Coq Survival Kit

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Tactics used in Lecture3.v

destruct

on a hypothesis of the form P/\Q

```
H : P /\ Q
=====================
R
```

destruct H →

```
H : P
H0 : Q
=====================
R
```

```
H : P /\ Q
=====================
R
```

destruct H → **as** [H1 H2]

```
H1 : P
H2 : Q
=====================
R
```

destruct

on a hypothesis of the form P\/Q

```
H : P \/ Q
====================
R
```

destruct H

first subgoal:

```
H : P
====================
R
```

second subgoal:

```
H : Q
====================
R
```

destruct

on a hypothesis of the form False

```
H : False
=====================
P
```

destruct H →

no more subgoal

destruct

on a term with an inductive type

```
n : nat
======================
P n
```

destruct n

first subgoal

```
======================
P 0
```

second subgoal

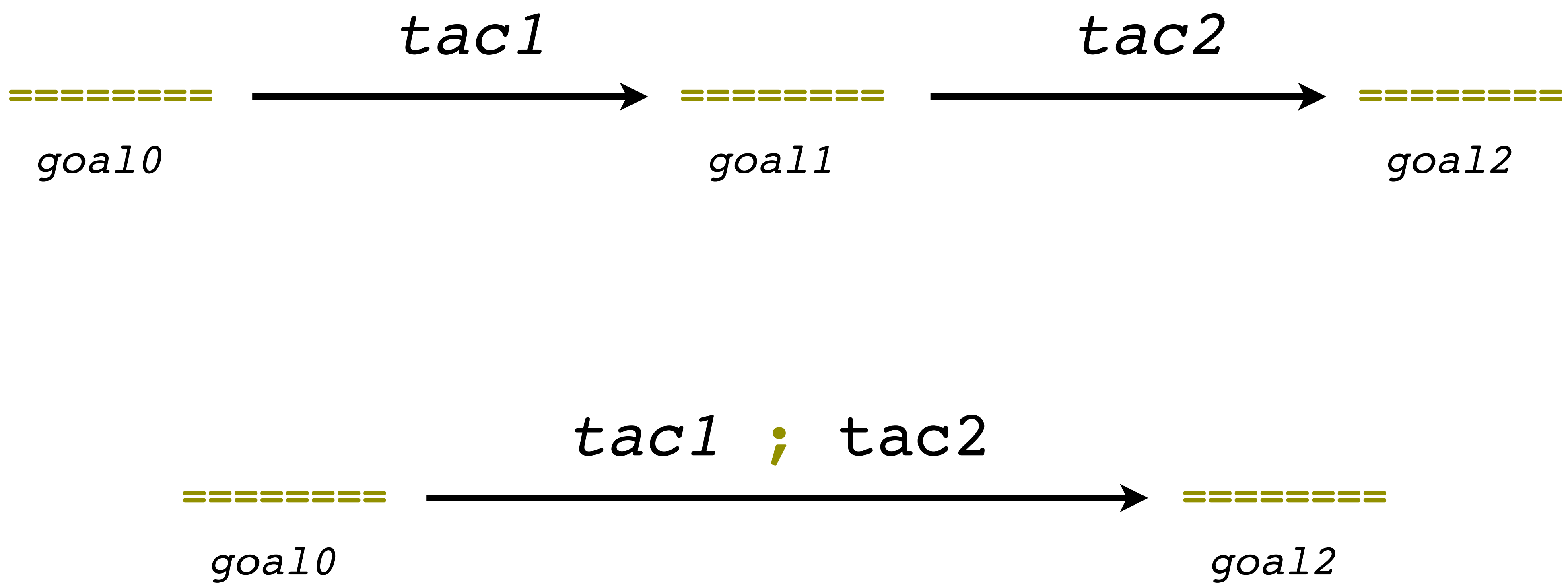
```
m : nat
======================
P (S m)
```

left / right

```
                              left
====================      ---------->      ====================
 P \/ Q                                     P


                              right
====================      ---------->      ====================
 P \/ Q                                     Q
```

tac1 ; tac2



If *tac1* generates several subgoals, *tac2* is applied on each of them.

simpl

see also simpl in *

```
Definition pred (n:nat) :=
  match n with
    | O => O
    | S m => m
  end.
```

```
====================        simpl         ====================
 ... pred 0 ...          ---------->        ... 0 ...

====================        simpl         ====================
 ... pred (S n) ...      ---------->        ... n ...
```

But the behavior of the command is not always that simple ...

intros

```
                                            a : A
=====================    intros --->    =====================
 forall (a : A), P                        P
```

You should think about a term of type **Prop** as a logical property

```
                                            P : Prop
 P : Prop                                   H : P
=====================    intros --->    =====================
 P -> Q                                     Q
```

```
                                            H : P
                                            H0 : Q
=====================    intros --->    =====================
 P -> Q -> R                                R
```

In Coq, we often use the form $P \Rightarrow Q \Rightarrow R$ instead of $P \land Q \Rightarrow R$

intros *names*

```
=====================
 forall (a : A), P
```

intros a →

```
a : A
=====================
 P
```

```
P : Prop
=====================
P -> Q
```

intros H →

```
P : Prop
H : P
=====================
 Q
```

```
=====================
P -> Q -> R
```

intros H H0 →

```
H : P
H0 : Q
=====================
 R
```

```
=====================
not P
```

intros H →

```
H : P
=====================
 False
```

`not P` is a macro for `P -> False`

admit

```
====================
 P
```

admit →

no more subgoal

- solve the current subgoal with an axiom
- *this is cheating!*

congruence

It solves automatically a subgoal using only the following deduction rules

$$\frac{}{x = x} \qquad \frac{x = y \quad (P\ x)}{(P\ y)} \qquad \frac{}{(C\ x) \neq (C'\ y)} \qquad \frac{(C\ x) = (C\ y)}{x = y}$$

where C and C' are constructors

Examples

```
n : nat
H : S n = S m
=====================
plus n p = plus m p
```

congruence → no more subgoal

```
n : nat
H : S n = O
=====================
False
```

congruence → no more subgoal

induction

on a term with an inductive type

```
n : nat
=====================
P n
```

induction n

first subgoal

```
=====================
P 0
```

second subgoal

```
m : nat
IHm : P m
=====================
P (S m)
```

assert

```
=====================
  Q
```

assert (H: P)

first subgoal

```
=====================
  P
```

second subgoal

```
  H : P
=====================
  Q
```

rewrite

```
H : a = b                         rewrite H         H : a = b
=====================           ------------>       =====================
 ... a ...                                           ... b ...
```

```
H : forall x y, f x y = x         rewrite H         H : forall x y, f x y = x
==========================      ------------>       ==========================
 ... (f a b) ...                                     ... a ...
```

Coq guesses how to instantiate the quantifiers

```
H : a = b                         rewrite <- H      H : a = b
=====================           ------------>       =====================
 ... b ...                                           ... a ...
```

omega

(do a `Require Import ZArith` before using it)

It solves automatically a subgoal using only arithmetic reasoning on `nat` and `Z`.
Beware, this is only for *linear arithmetic*: multiplication is only understood if one of the arguments is a numerical constant.

Examples

```
H : x <= y + 1
H0 : 2 * y <= z - 3
=====================
2 * x +1 <= z
```

omega → no more subgoal

```
===========================
 x + (y + z) = (x + y) + z
```

omega → no more subgoal

apply

```
H : P -> Q                          H : P -> Q
=====================   apply H    =====================
 Q                      ------->     P
```

```
H : forall x y, P y -> Q x y                H : forall x y, P y -> Q x y
=============================   apply H    =============================
 Q a (f a)                      ------->     P (f a)
```

Coq guesses how to instantiate the quantifiers

```
H : forall x y, P x y -> Q y                H : forall x y, P x y -> Q y
=============================   apply H    =============================
 Q (f a)                        ------->     P a (f a)
                                with a
```

We have to help Coq and give him the missing instantiation

Other useful tactics

assumption

```
H1 : P
H2 : Q
H3 : R
====================
P
```

P appears in the current hypotheses

assumption →

no more subgoal

unfold

replace a name by its definition

```
Definition succ (n:nat) :=  S n.
```

unfold succ

```
=====================        =====================
 ... succ x ...      ---->     ... S x ...
```

unfold not

```
=====================        =====================
 not P               ---->     P -> False
```

split

```
=====================                  =====================
  P /\ Q          --split-->            P
                  first subgoal
                                       =====================
                  second subgoal        Q
```

```
=====================                  =====================
  P <-> Q         --split-->            P -> Q
                  first subgoal
                                       =====================
                  second subgoal        Q -> P
```

exists

```
================
 exists x, P x
```

```
exists t
───────→
```

```
===================
 P t
```

`inv`

can be loaded with the library MSMLib given

or by inserting `Ltac inv H := inversion H; clear H; try subst.`

```
Inductive le (n : nat) : nat -> Prop :=
  | le_n :
  (* ==== *)
    le n n
  | le_S m
    (Hle: n <= m):
  (* ==== *)
    le n (S m).
```

```
H: le n O
====================
P n
```

inv H →

```
====================
P O
```

```
H: le n m
====================
Q n m
```

inv H

first subgoal →

```
====================
Q m m
```

second subgoal →

```
m' : nat
H' : le n m'
====================
Q n (S m')
```