

Invariants for One-Counter Automata with Disequality Tests



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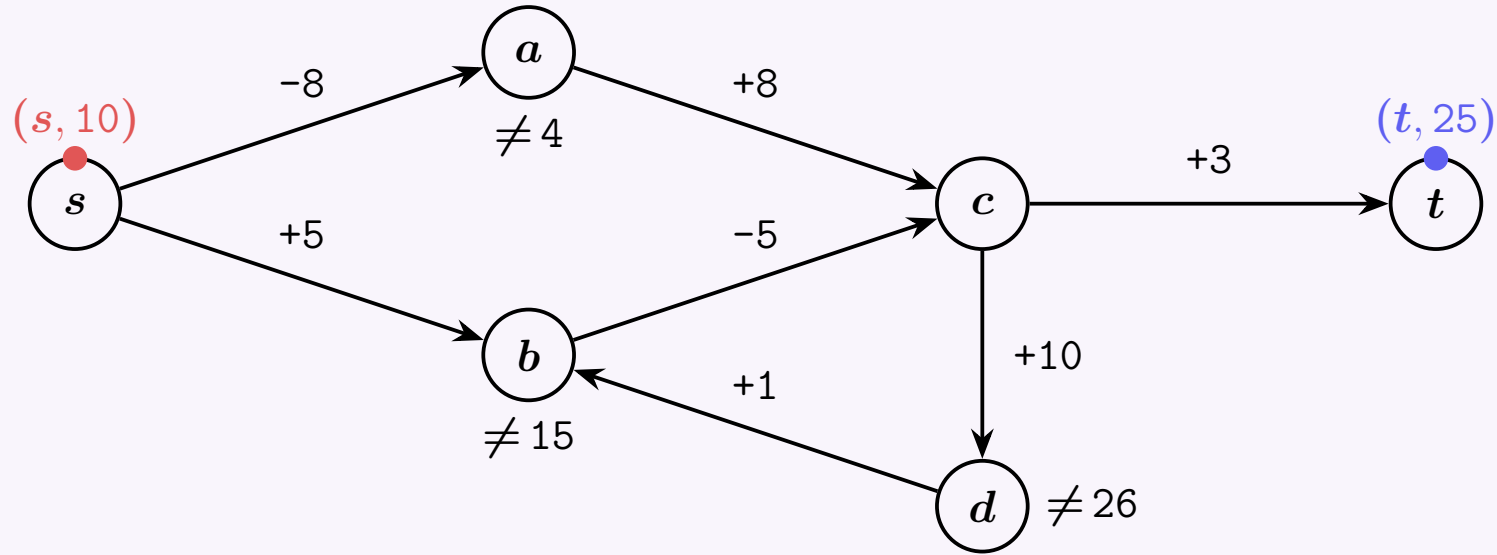
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One-Counter Automata with Disequality Tests



A **configuration** consists of current state and counter value that respects the disequality tests.

A **run** is a sequence of configurations.

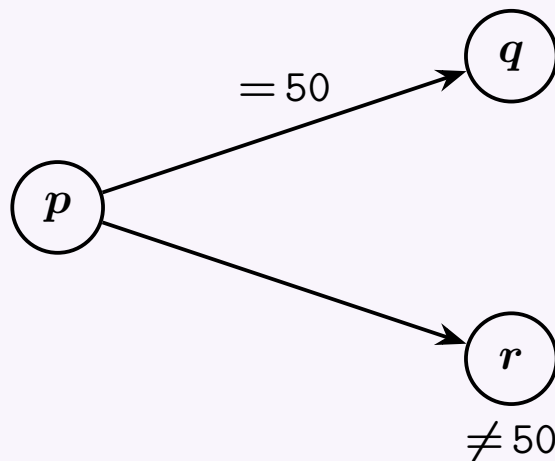
Reachability: is there a run from **the initial configuration** to **the target configuration** ?

Motivation

Original motivation. Reachability in one-counter automata with disequality tests can be used for model checking in LTL extended with *flat freeze operators*. [Demri and Sangnier '10]

[Lechner, Mayr, Ouaknine, Pouly, and Worrell '18]

Standalone motivation. Equality and disequality tests allow for *if-then-else* conditionals.



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Theorem. Reachability in one-counter automata (with equality tests) is NP-complete.

[Haase, Kreutzer, Ouaknine, and Worrell '09]

Simulating disequality tests with equality tests is inefficient.

Theorem. Reachability in one-counter automata with $\leq$ tests is PSPACE-complete.

[Fearnley and Jurdziński '13]

Our Main Contribution and Prior Work

Theorem. Reachability in one-counter automata (with equality tests) is NP-complete

[Haase, Kreutzer, Ouaknine, and Worrell '09]

Theorem. Coverability (and boundedness) in one-counter automata with disequality tests is in P.

[Almagor, Cohen, Pérez, Shirmohammadi, and Worrell '20]

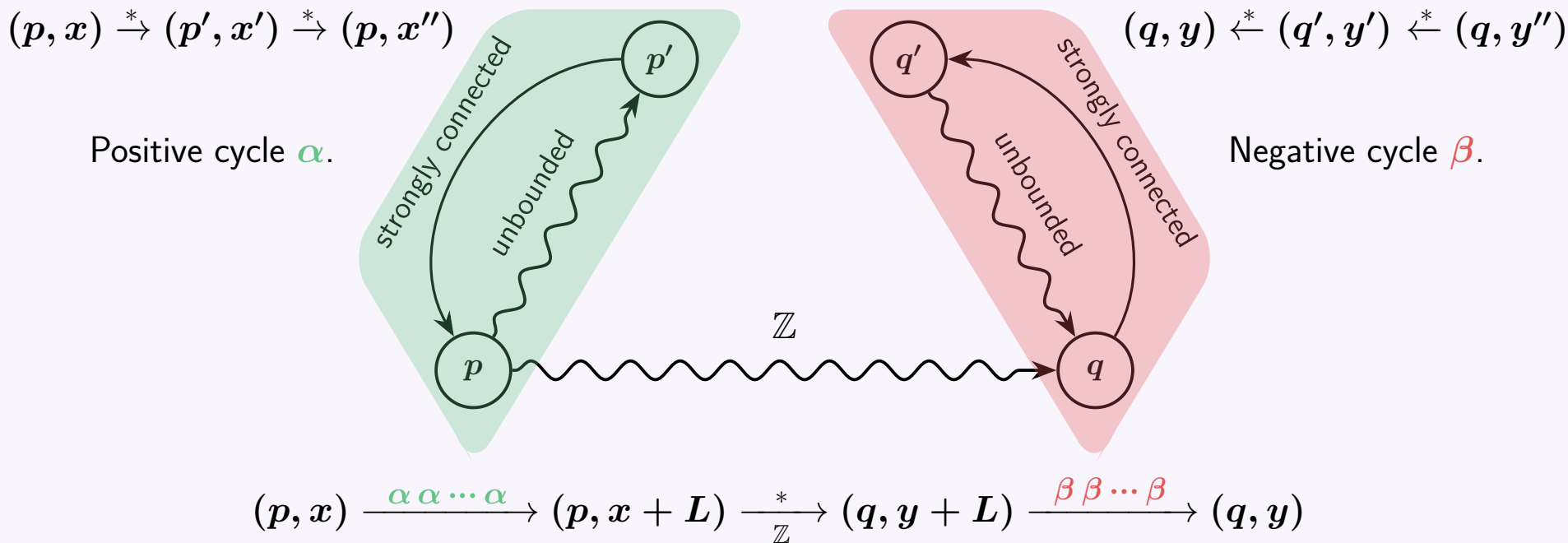
*“The complexity of reaching a given configuration in this model is open,
lying between NP and PSPACE”*

Theorem. Reachability in one-counter automata with disequality tests is in coNP^{NP} . [This paper]

Reachability in Strongly Connected OCA with Disequality Tests

Case 1: (p, x) is unbounded and (q, y) is “reverse unbounded”.

Claim: There is a run from (p, x) to (q, y) if and only if there is a $\mathbb{Z}$ -run from (p, x) to (q, y) .



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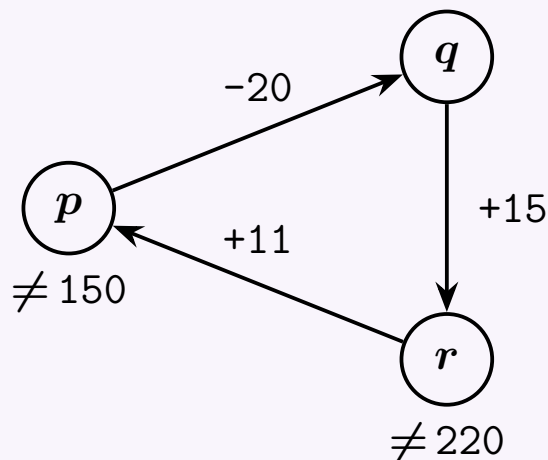
$\implies$ Case 1 is in NP.

Case 2: (p, x) is bounded (symmetric to (q, y) is “reverse bounded”).

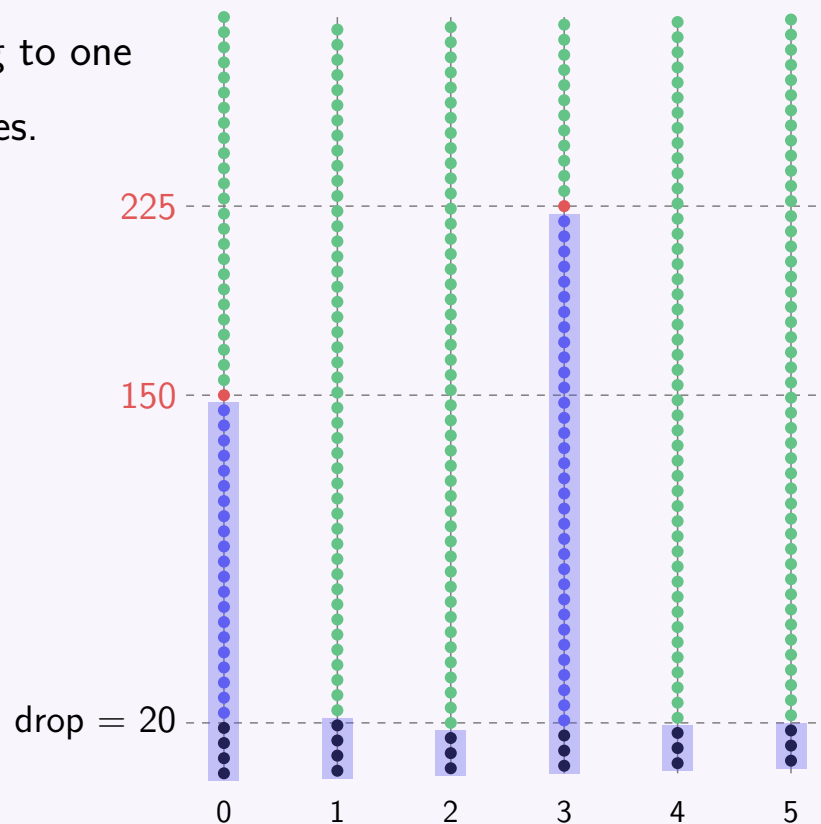
Mission for the remainder of this presentation: Case 2 is in coNP^{NP} .

Setting up an Invariant for Reachability

For each state, pick a cycle* with effect $e > 0$. In the example, for state p , $e = 6$.



Configurations belong to one of $e = 6$ many classes.



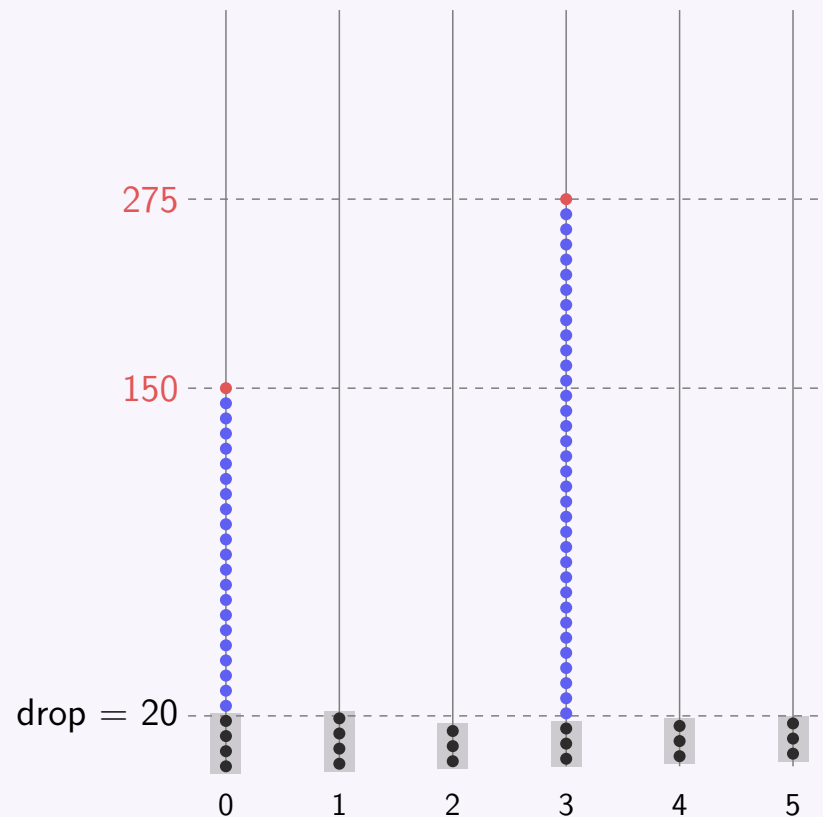
Key:

- Black = cannot afford cycle.
- Red = violate disequality test.
- Green = unbounded.
- Blue = bounded (by a disequality test).

Reachability Properties Below and in Chains

Reachability under the drop of all cycles can be checked in NP.

"Pessimistic Reachability"



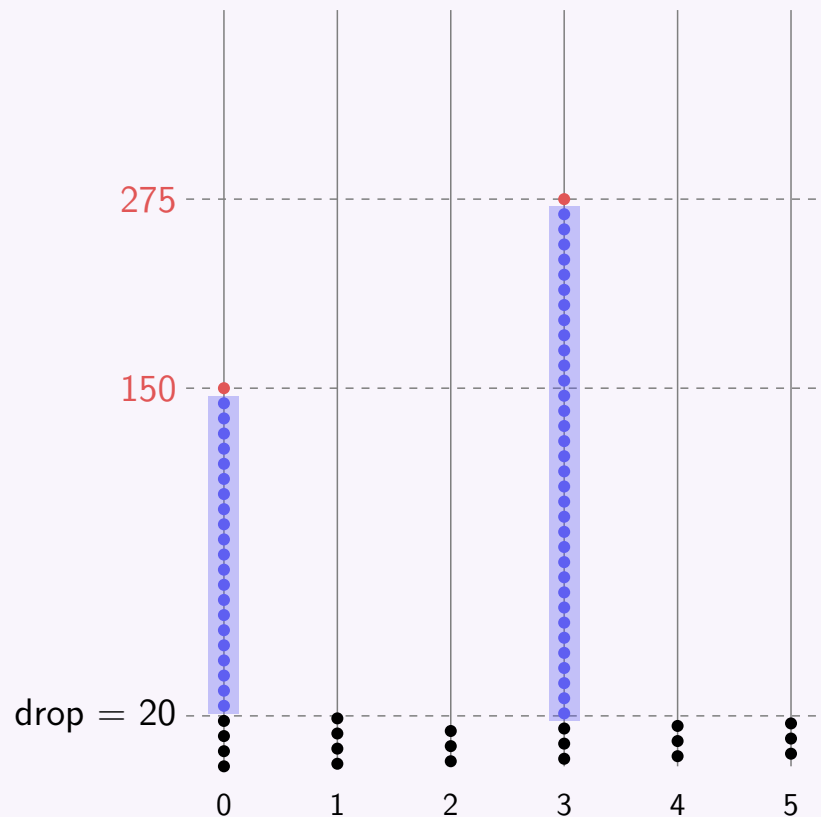
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“Pessimistic Reachability”

Bounded chains are upwards closed.

Let $R = \{ \text{reachable points in bounded chains} \}$.



Invariants that Witness Non-reachability

Theorem. Let $\mathcal{A}$ is a strongly connected one-counter automata with disequality tests.

Let $(p, x), (q, y)$ be two configurations and suppose (p, x) is bounded in $\mathcal{A}$.

Then, there *does not exist* a run from (p, x) to (q, y) in $\mathcal{A}$ if and only if

(Cond1) $(p, x) \in R$,

(Cond2) $(q, y) \notin \text{Post}_*(R)$, and

reachable via configurations under
the drop of all cycles (pessimistic)

(Cond3) $\text{Post}(\text{Post}_*(R)) \cap \{\text{bounded chains}\} \subseteq R$.

Theorem. Non-reachability in strongly connected one-counter automata with disequality tests is in NP^{NP} .

Proof idea. Guess R concisely. Check violation of (Cond1), or (Cond2), or (Cond3) in coNP.

Remember that $\text{NP}^{\text{coNP}} = \text{NP}^{\text{NP}}$.

Corollary. Reachability in strongly connected one-counter automata with disequality tests is in coNP^{NP} .

Invariants for One-Counter Automata with Disequality Tests

Corollary. Reachability in strongly connected one-counter automata with disequality tests is in coNP^{NP} .
[This presentation]

Theorem. Reachability in one-counter automata with disequality tests is in coNP^{NP} . [In the paper]

Theorem. Reachability in one-counter automata with *equality and disequality tests* is in $\text{P}^{\text{NP}^{\text{NP}}}$.
[In the paper]

Thank You!



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