Bisimulational Categoricity

Jędrzej Kołodziejski

Highlights 2019
Warszawa

we consider modal logic

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$$a \mid \neg\varphi \mid \varphi \vee \psi \mid \Diamond\varphi$$

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atomic propositions

$\exists \rightarrow p$ s.t. $p \models \varphi$

semantics

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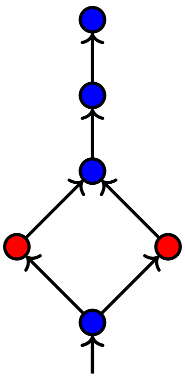

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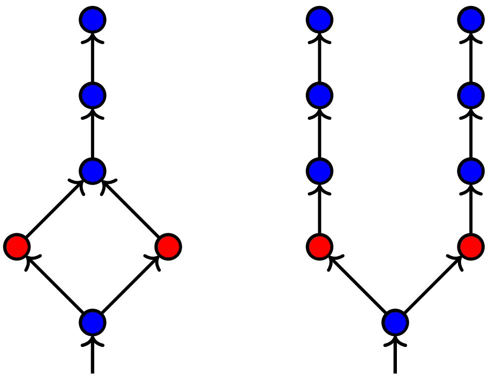

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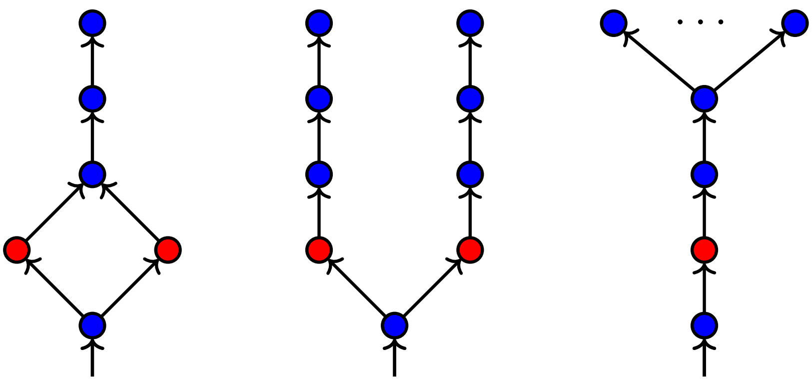

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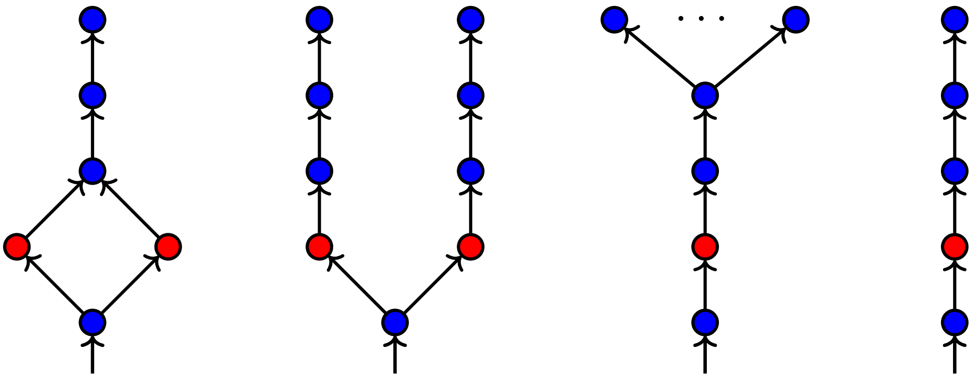

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$$\{\Box^n(\text{blue} \wedge \Diamond\top) \mid n \in \{0, 1, \ldots\}\}$$

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syntactic sugar for $\neg\Diamond\neg$
i.e. "for every son"

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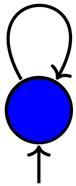

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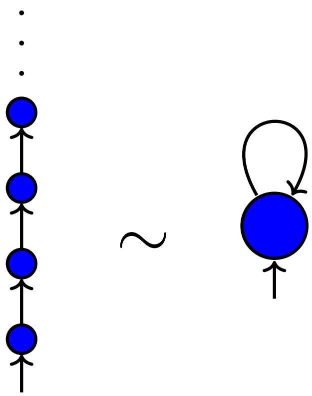

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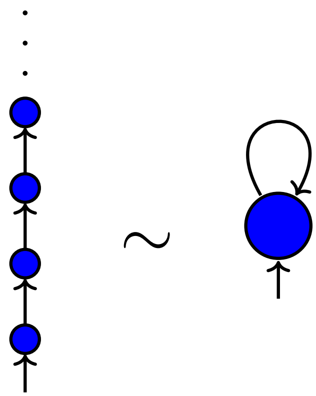

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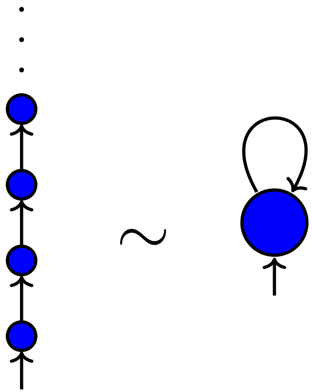

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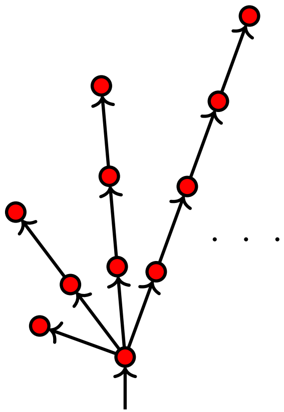

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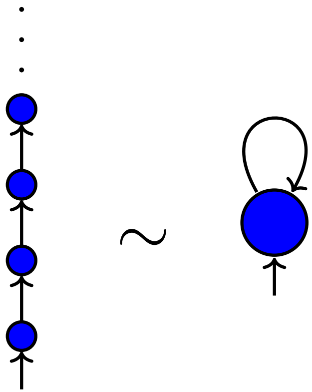

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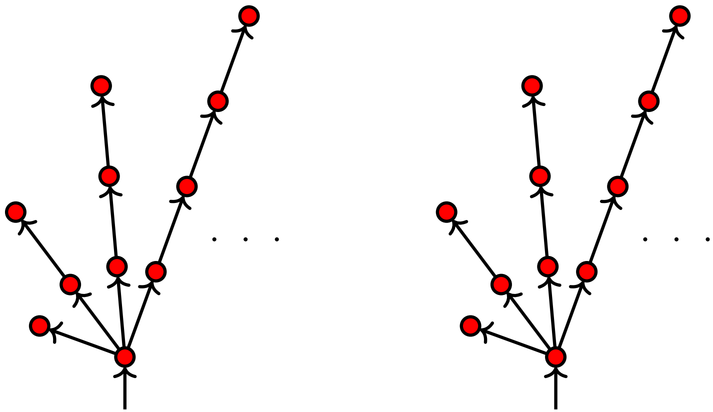

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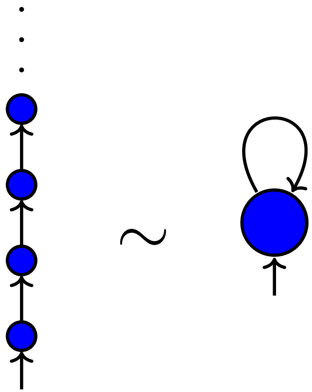

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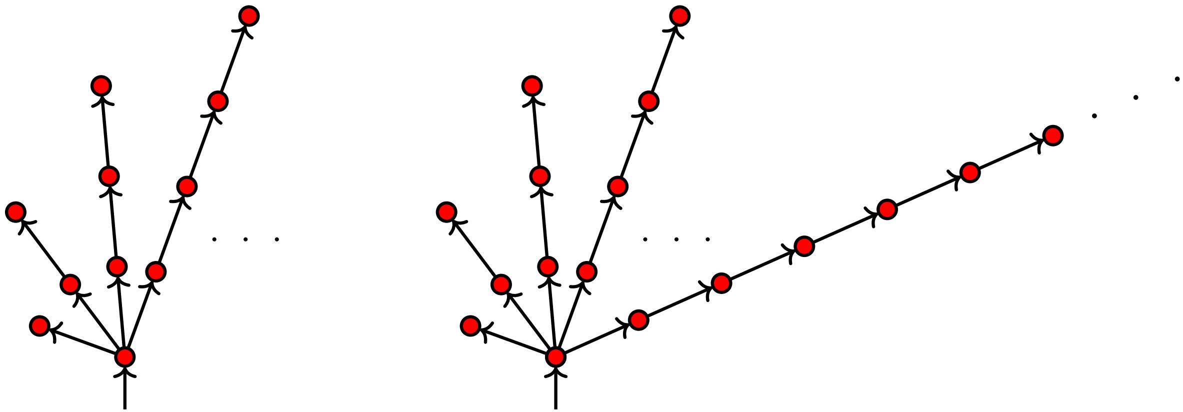

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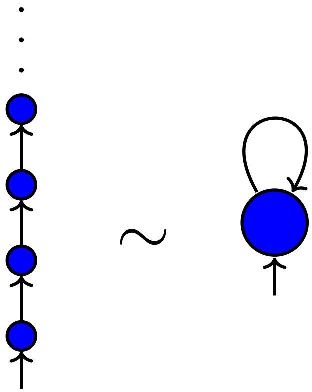

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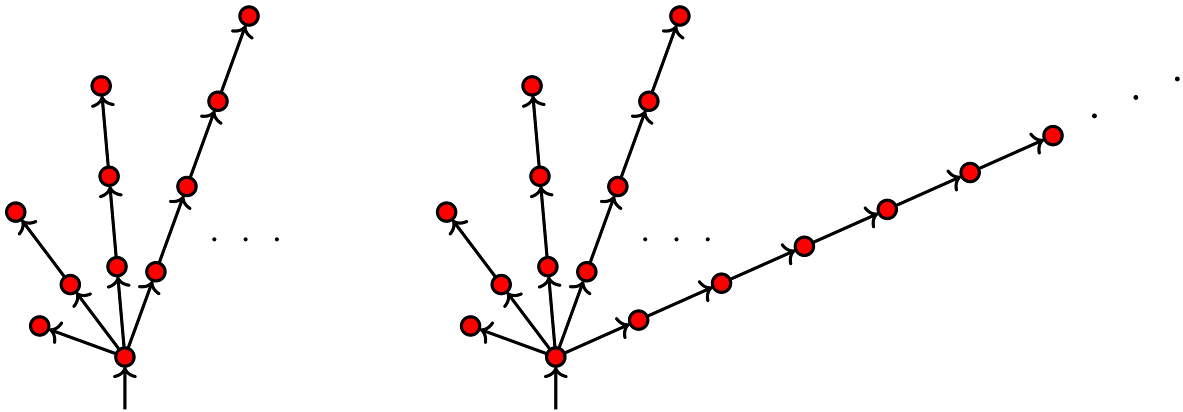

satisfy the same modal formulae, but are not bisimilar!

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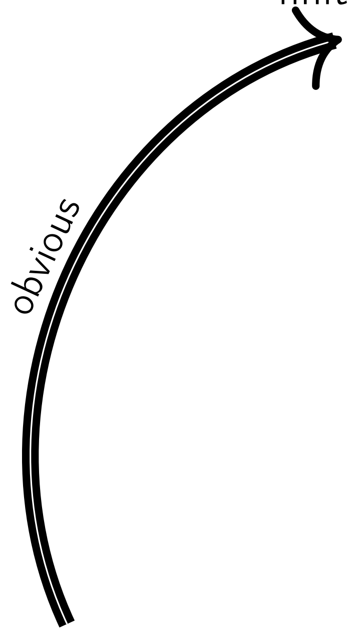

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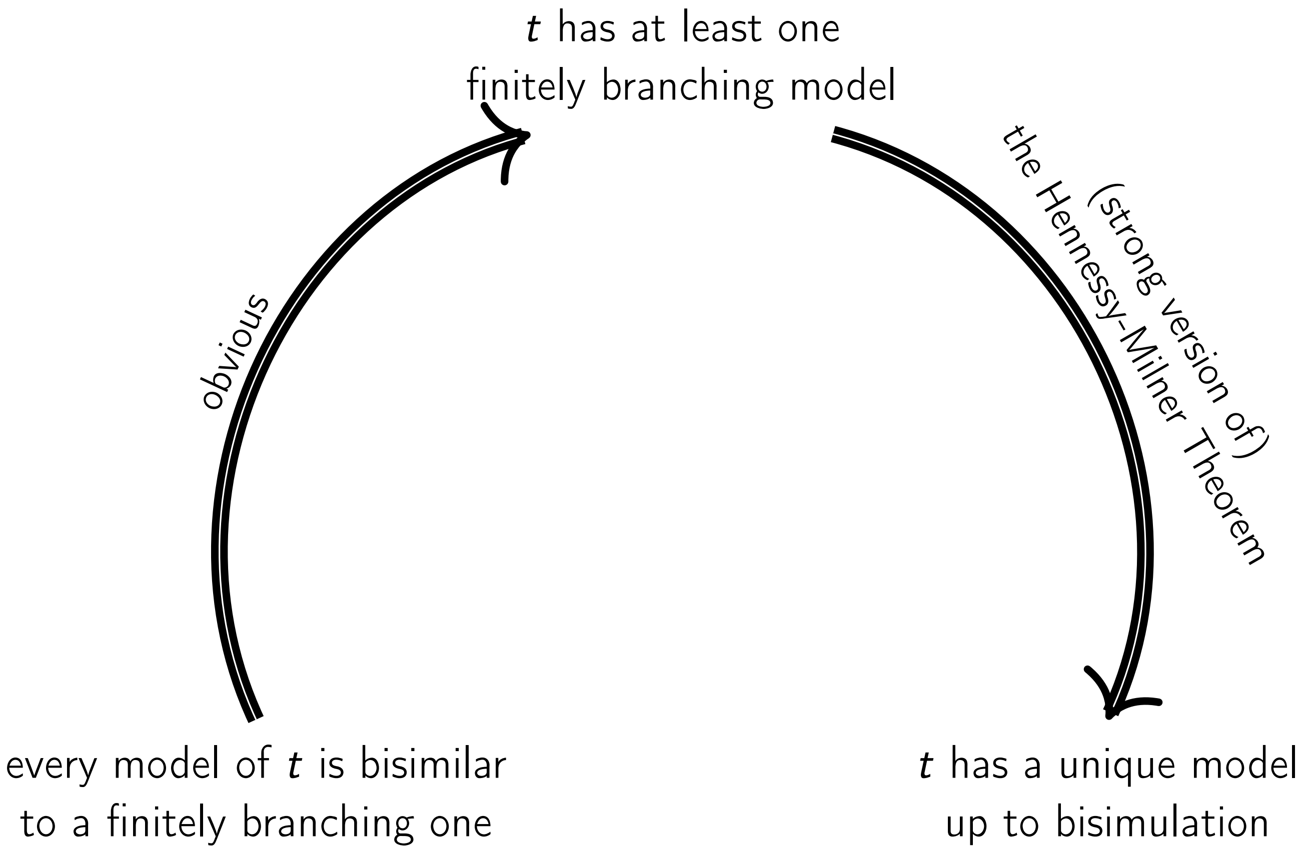

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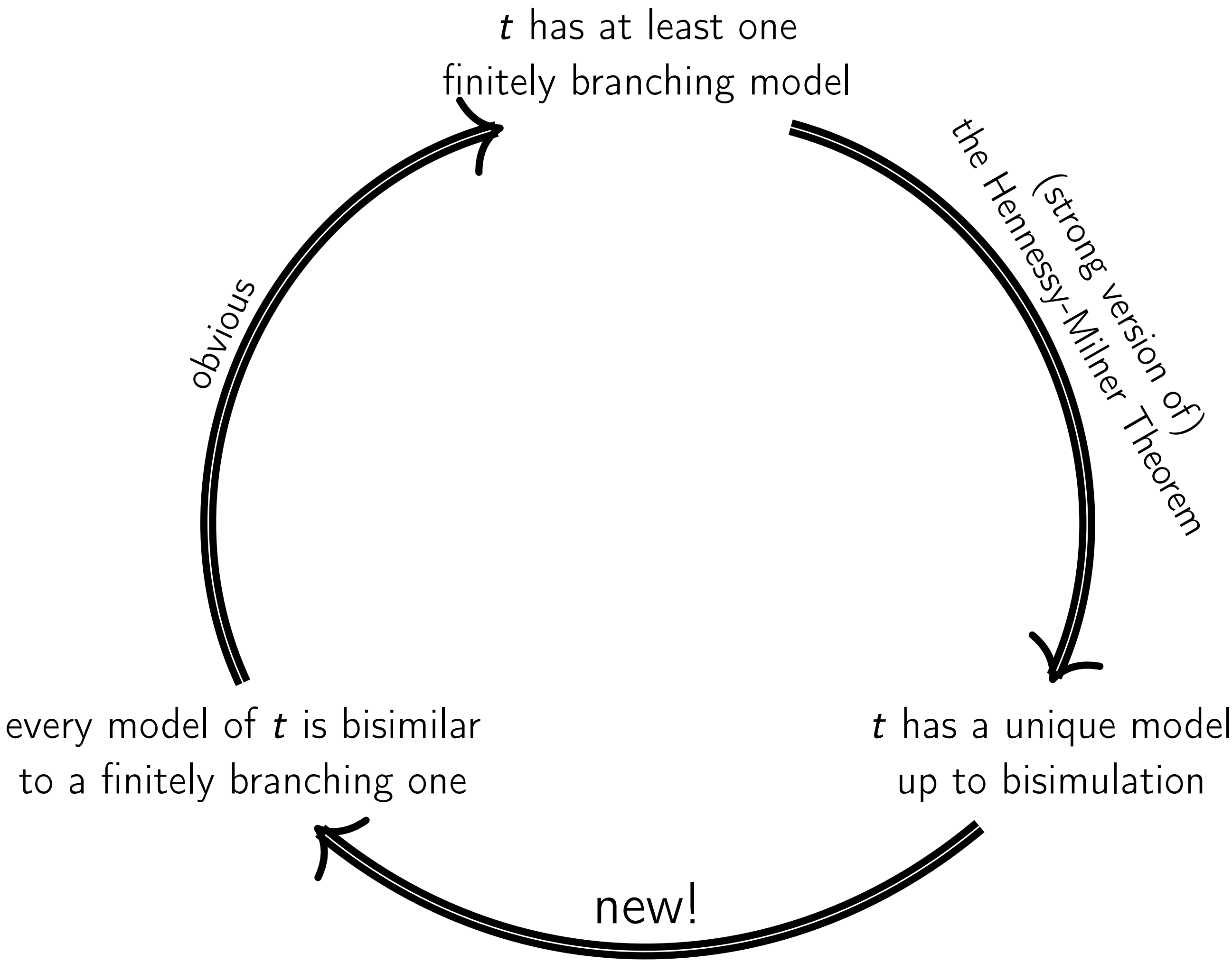

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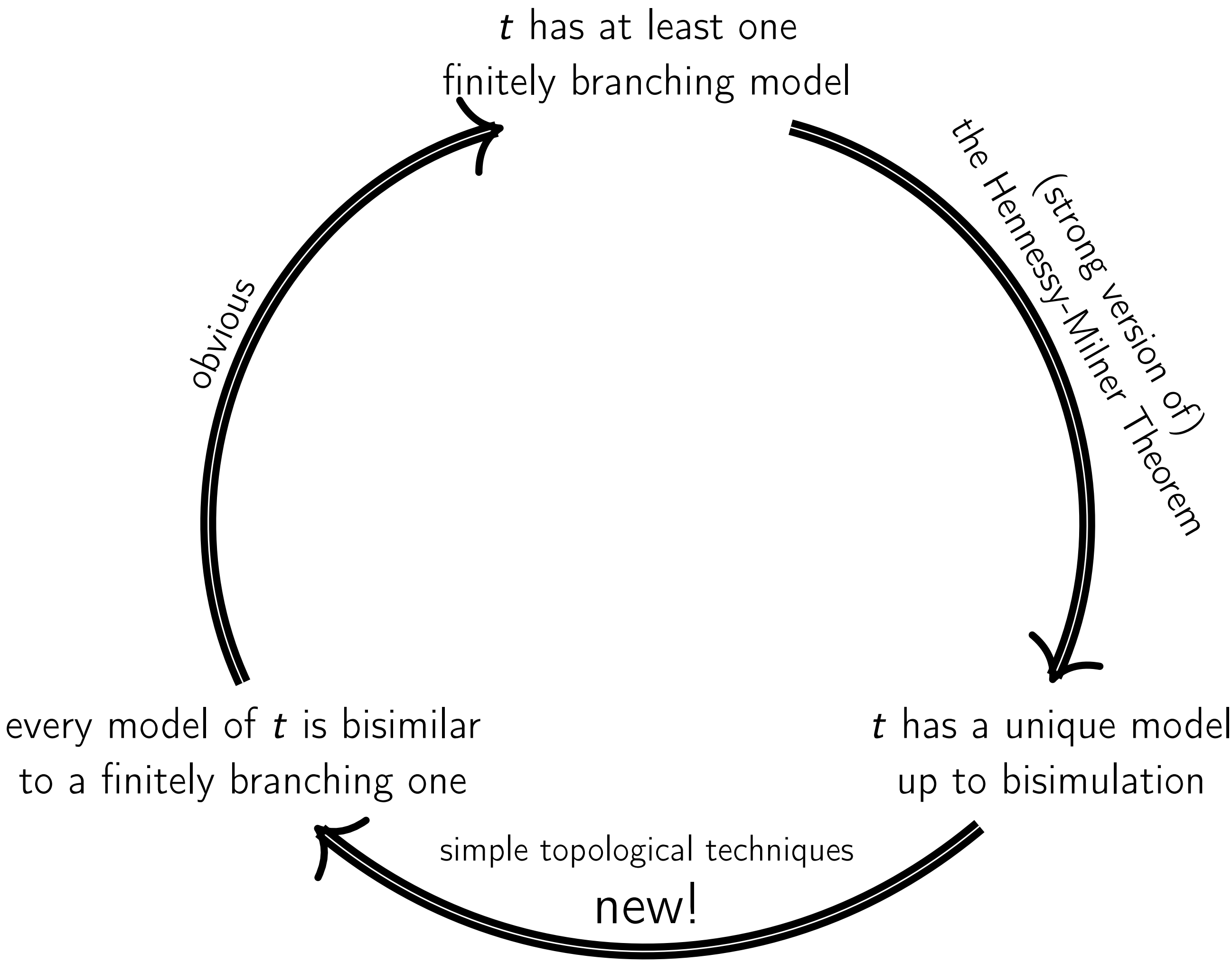

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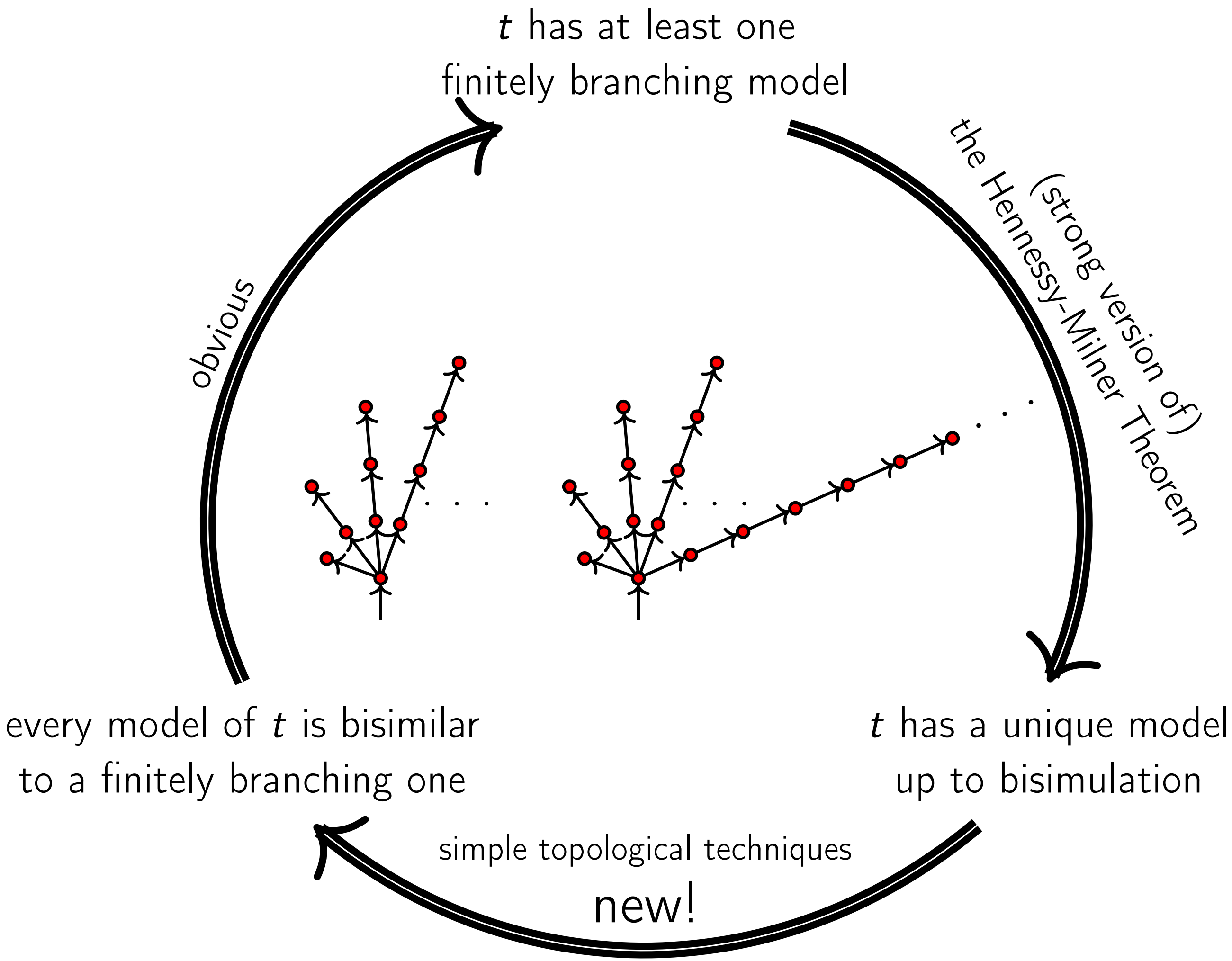

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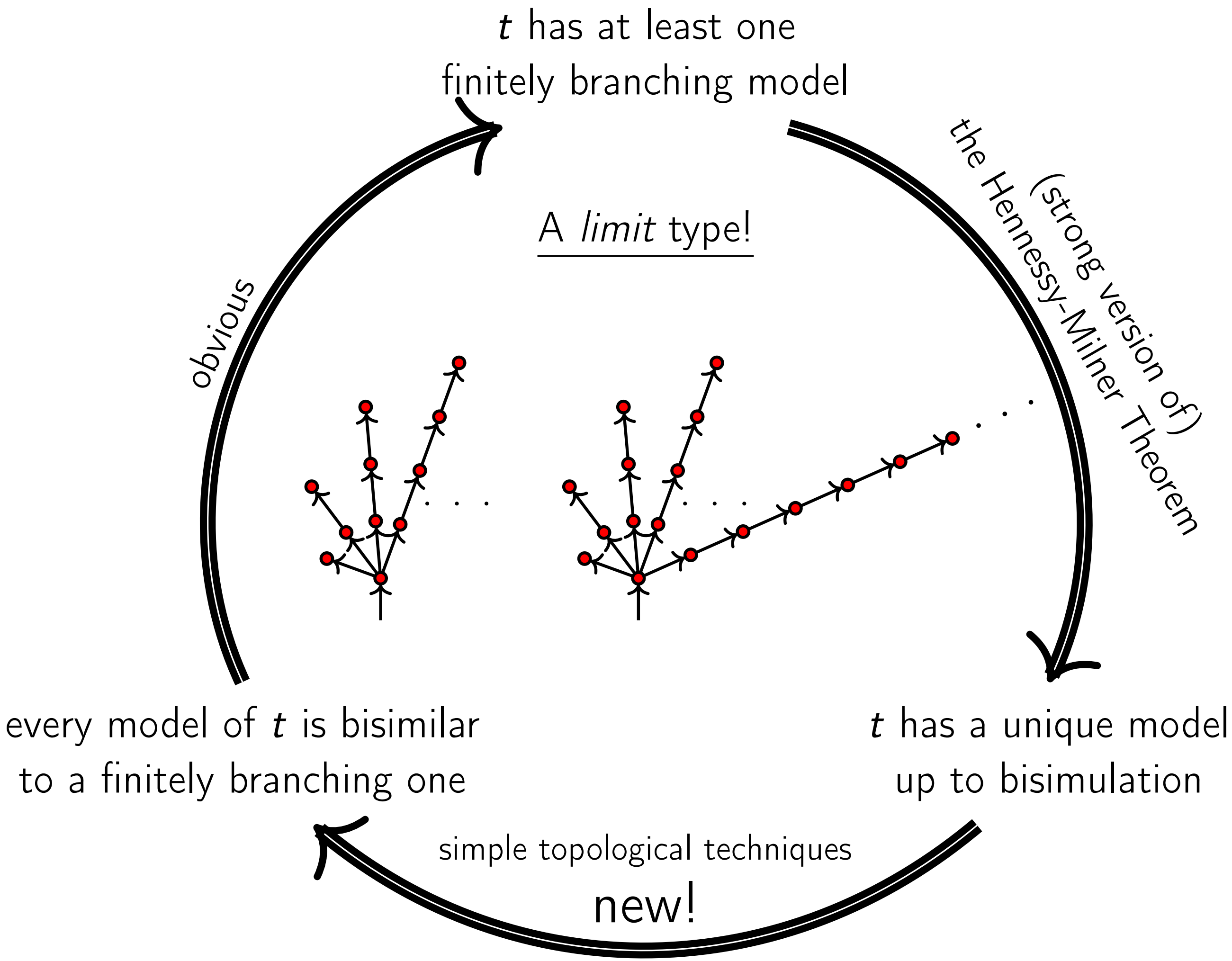

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- modal μ-calculus seems challenging, as not much is known about (infinitary) model theory for MSO