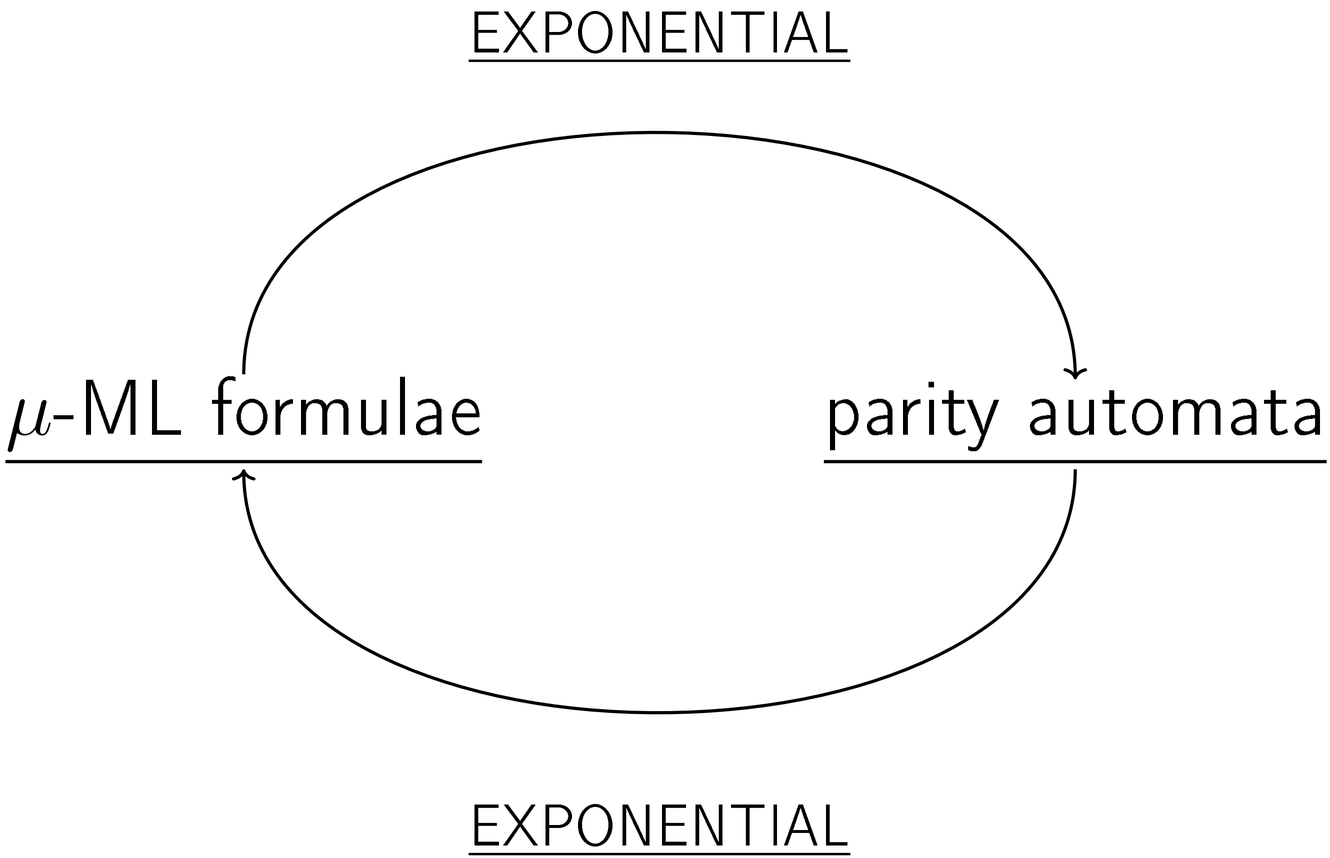

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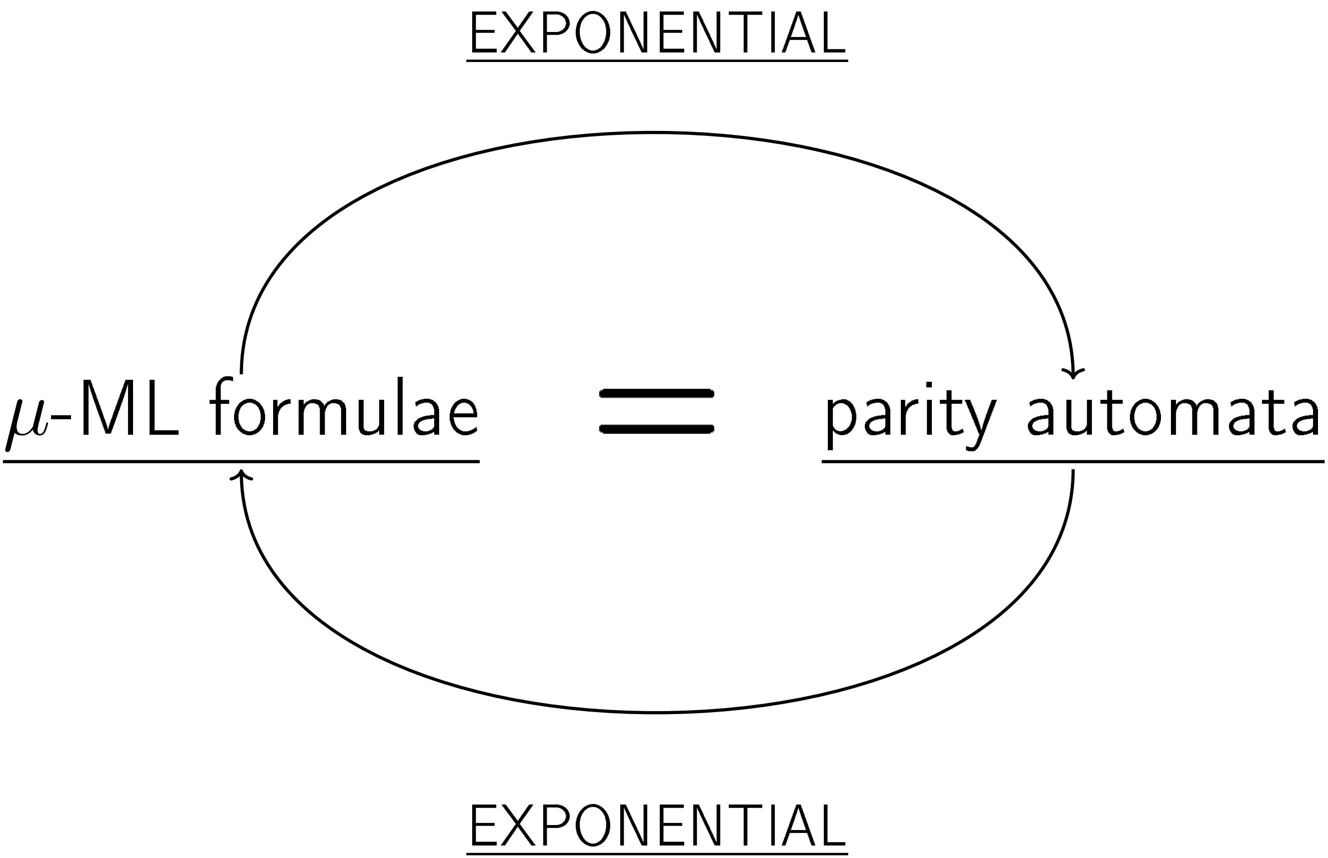

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φ entails no modal formulae!

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- reduces to $\mathcal{L}$-separability.

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- Hence, separators of modal depth n exponential in k suffice.

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- (ML^n = modal formulae of modal depth n)

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- If φ, φ' have (any) separator of depth n then ψ^n is a separator.

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- This is **optimal**: μ-ML is 2EXP **succinct** compared to ML

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- Automata are simpler, hence separability is PSpace complete.
- As before: exponential bound on needed modal depth of separators
- More efficient n-uniform consequences!

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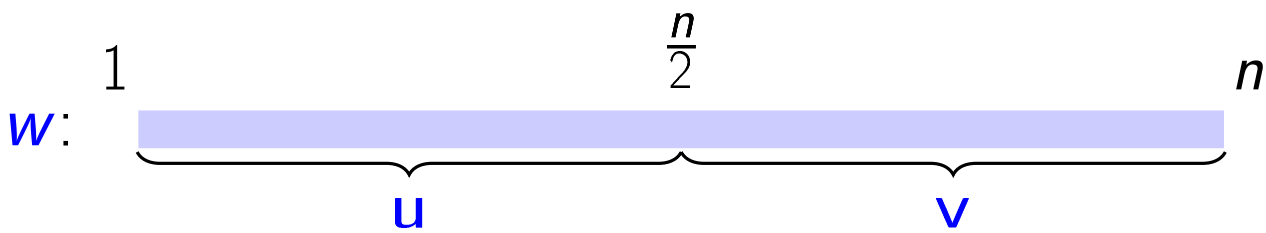

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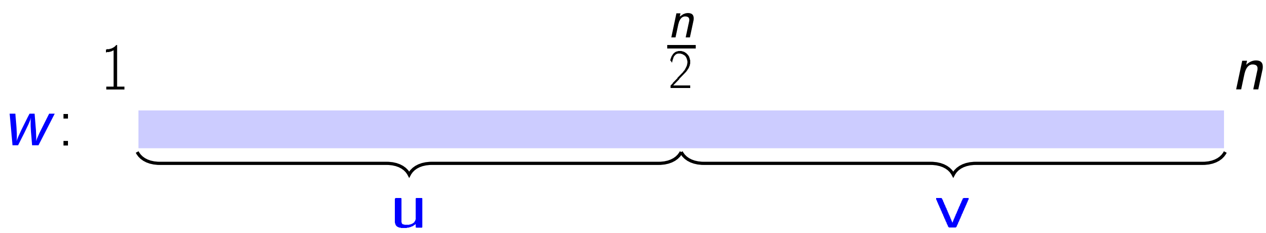

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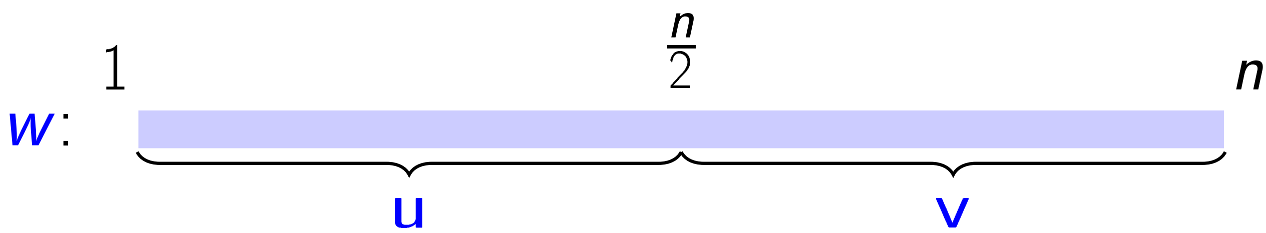

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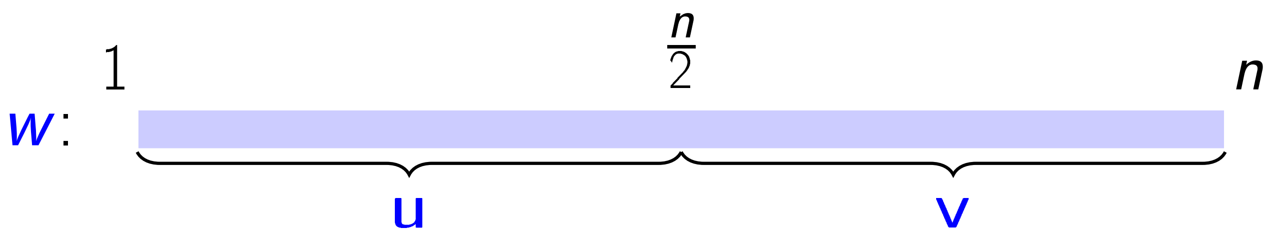

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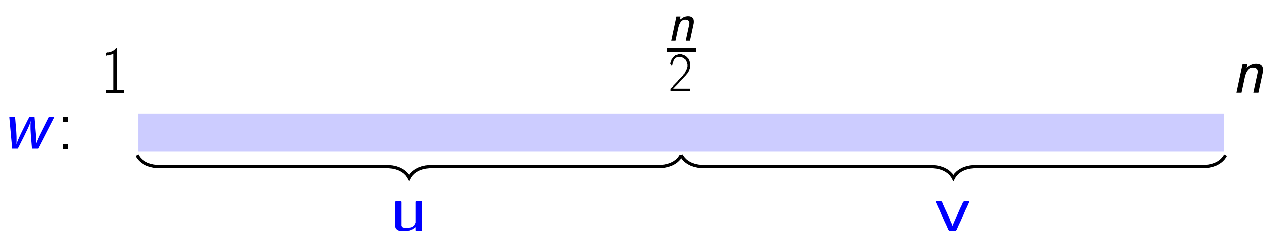

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- For arbitrary automaton: run on **w** = part on **u** and on **v**

Relativization

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- Assume classess of models $\mathcal{C}$ and $\mathcal{D}$ and formula θ such that
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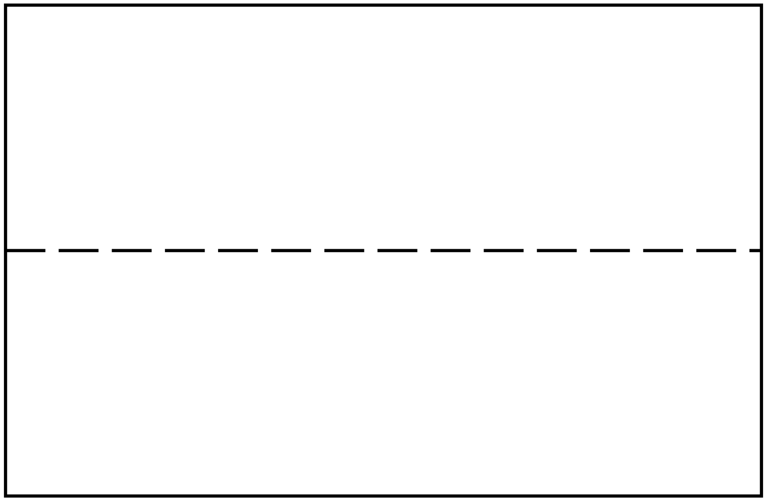

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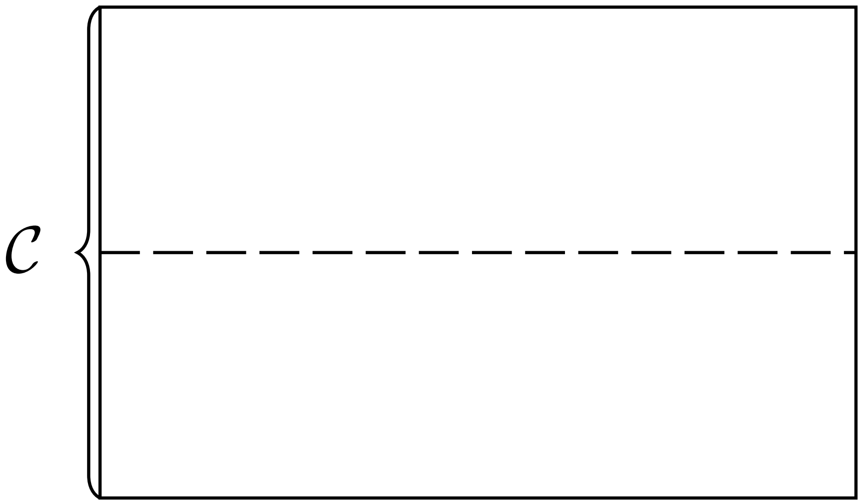

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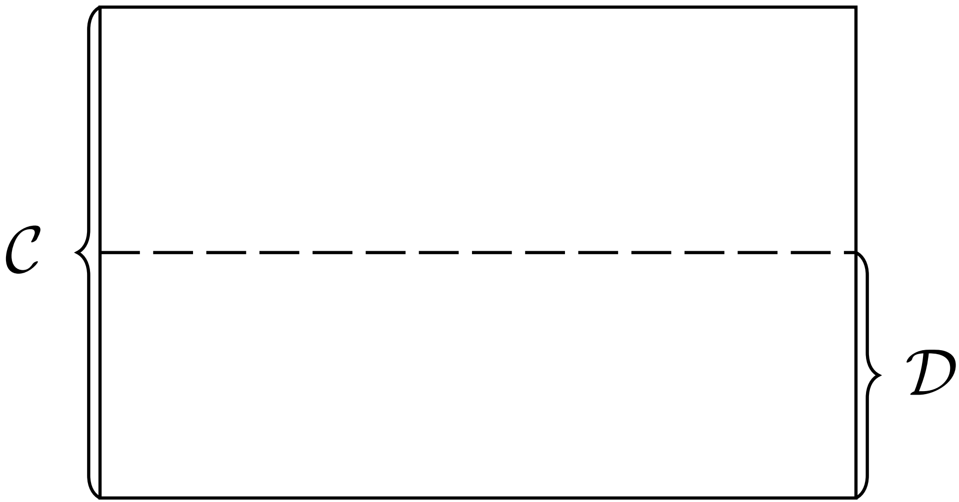

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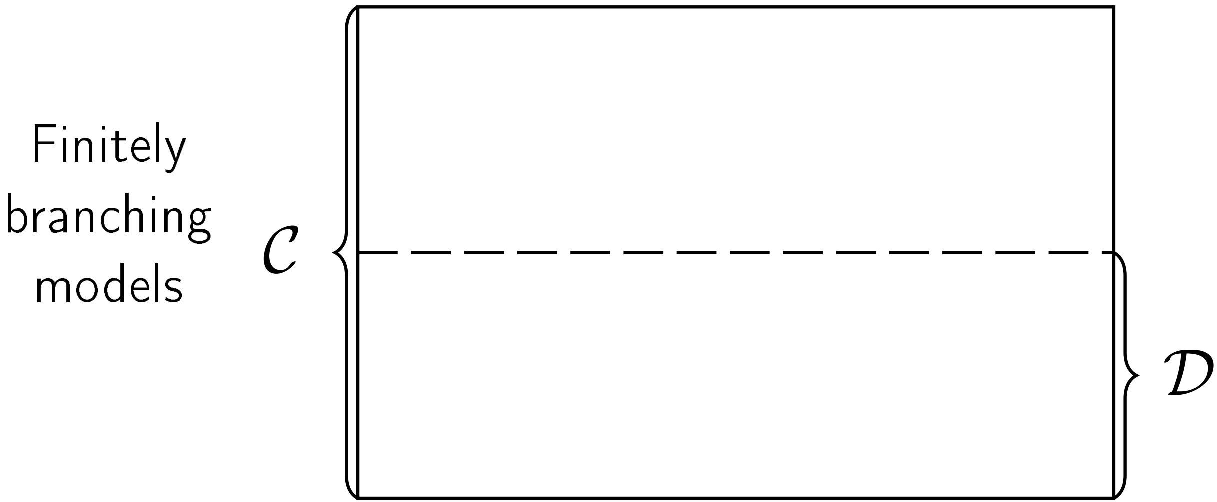

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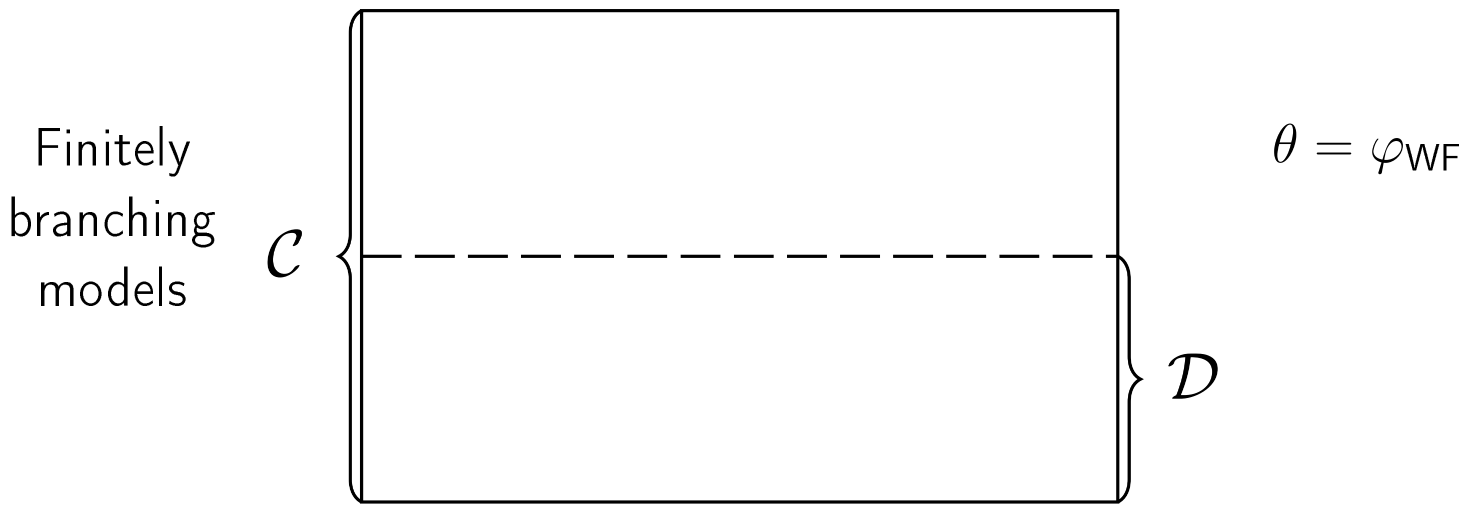

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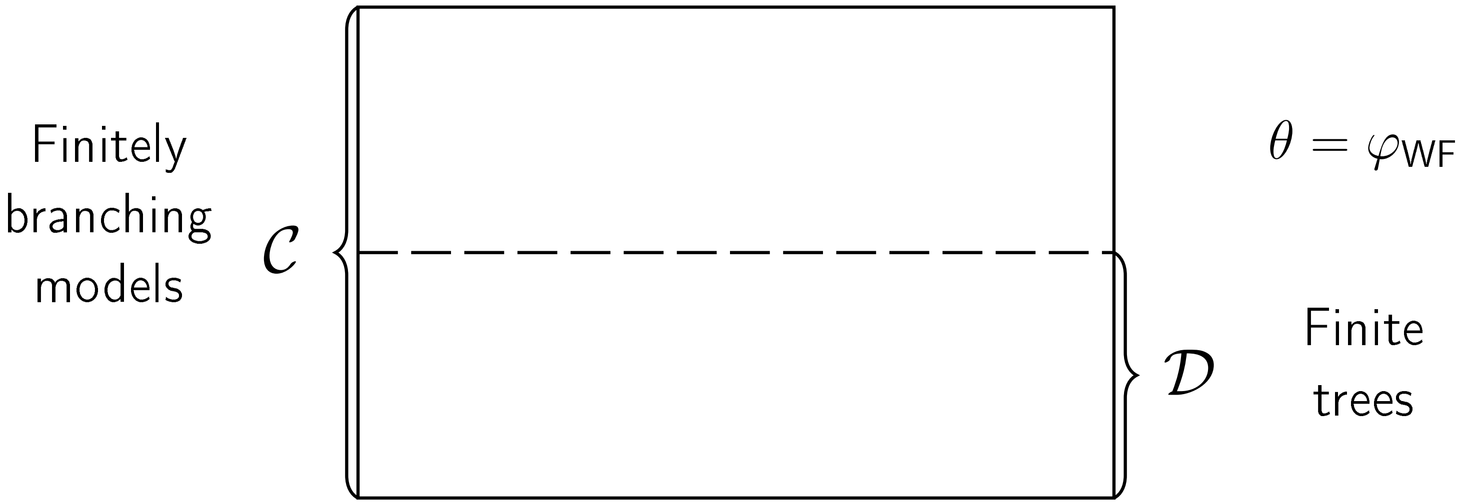

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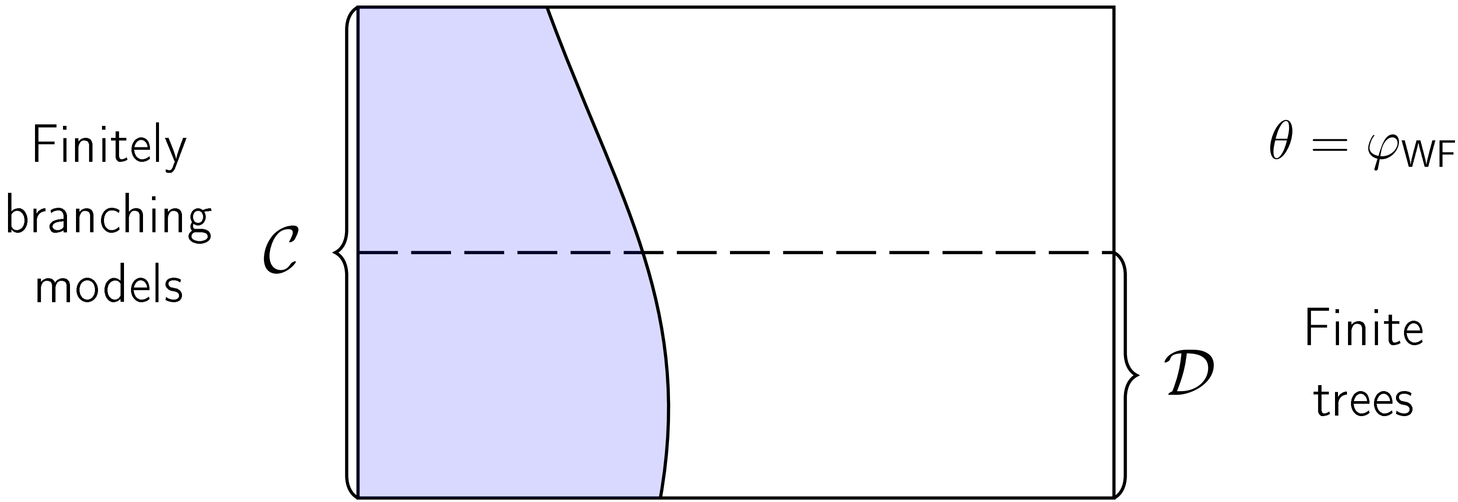

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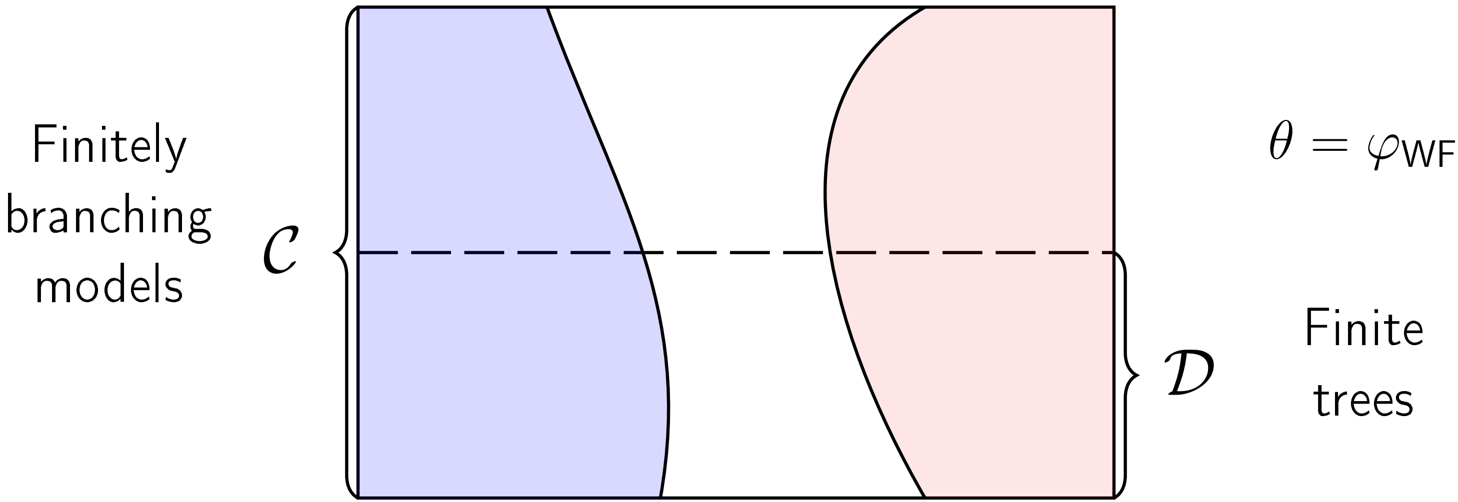

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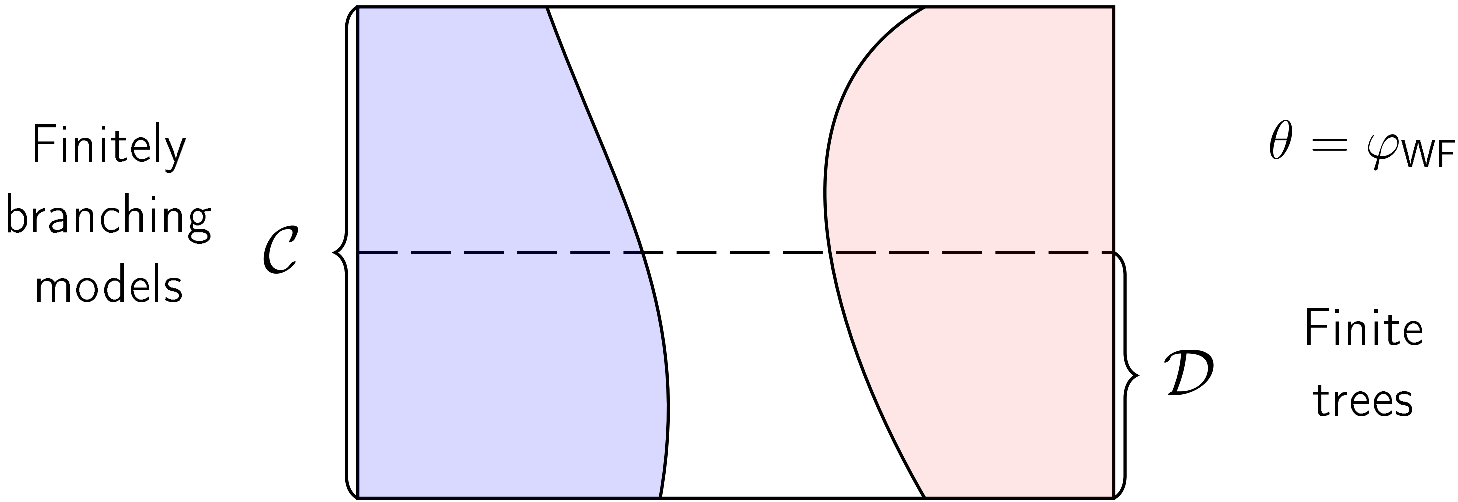

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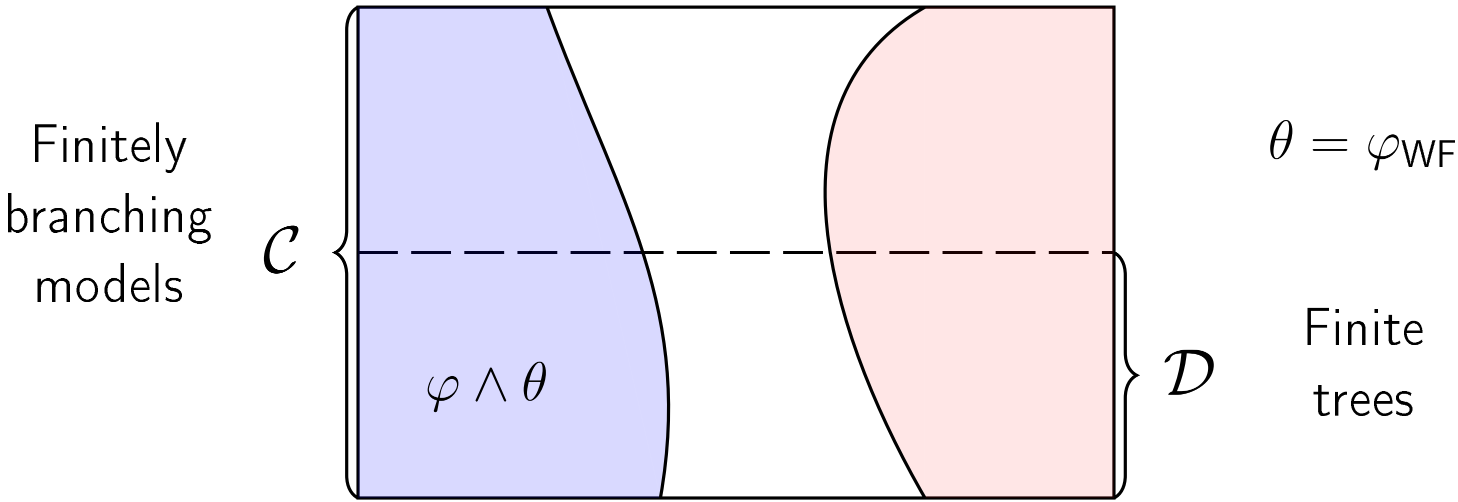

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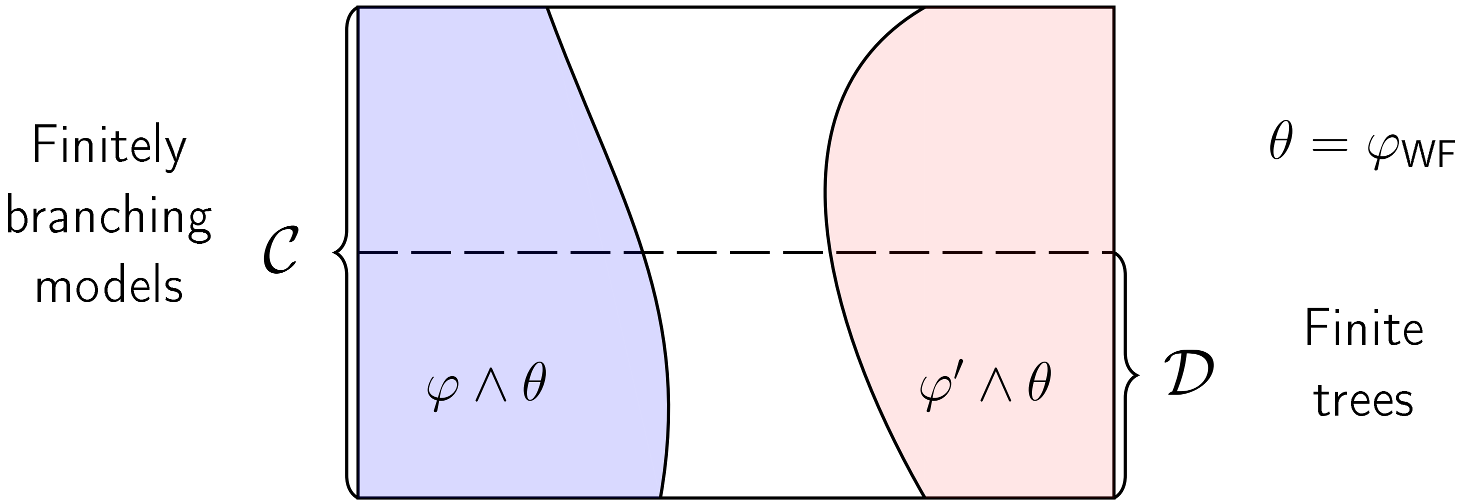

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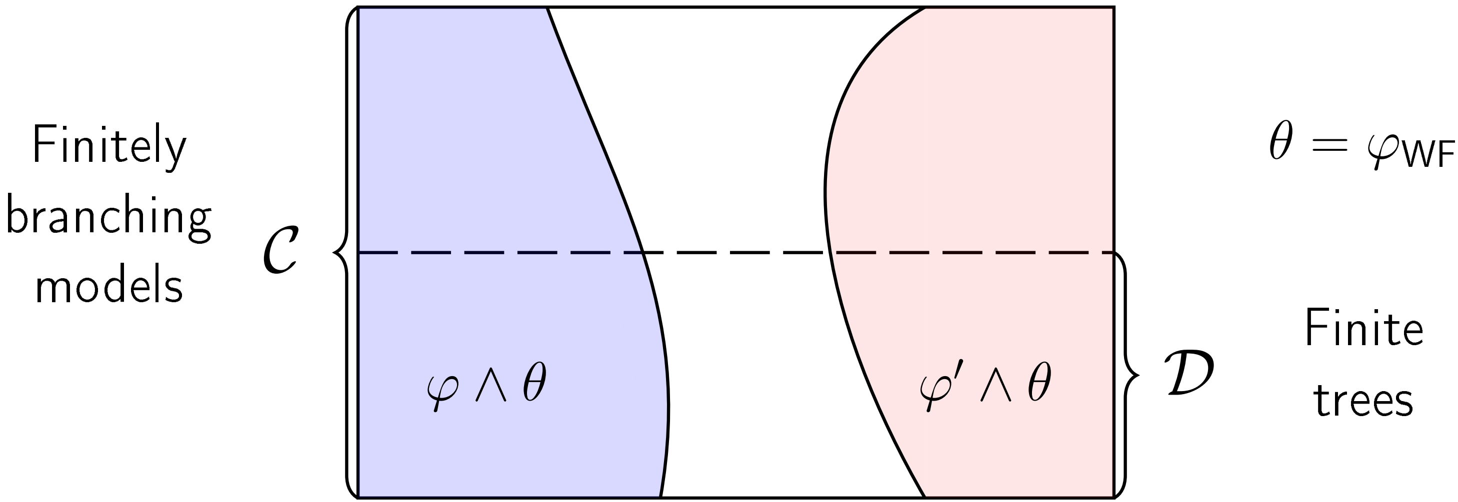

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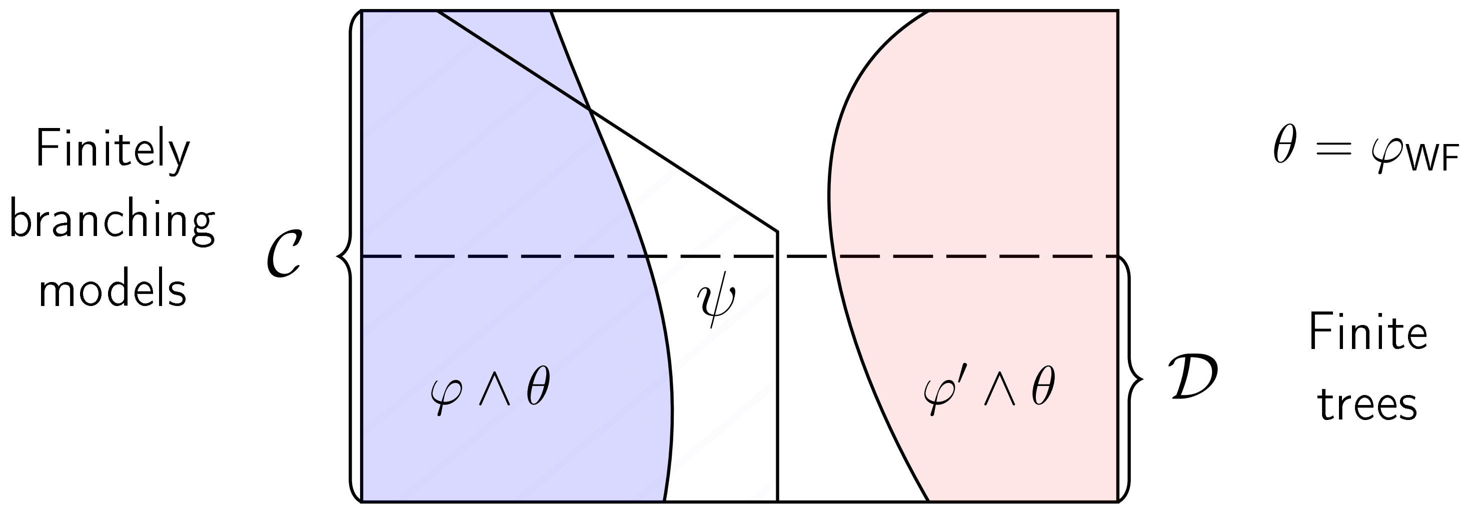

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Thank you!