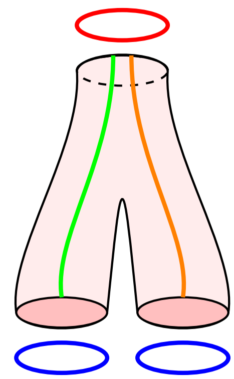
Birational geometry and elliptic cohomology invariants

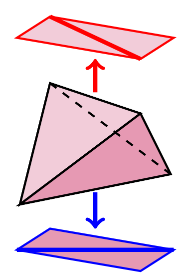
Description for General Public

The picture presents a popular illustration of Marston Morse theory: a pair of pants representing *topological cobordism* which links a circle at the top with a pair of circles at the bottom. Two drops of water starting nearby and flowing down the trousers may eventually end in two disjoint circles, a phenomenon which shows that in certain circumstances a small change of parameters may have dramatic consequences.



The items in the upper drawing are *topological manifolds* so from the mathematical standpoint they are very crude objects which make foundations on which much finer structures can be built. The present project is focused on *algebraic manifolds* and *varieties* which bear much richer structure and can describe more complex phenomena.

Algebraic geometry is a central branch of modern mathematics. Its roots are in the classical and differential geometry, analysis, topology, and commutative algebra. The research in algebraic geometry is strongly motivated by its vital applications in other branches of mathematics, theoretical physics, statistics, computational biology, computer science and engineering.



On the other hand, algebraic geometry applies plenty of methods stemming, among others, from topology, analysis and discrete mathematics. In fact, the present project is focused on understanding special algebraic structures by calculating their topological invariants, *characteristic classes* and using methods modeled on those from topology.

Geometric operations, far-reaching generalizations of the topological cobordism can be organized into an algebraic structure. The atoms (*Demazure-Lusztig operations*) are recipes how to build more complicated higher dimensional manifolds. Another modification is induced by a *torus action*. Whole algebra of operations is an incarnation of so-called *Hecke algebra*. This kind of algebra can serve as a tool to compute invariants of geometric objects. We wish to extend its application beyond the realm of well known *homogeneous spaces* and apply it in new territories, to compute elliptic characteristic of *nilpotent orbits*. Relations between various compositions of Demazure-Lusztig operations will be given by algebraic cobordisms. In this way the inaccessible high dimensional geometric objects can be ordered in a hierarchy governed by combinatorics. The algebraic methods allow to represent the objects in a relatively easy way.

For example, the lower part of the included picture represents Morelli–Włodarczyk *algebraic cobordism* the algebraic counterpart to the pair of trousers. The tetrahedron in the middle stands for 4-dimensional space while the upper and lower rhombuses to which the tetrahedron is projected represent pieces of three-dimensional algebraic manifolds. They represent two different resolutions of a three dimensional singularity $x_1y_1 + x_2y_2 = 0$ and the modification represented by a different choice of the diagonal in the rhombus is a basic *birational modification* of algebraic manifolds, the Atiyah *flop*.