1 Tree automata

What is a Tree Automaton?
Decision Problems

2 Logic

Logic for Words
Logic for Trees
Transitive Closure Logic

3 Temporal Logics

Temporal Logic for Words
Temporal Logic for Trees
XPath

4 Tree-Walking Automata, 1

Tree-Walking Automata
Expressive Power
Pebble Automata

5 Tree-Walking Automata, 2

Tree-Walking Automata Cannot Be Determinized

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Plan

Tree-Walking Automata

definition
some examples
problems

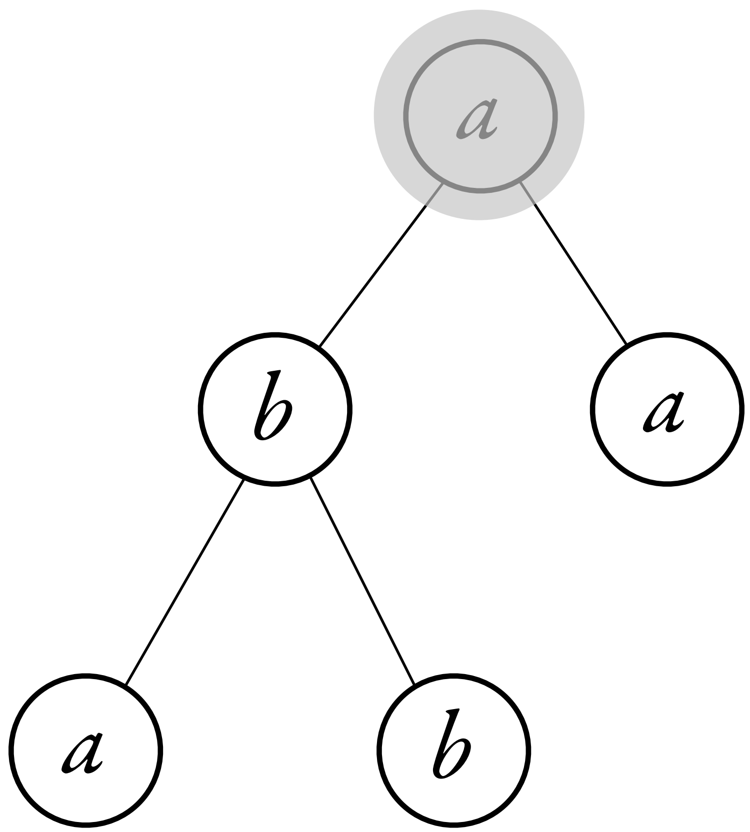
Expressive Power

comparison with tree automata
complexity
determinization

Pebble Automata

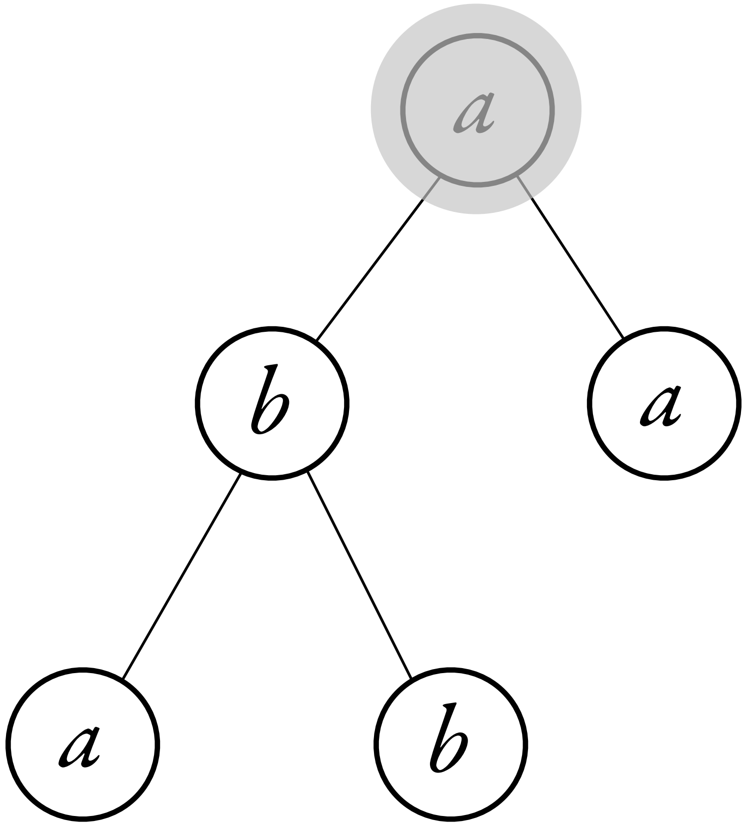
definition
stack discipline
transitive closure logic

Trees are finite, binary and labeled



Trees are finite, binary and labeled

A tree-walking automaton is sequential and two-way.



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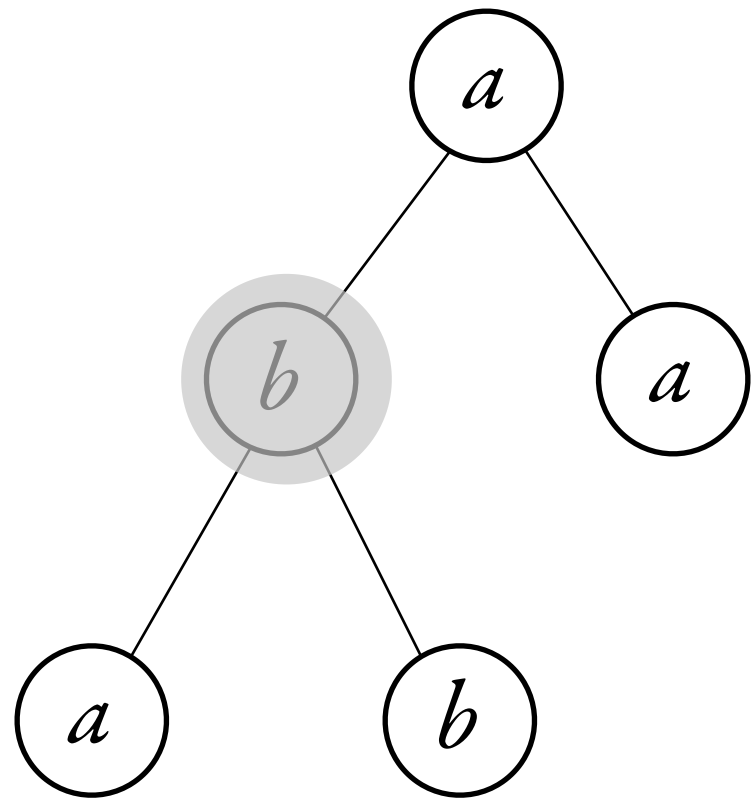

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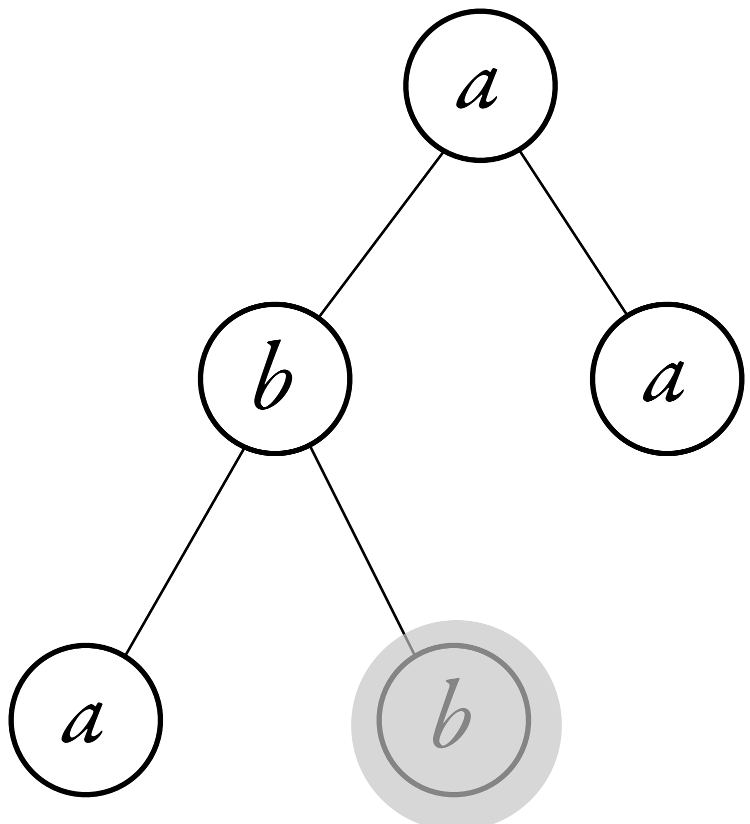

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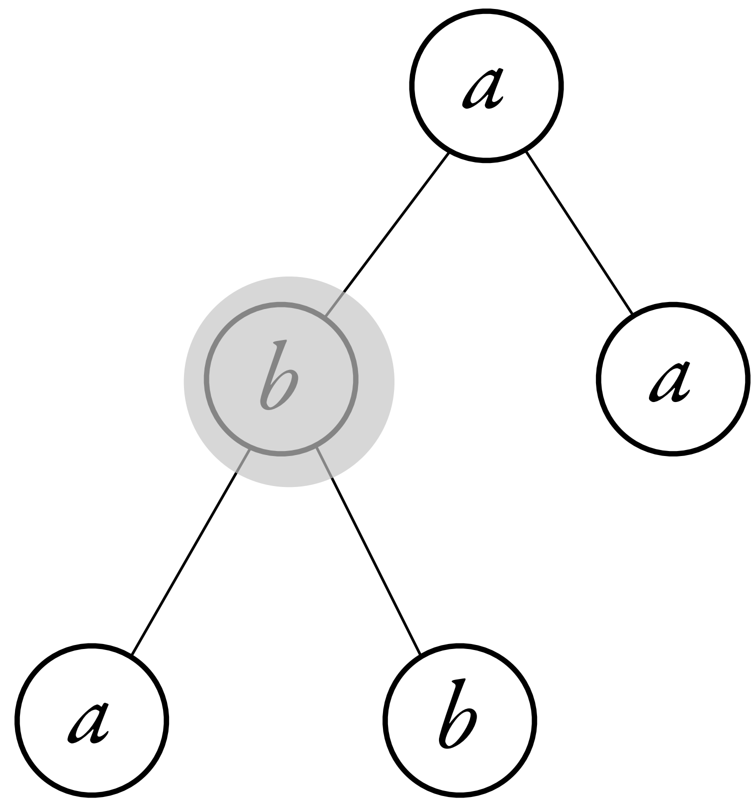

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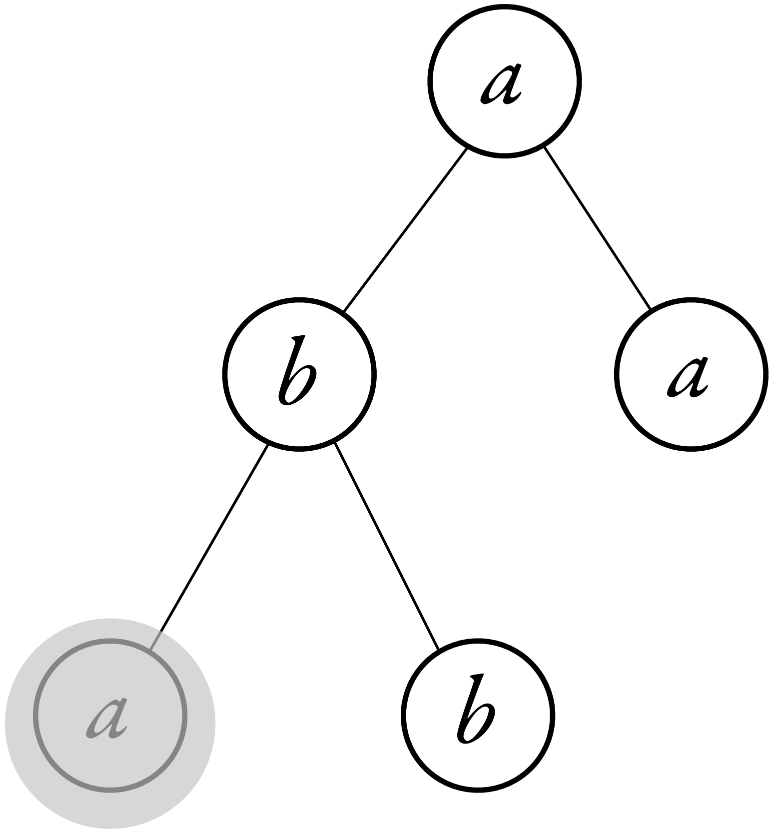
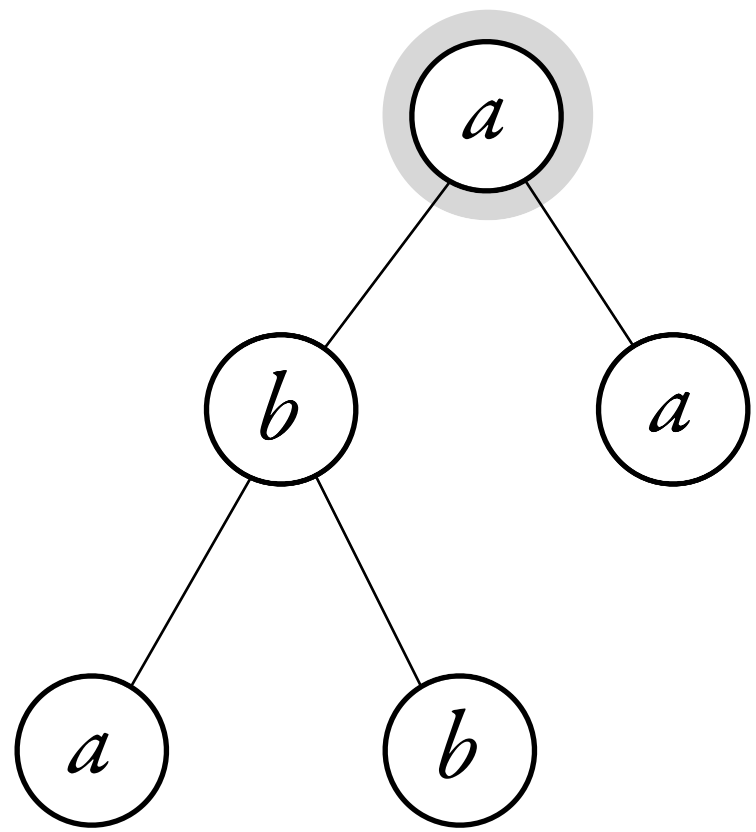


If the state is p and the node is the root with label a, then move to the left child and change state to q.



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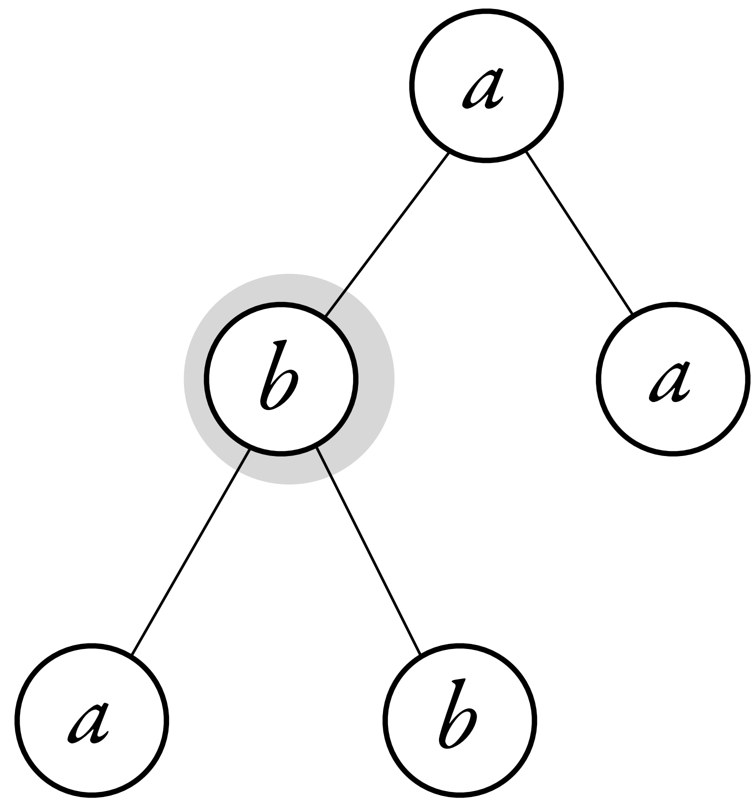
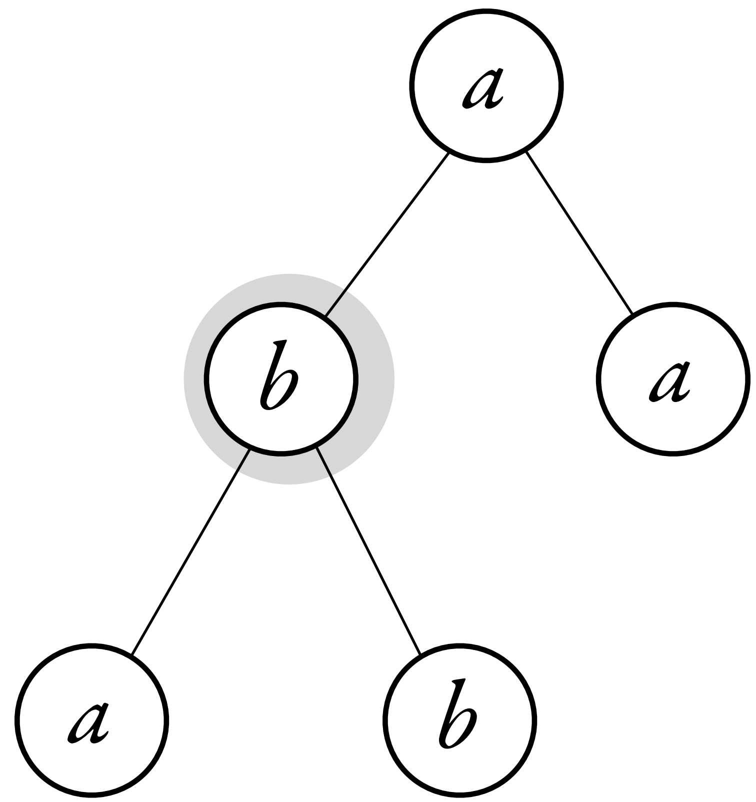


test

If the state is p and the node is the root with label a, then move to the left child and change state to q.

command



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If the state is p and the node is the root with label a,
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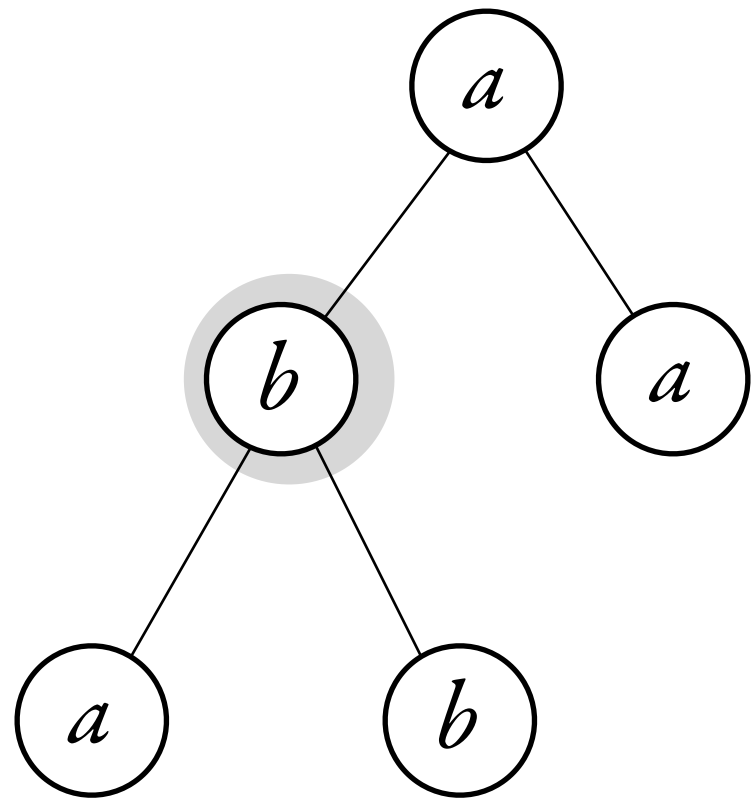
Tests are boolean combinations of:
has label a,
is right/left child,
is leaf



Commands are:
move left/right/up,
accept,
reject

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If the state is p and the node is the root with label a,
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Tests are boolean combinations of:
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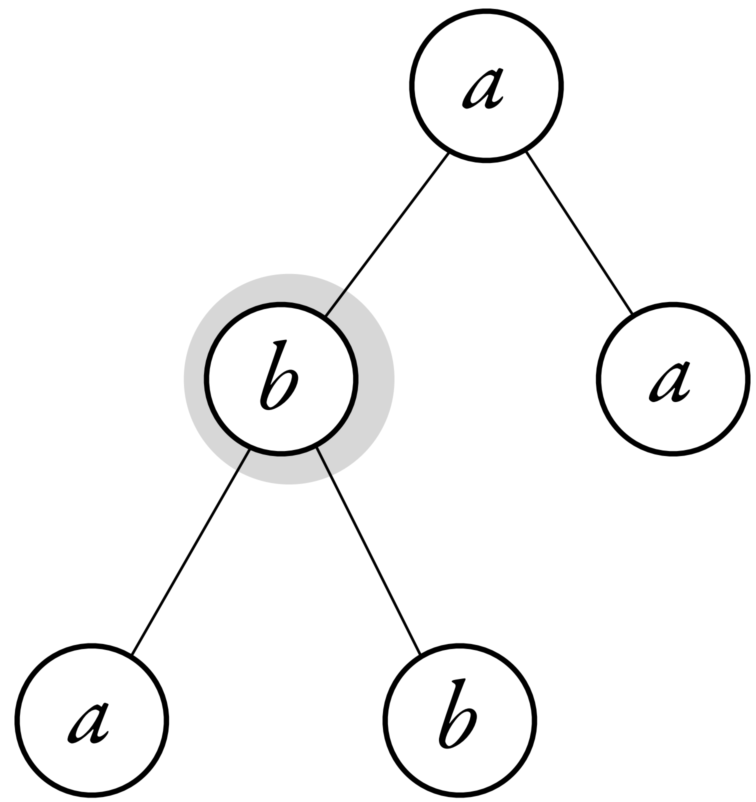

Commands are:
move left/right/up,
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Def. A tree walking-automaton is a tuple $\langle Q, q_I, \Sigma, \delta \rangle$

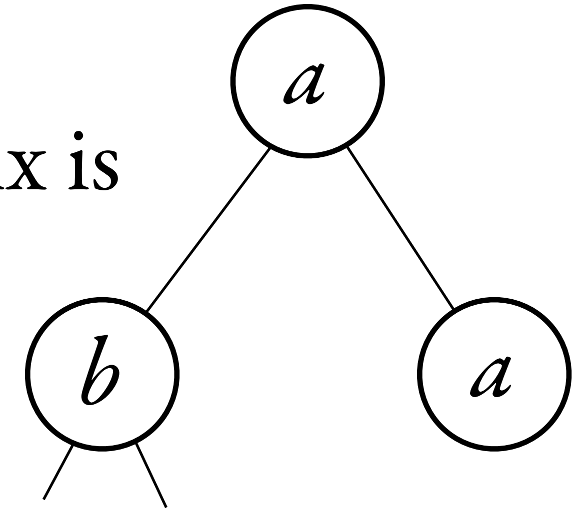
states

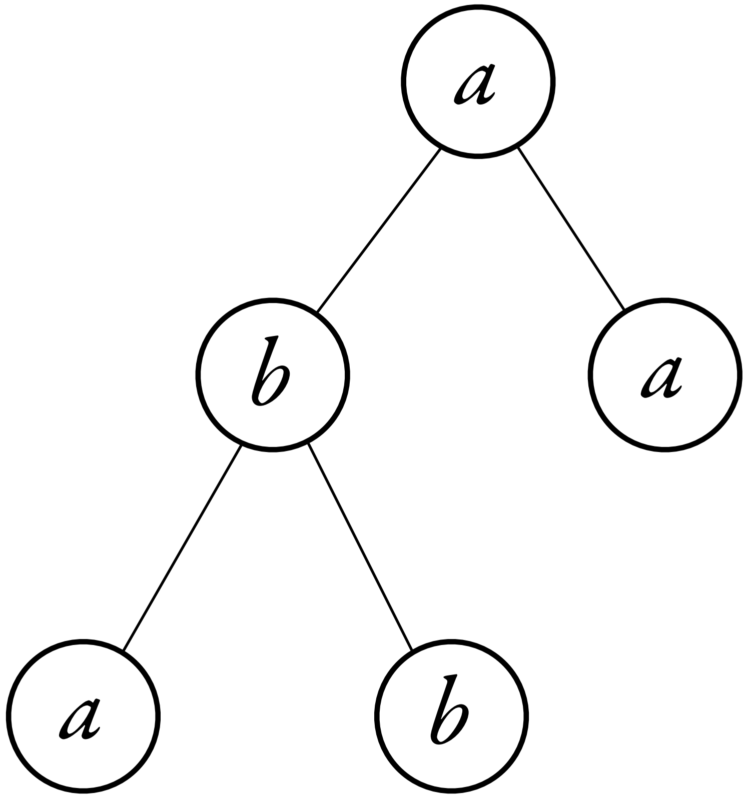
initial state

alphabet

transitions

Example.

Check if the prefix is 



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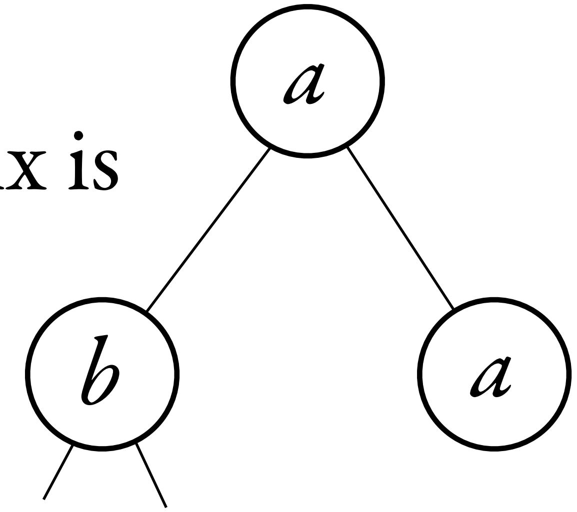

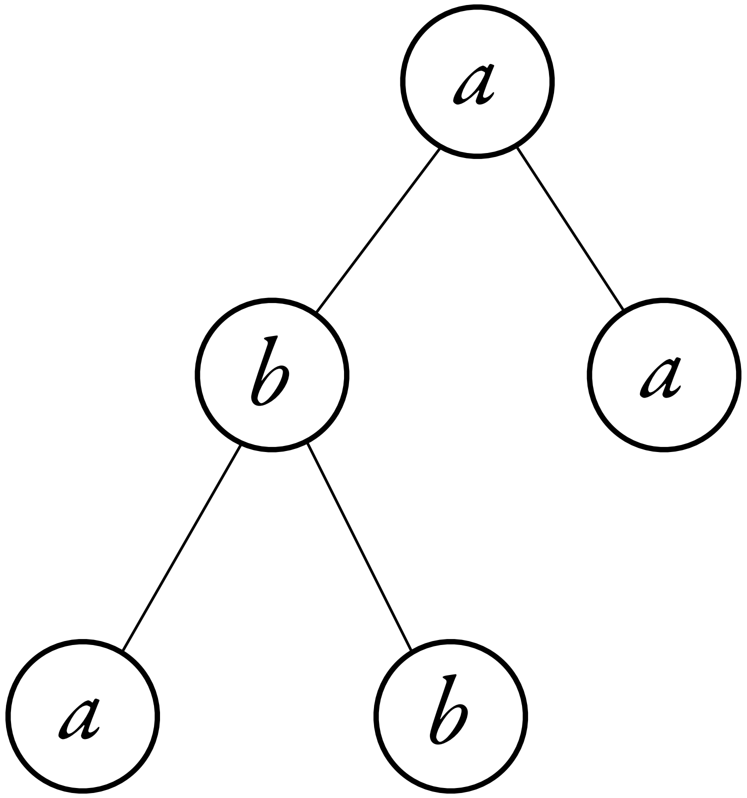

In state p, label a and root, move left, change state to q
In state q, label b and not leaf, move up, change state to r
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In state s, label a and leaf, accept.

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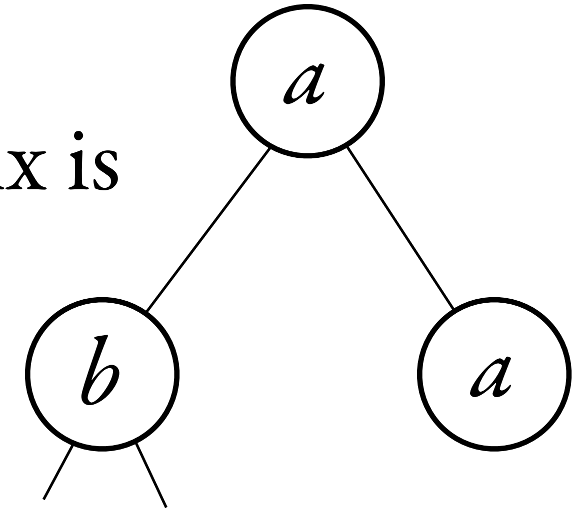

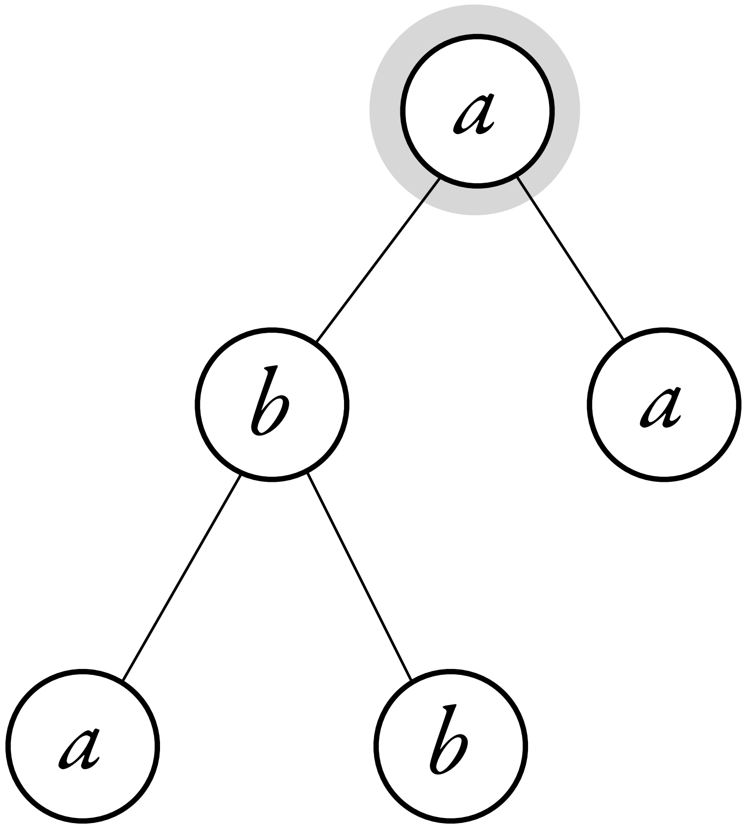

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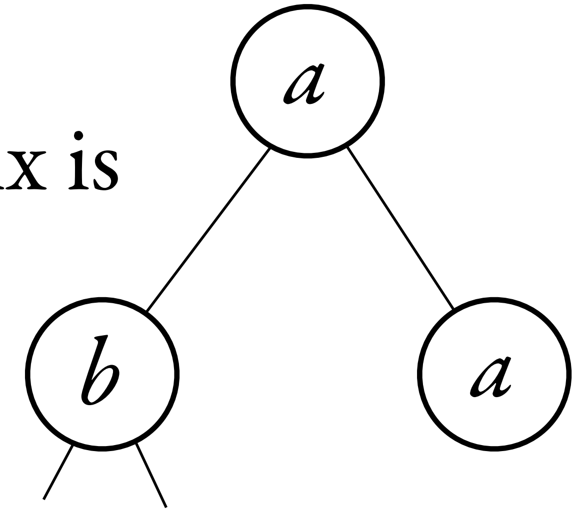

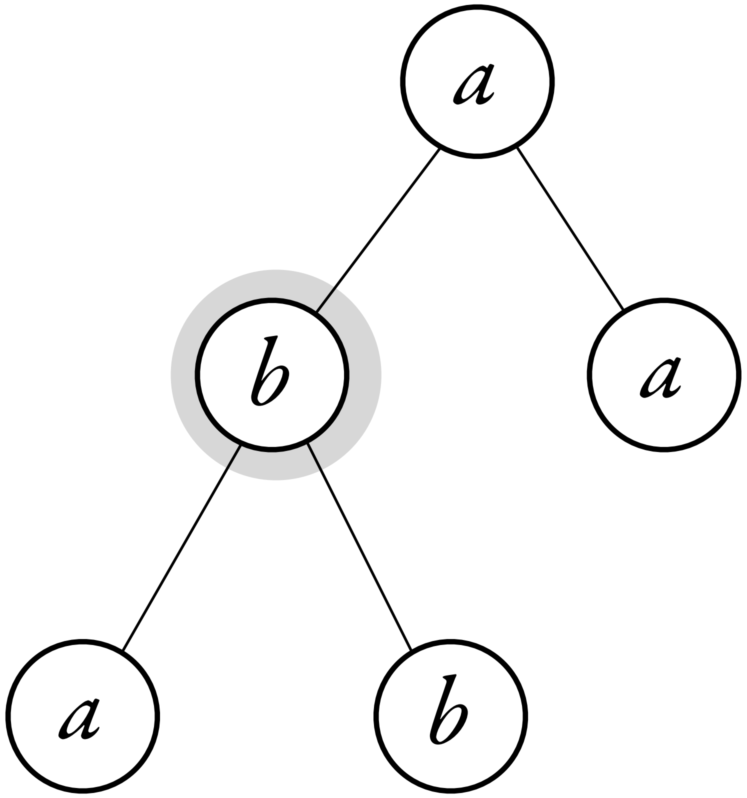

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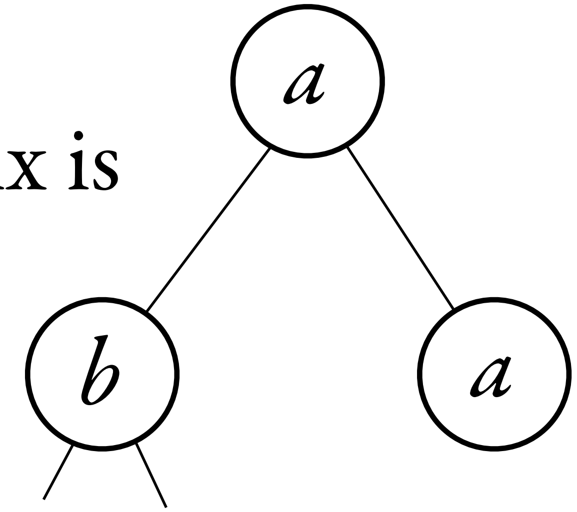

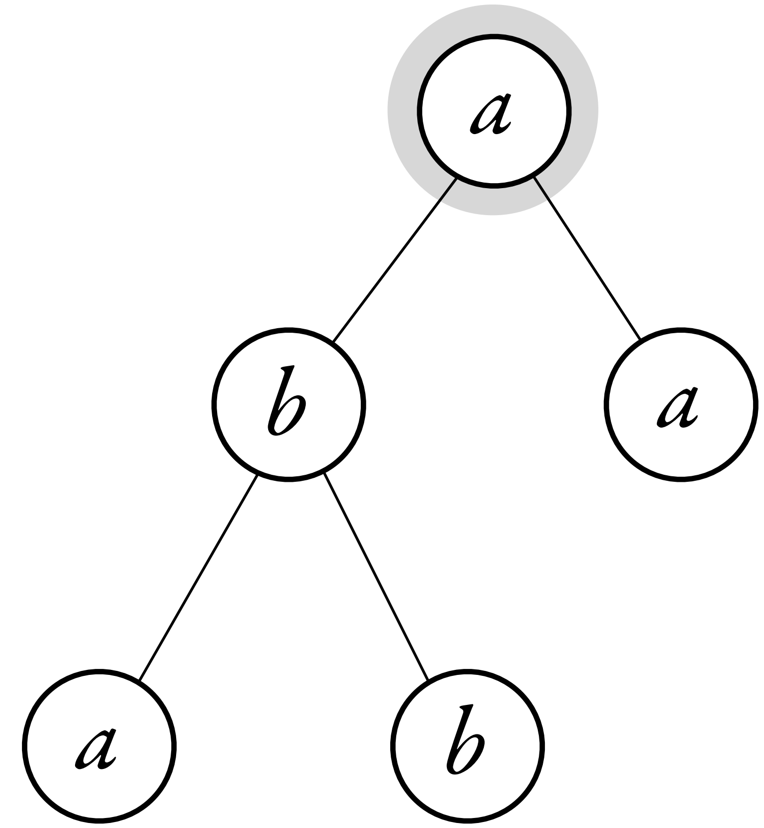

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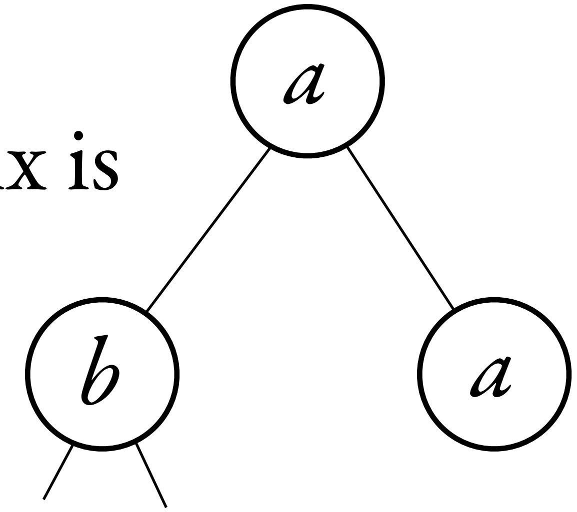

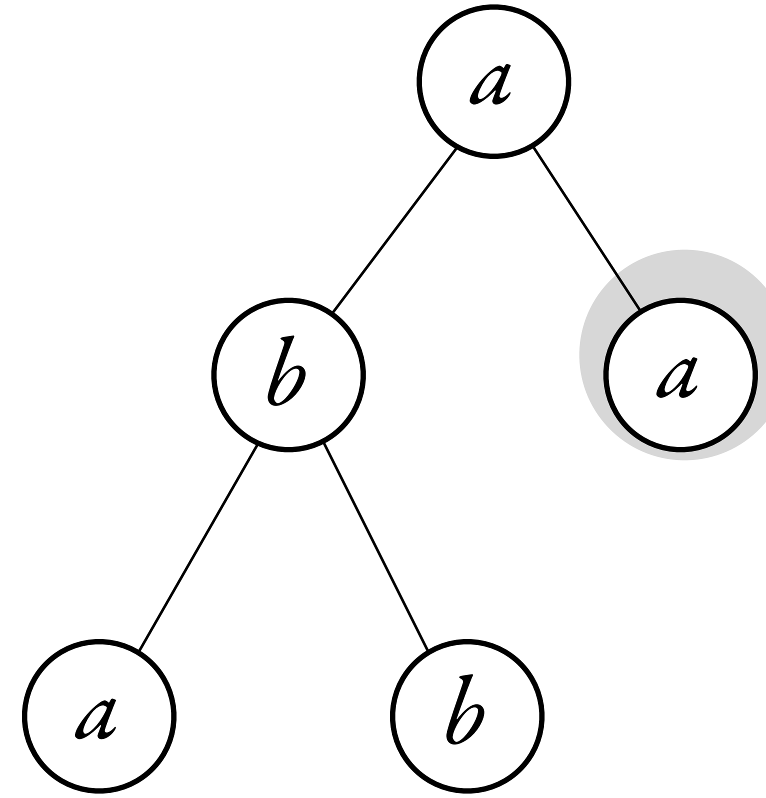

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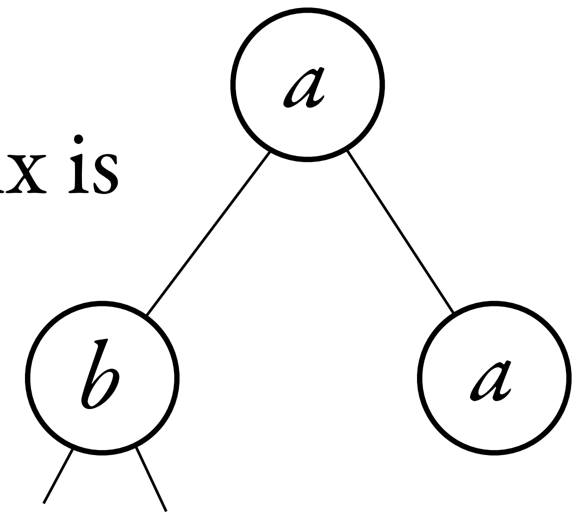

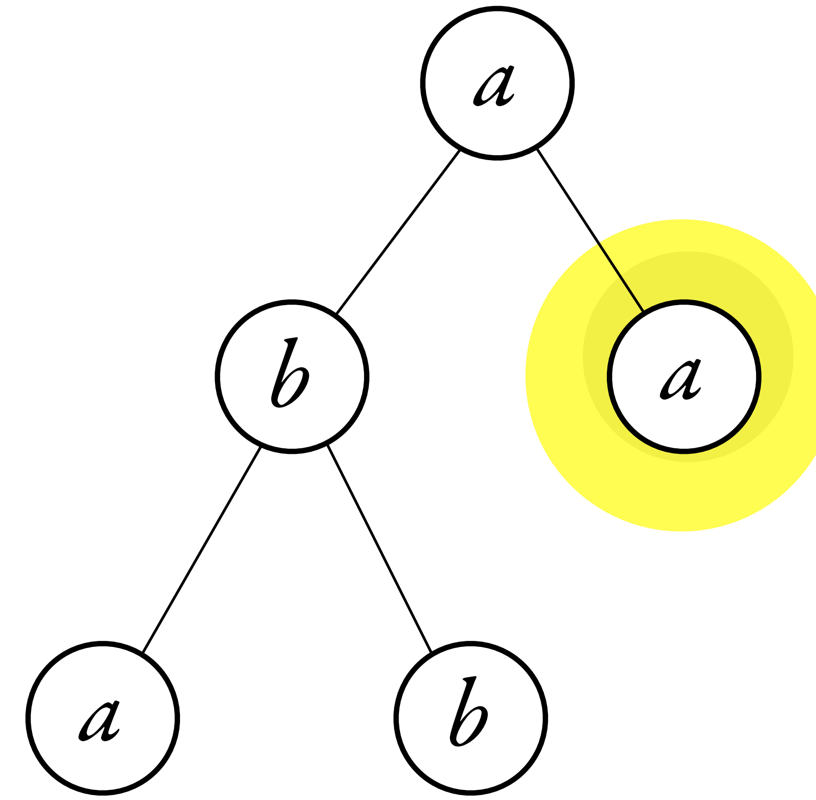

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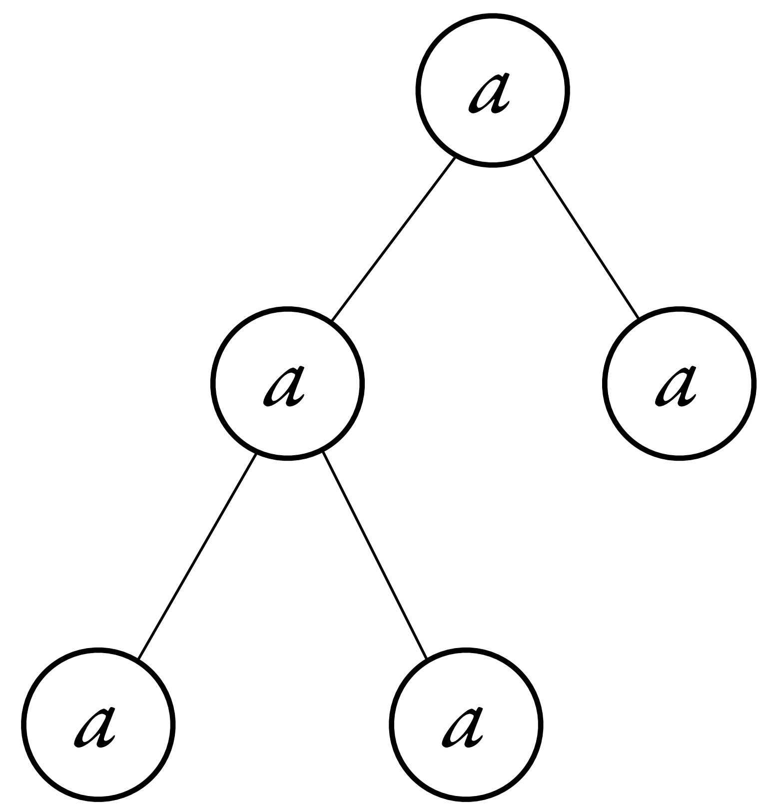
Example.
Some node has label b

In state p, move left.
In state p, move right.
In state p, label b, accept.



Example.
All nodes have label a

Example.
Some node has label b

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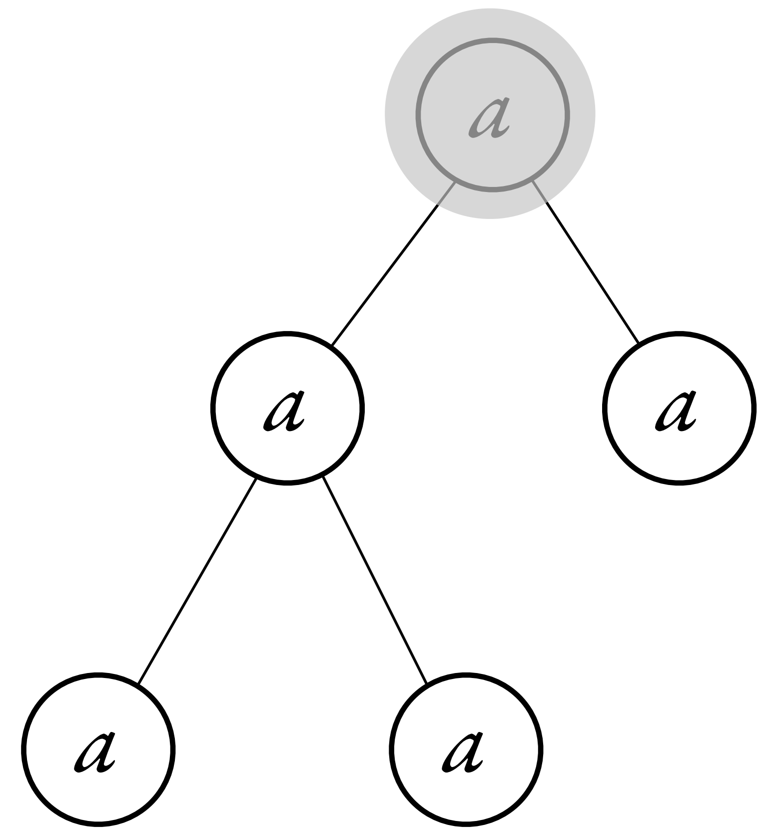

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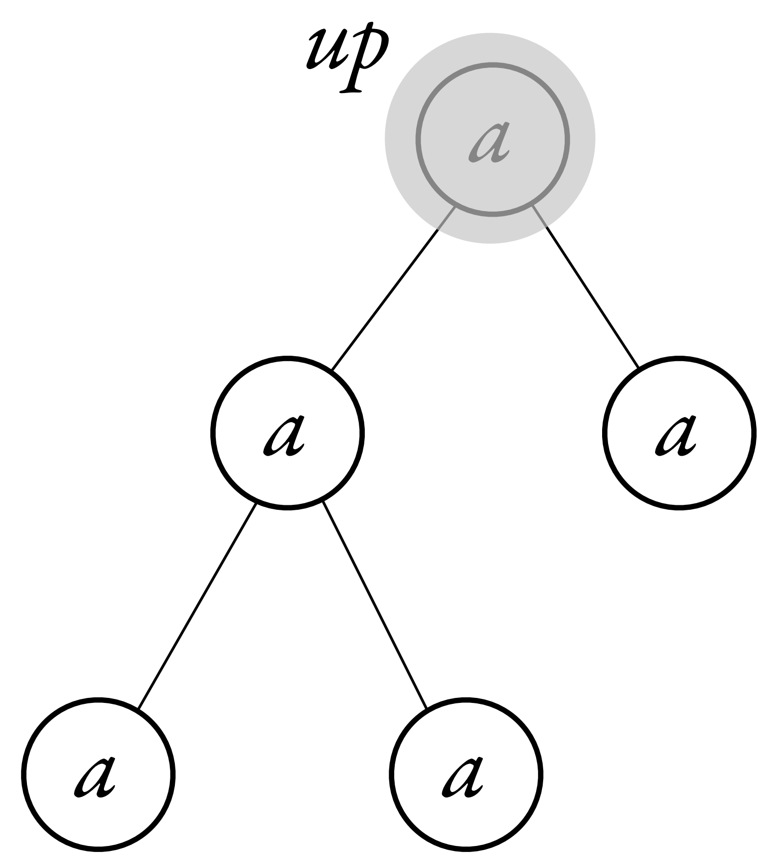

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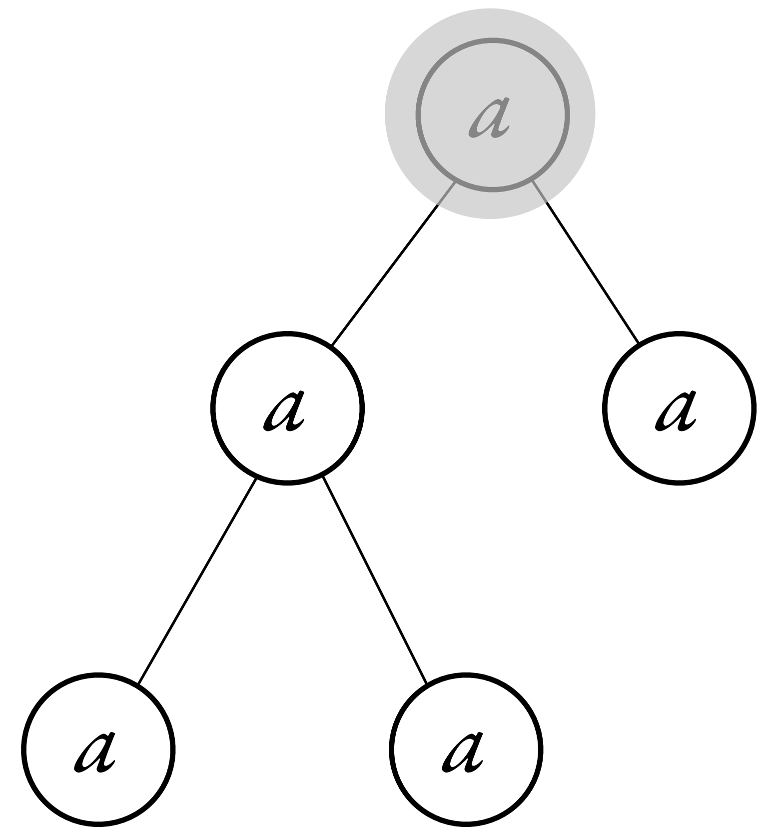

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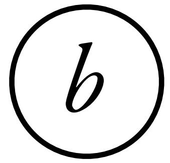
Some node has label 

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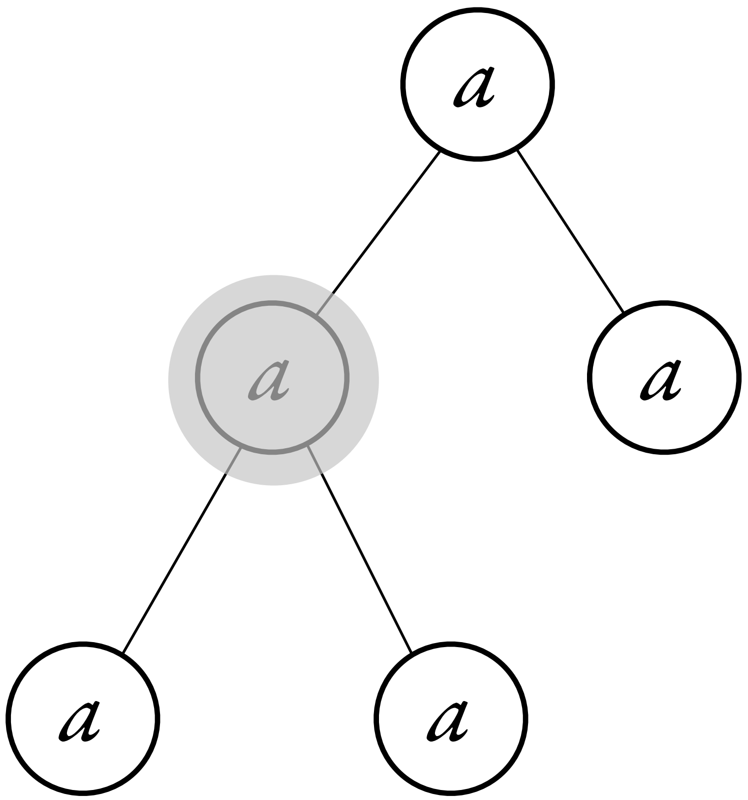

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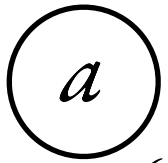
All nodes have label 

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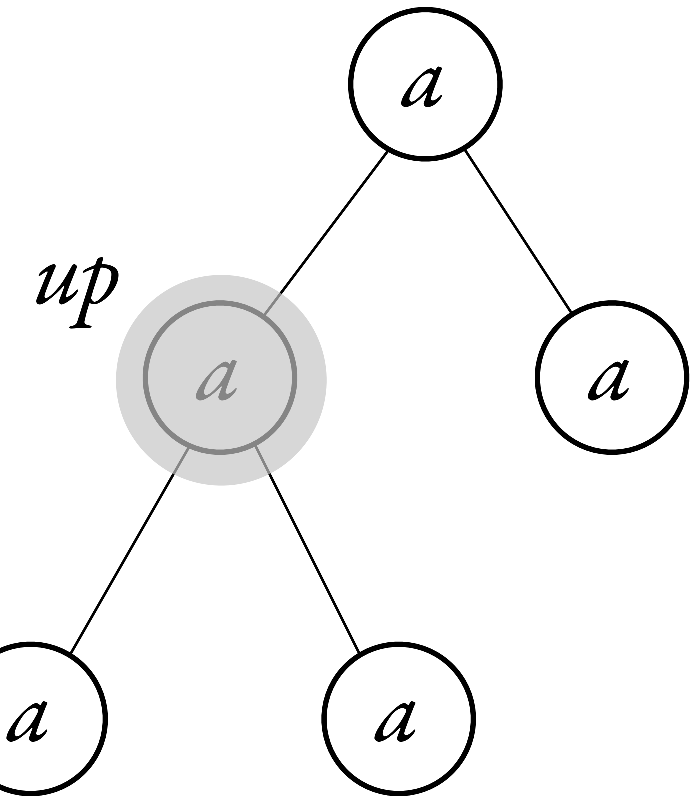

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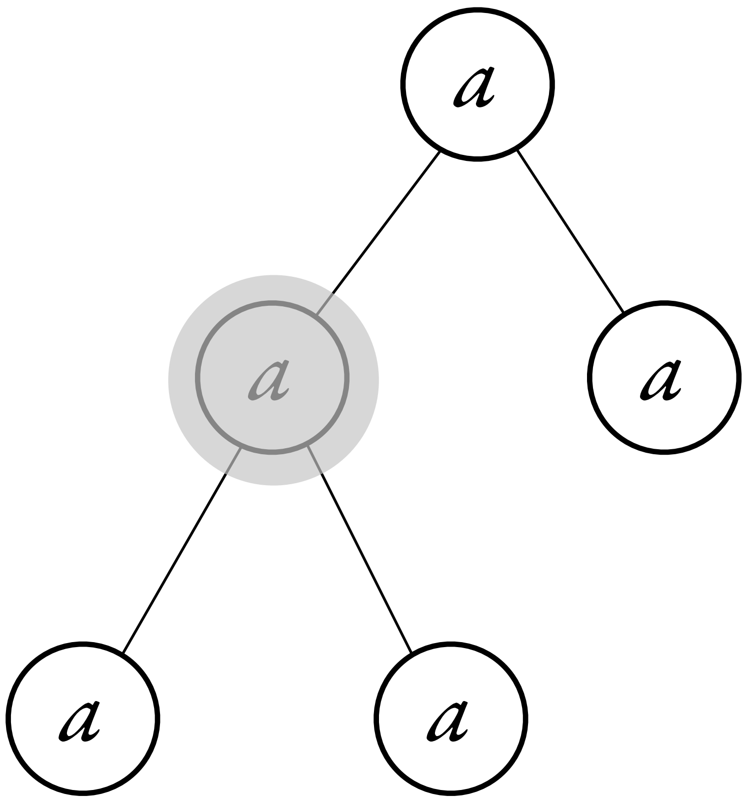

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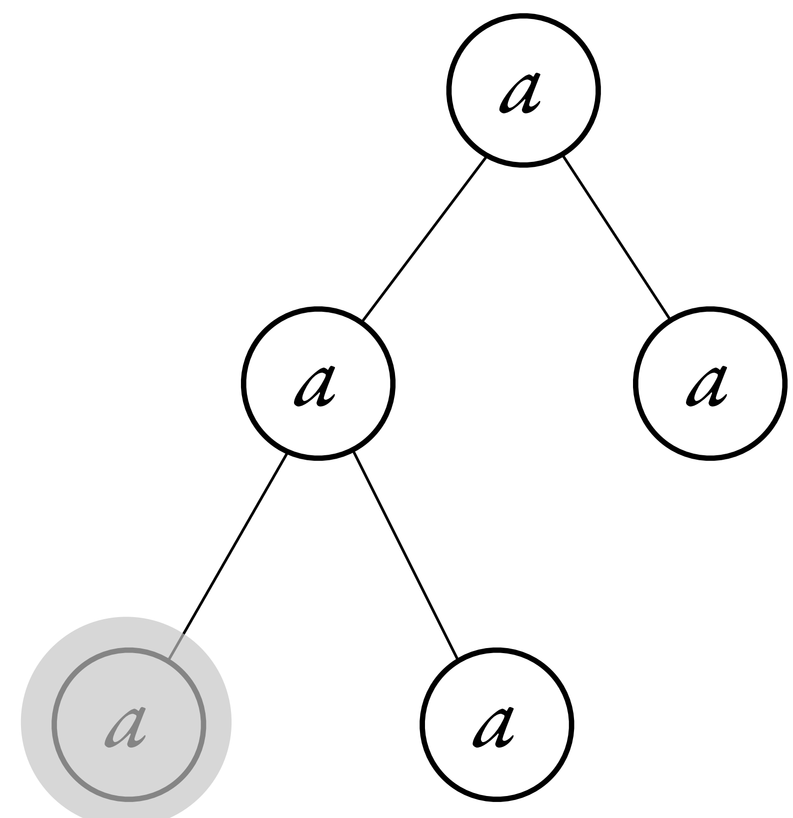

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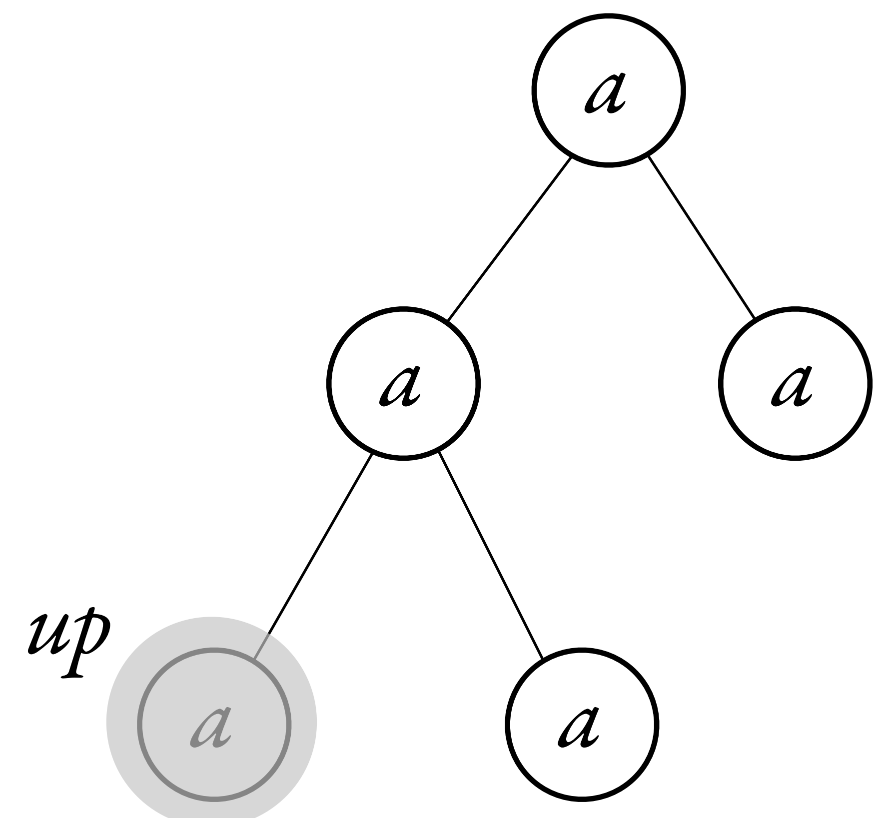

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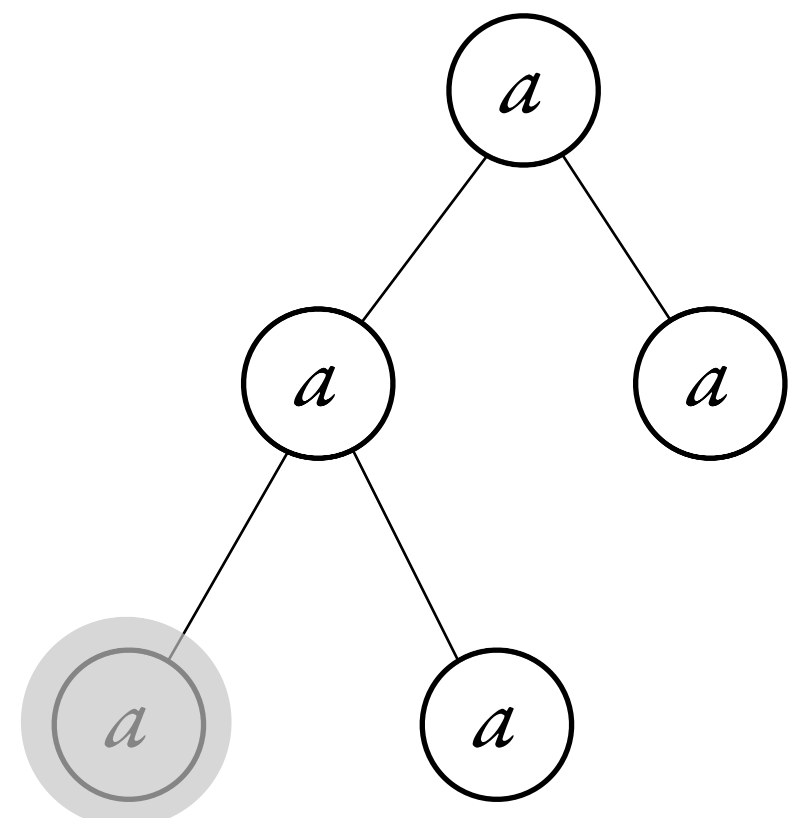

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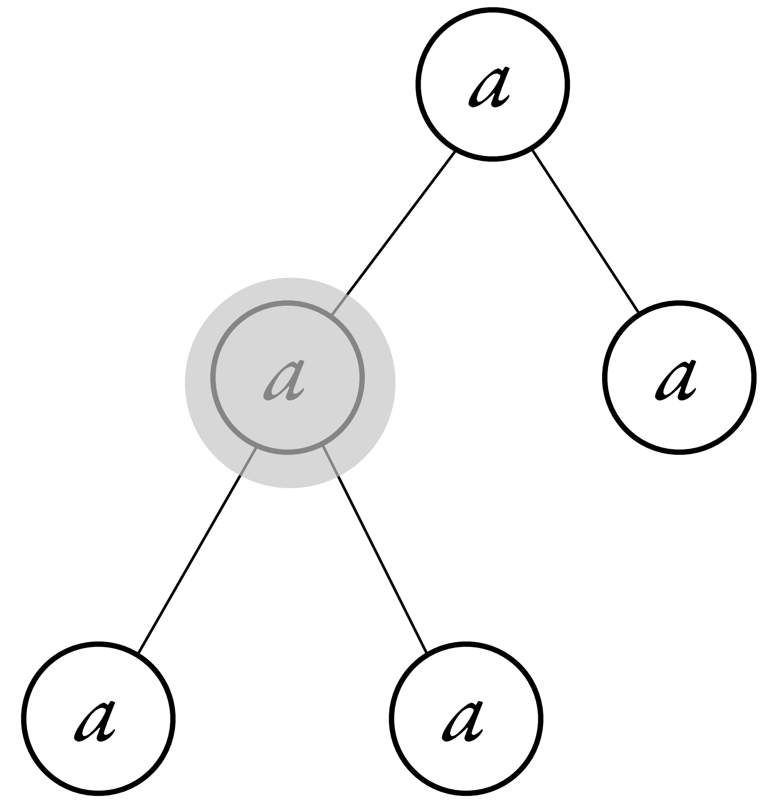

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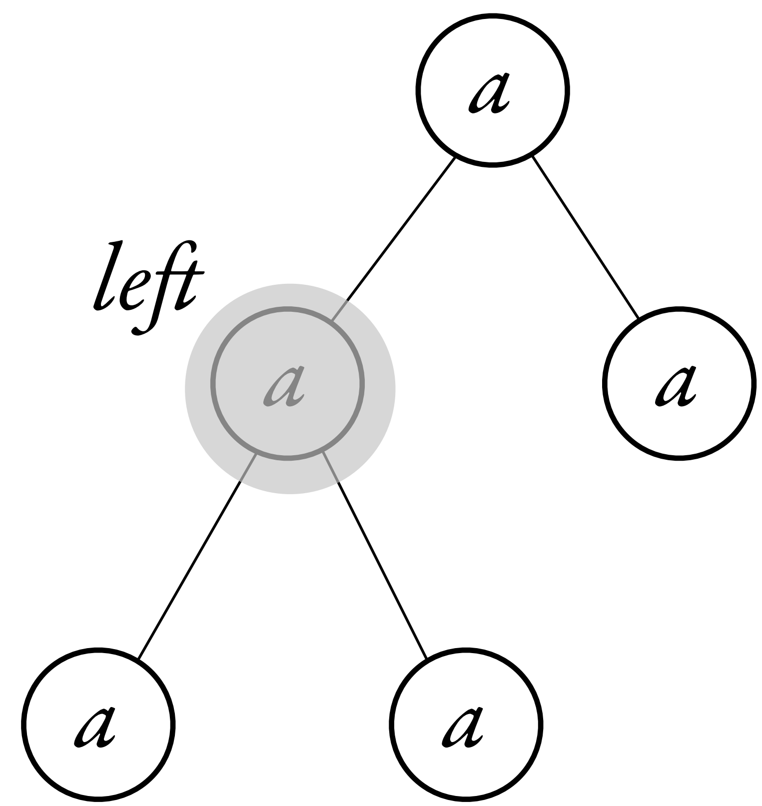
In state p, move right.

In state p, label b, accept.



Example.

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Example.
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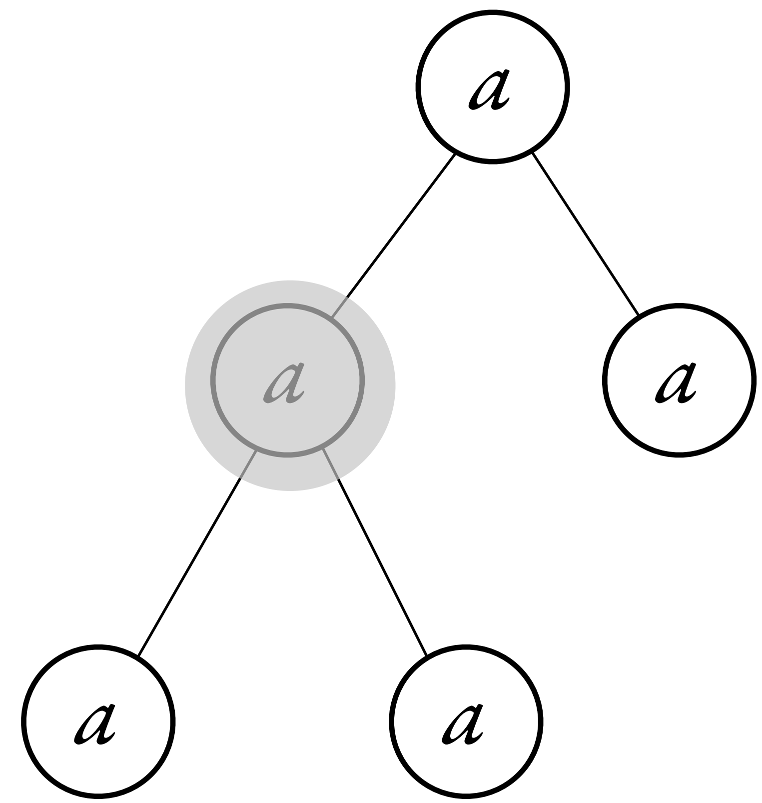

Example.
All nodes have label a

Example.
Some node has label b

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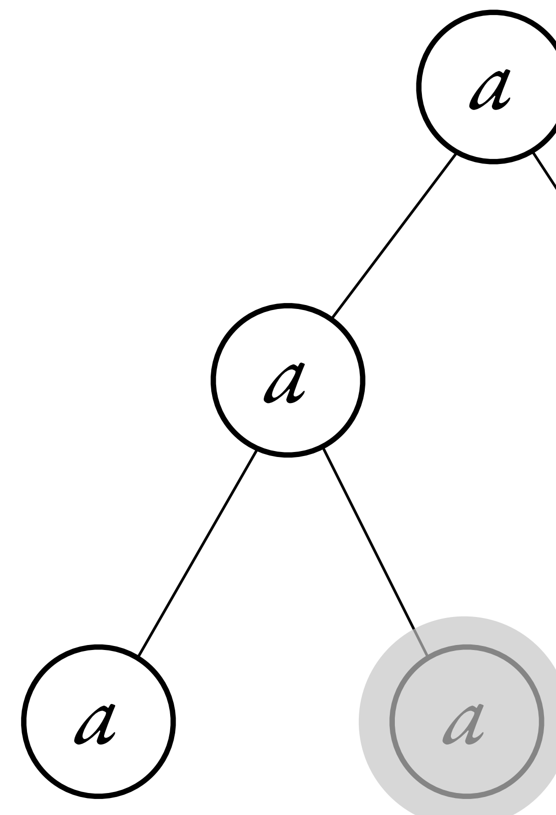
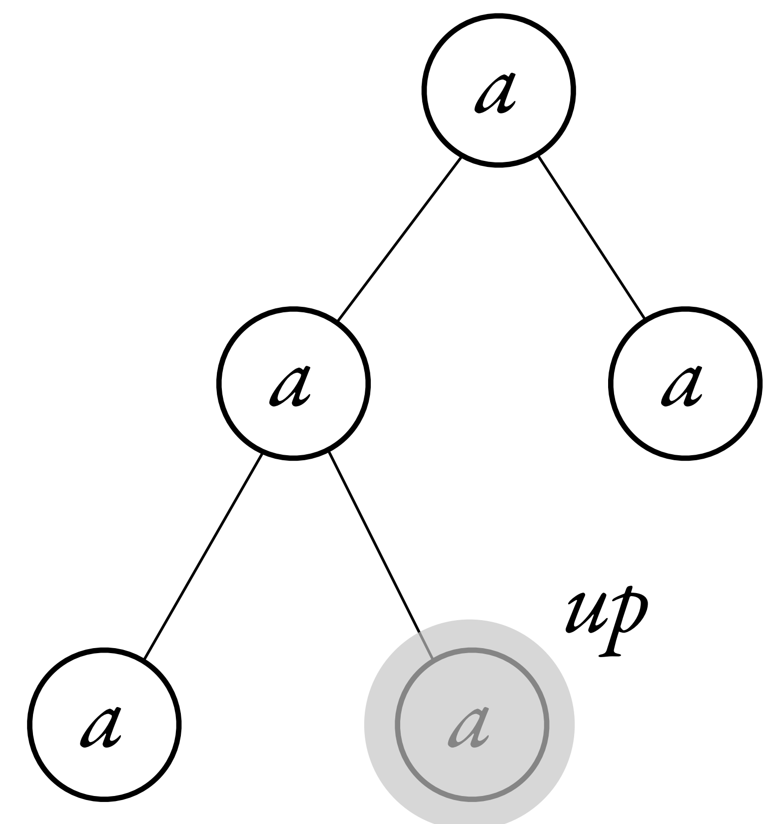


Example.
All nodes have label a

Example.
Some node has label *b*

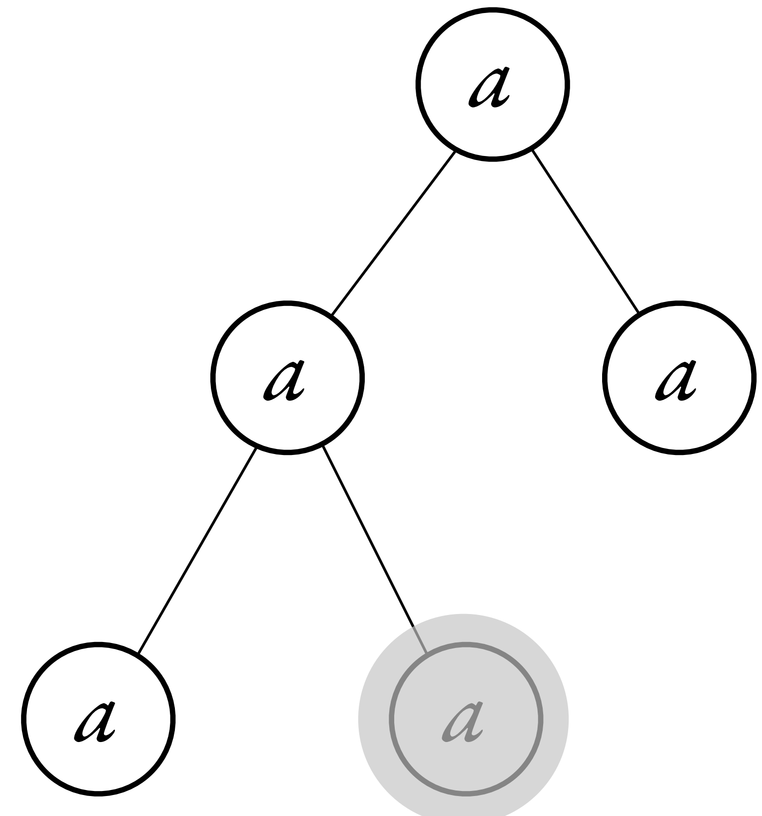
In state *p*, move left.
In state *p*, move right.
In state *p*, label *b,* accept.



Example.
All nodes have label *a*

Example.
Some node has label b

In state p, move left.
In state p, move right.
In state p, label b, accept.



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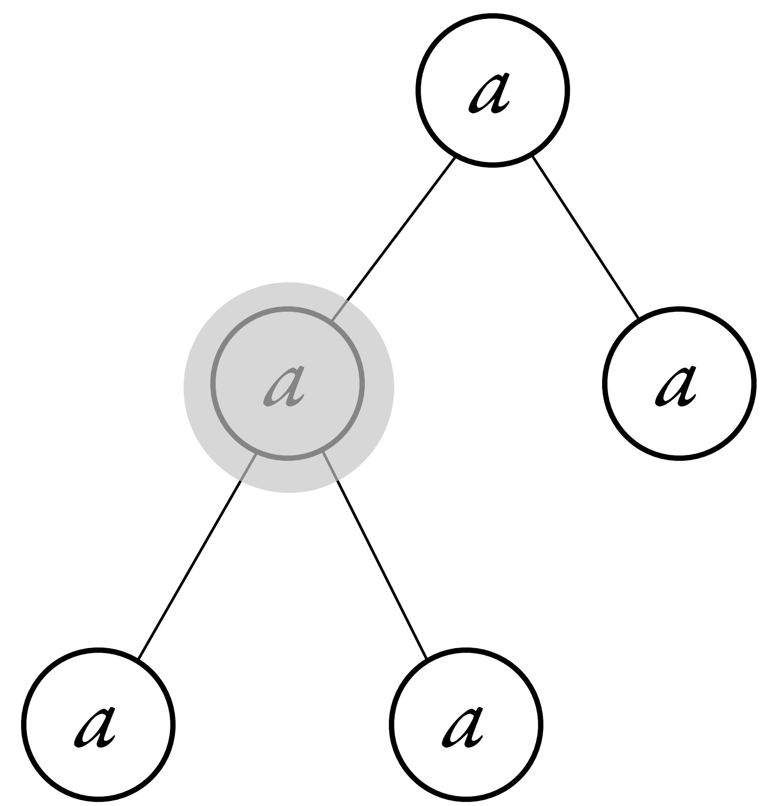

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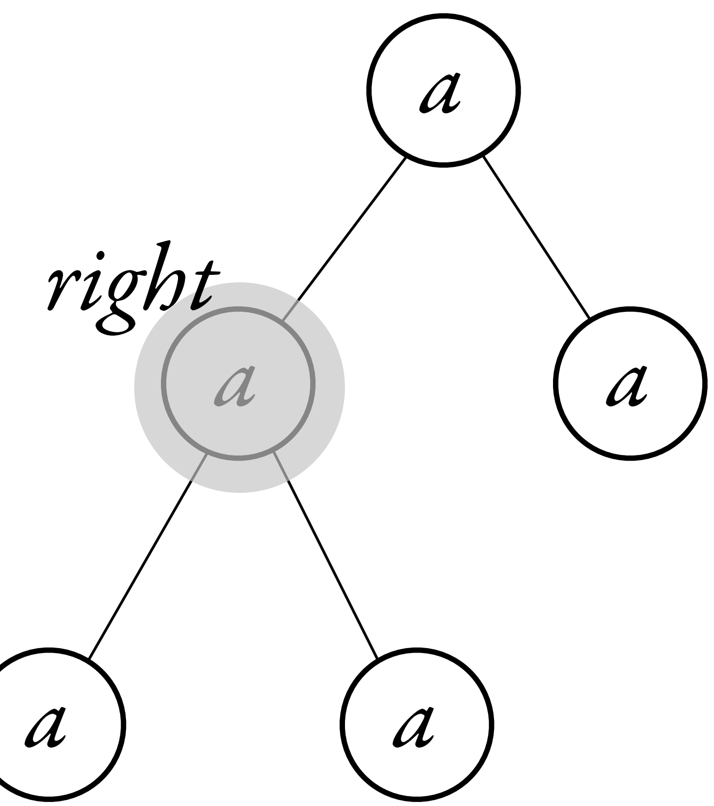

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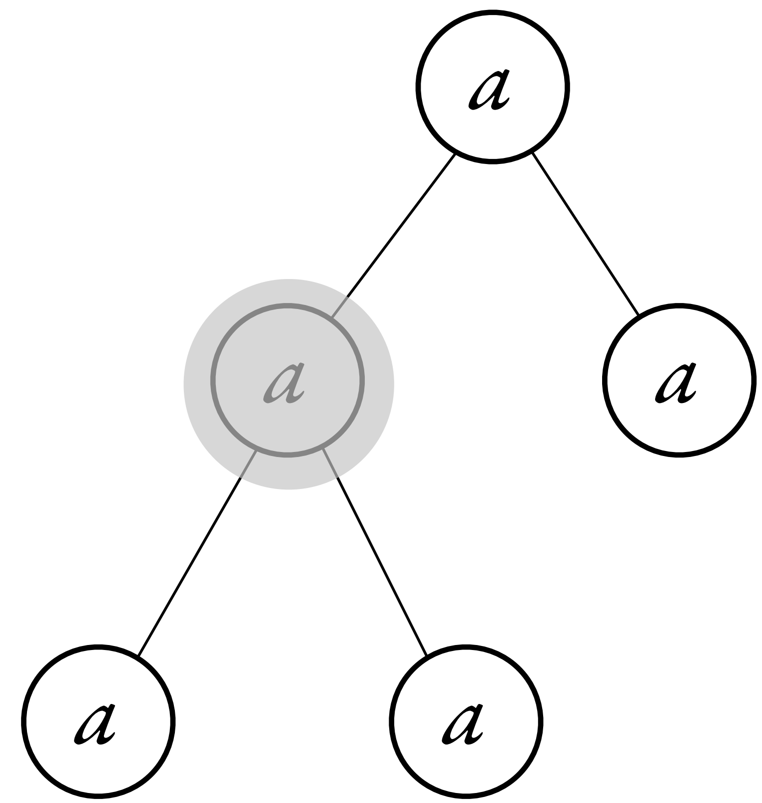

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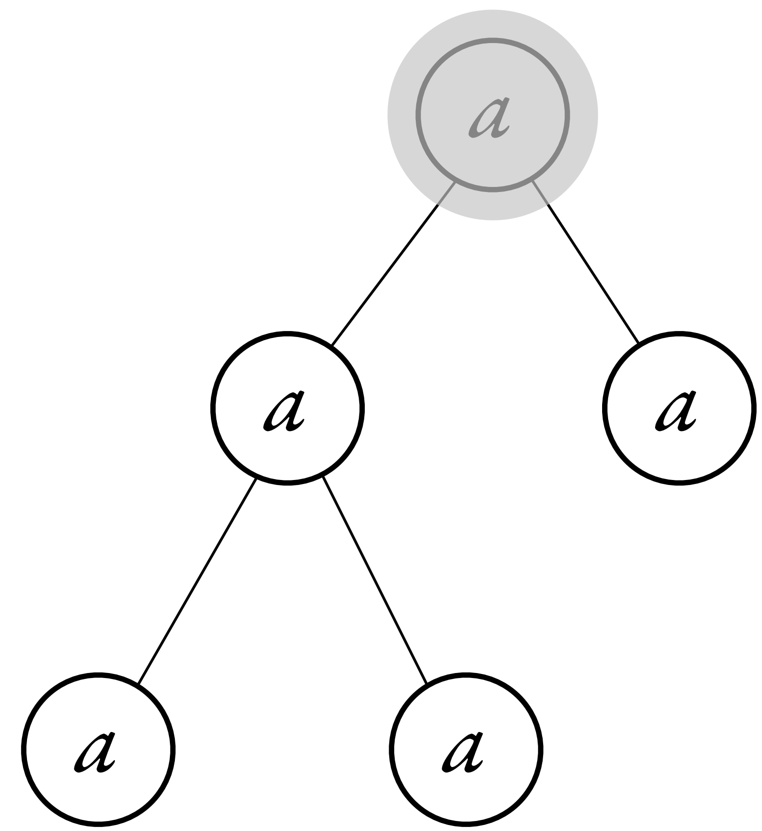

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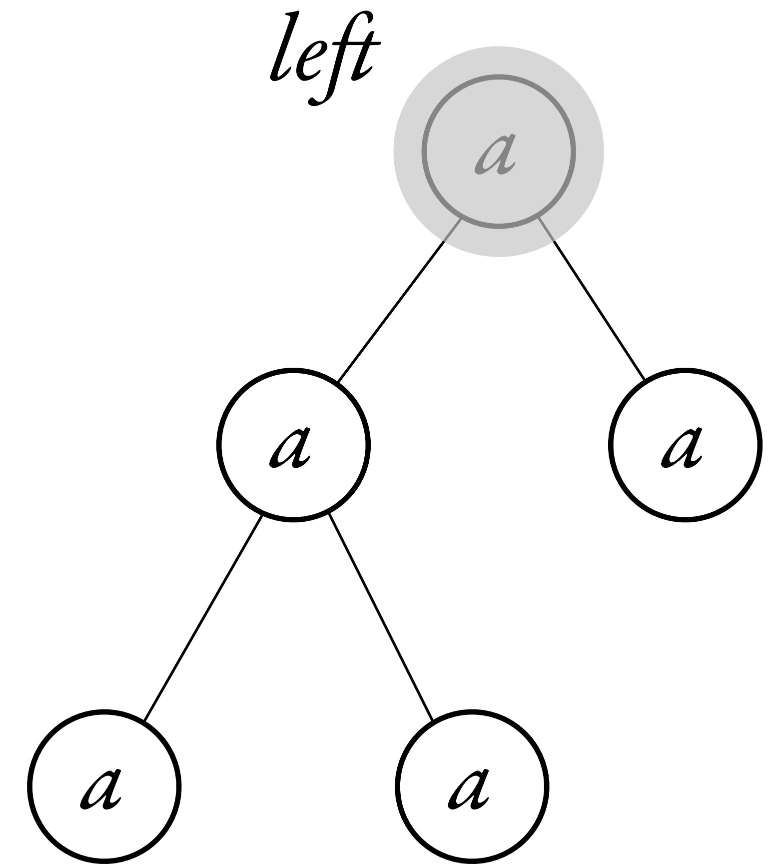

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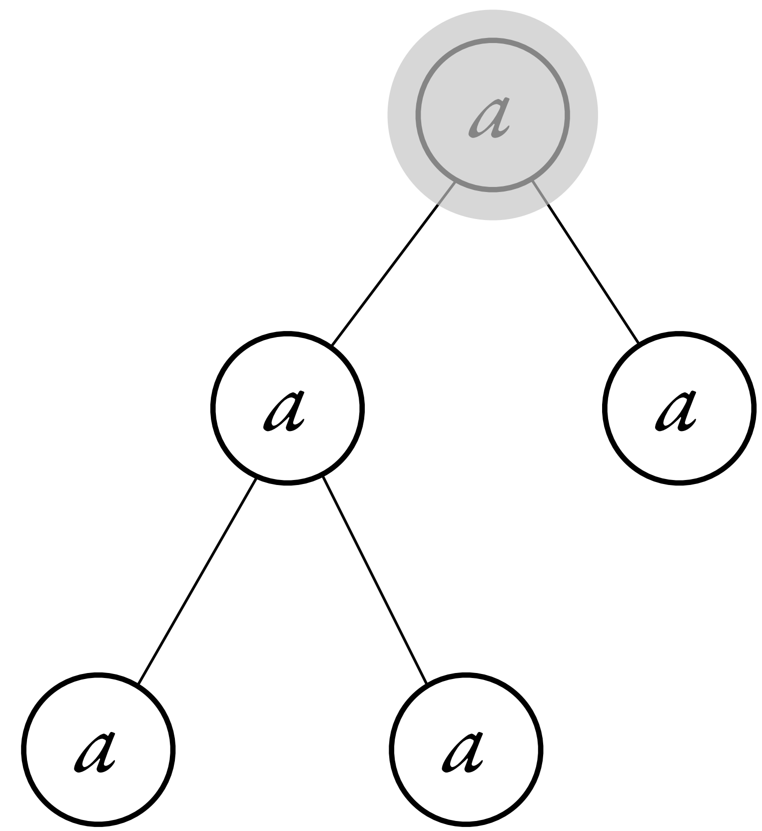

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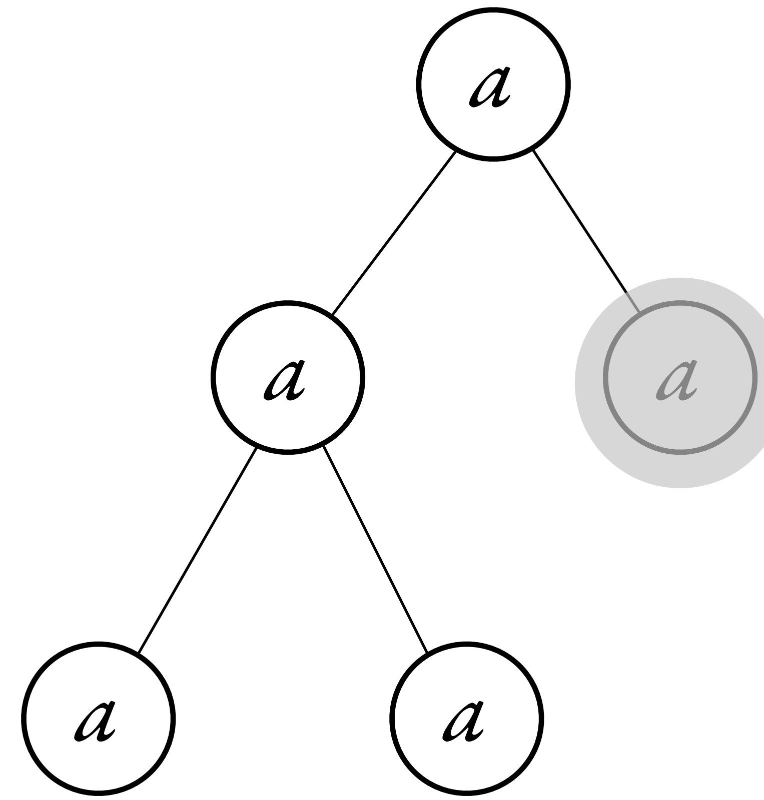

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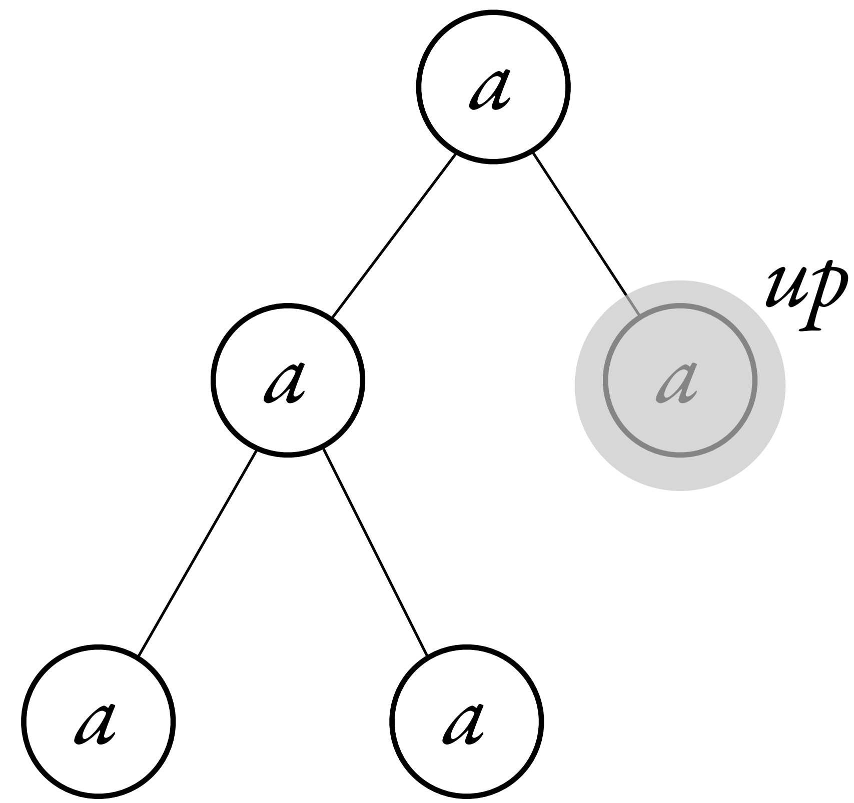

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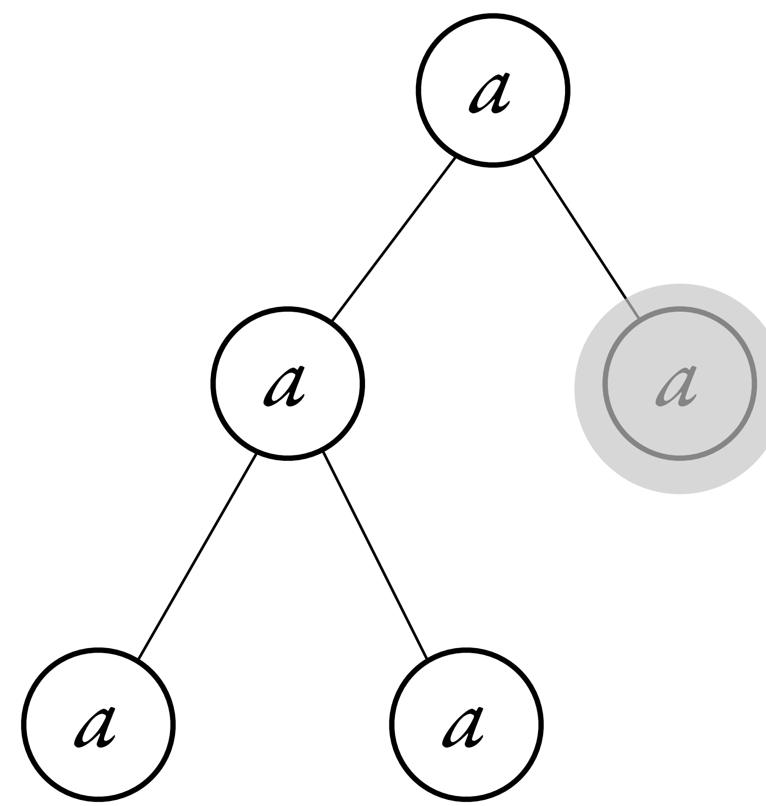

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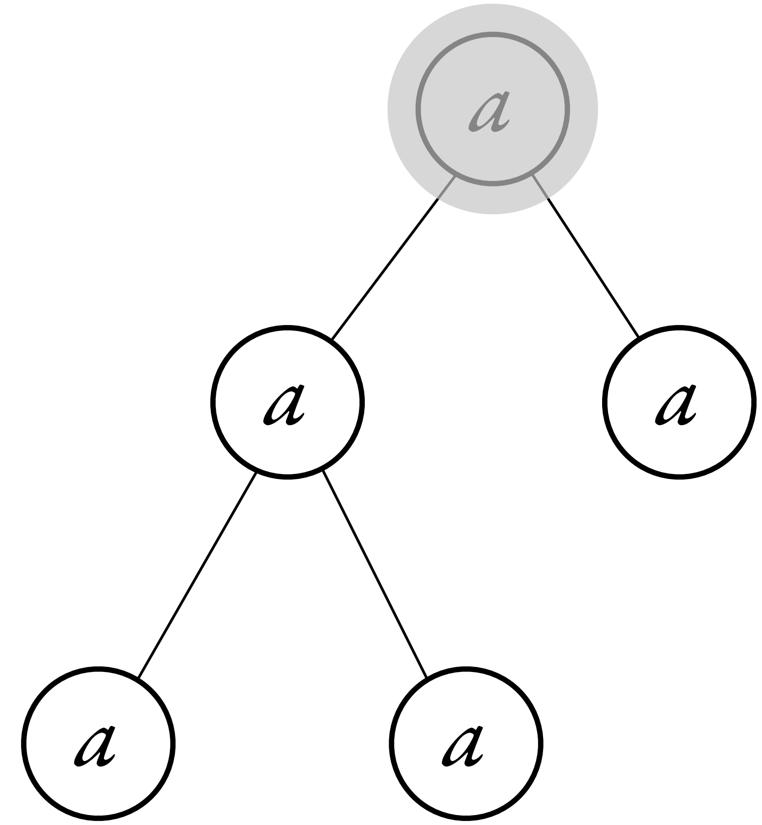

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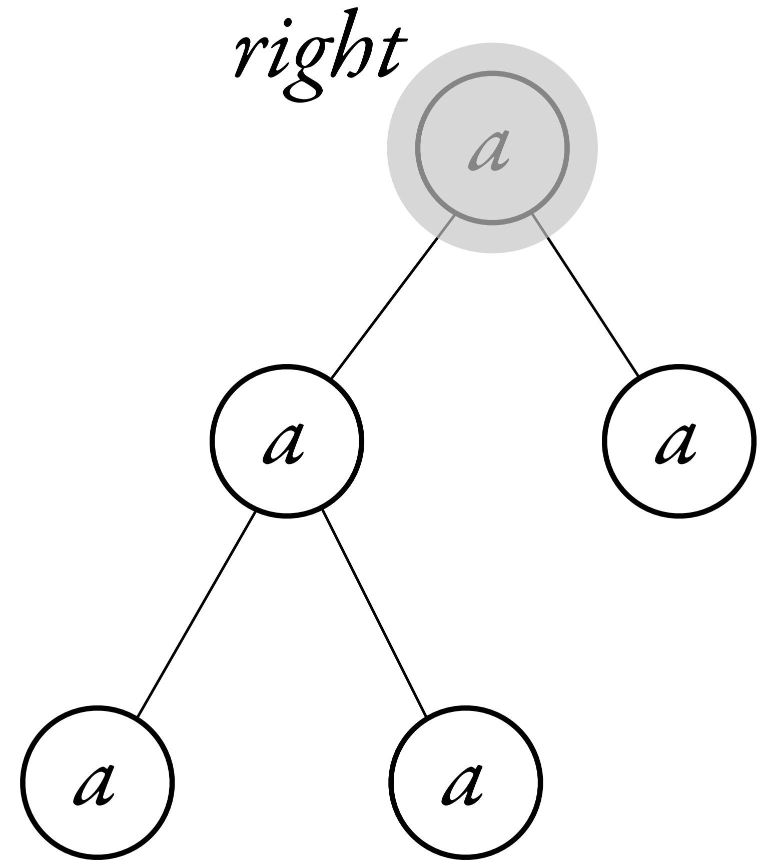

Example.
All nodes have label a

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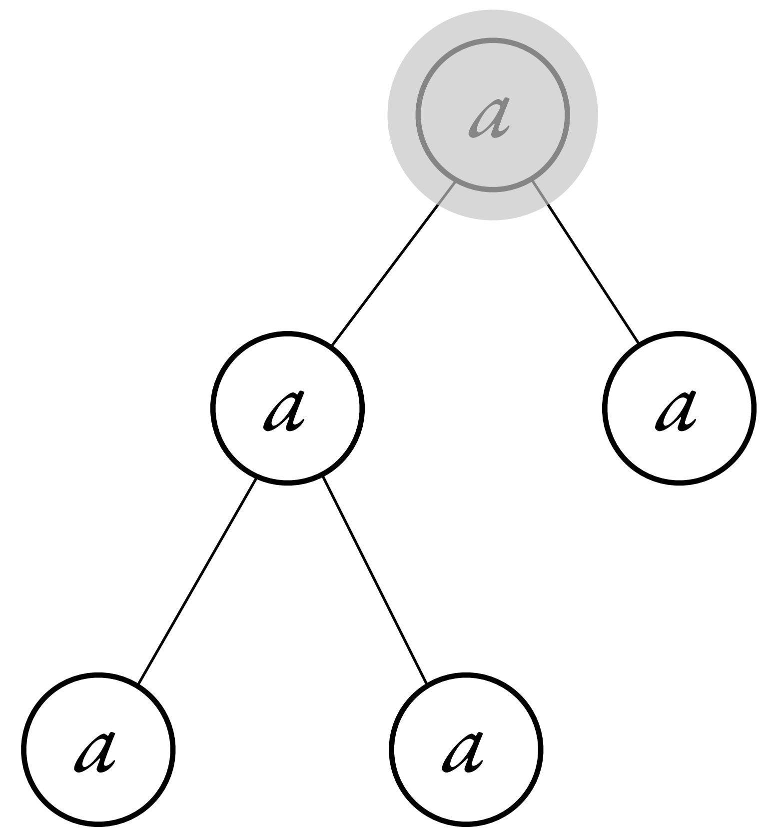

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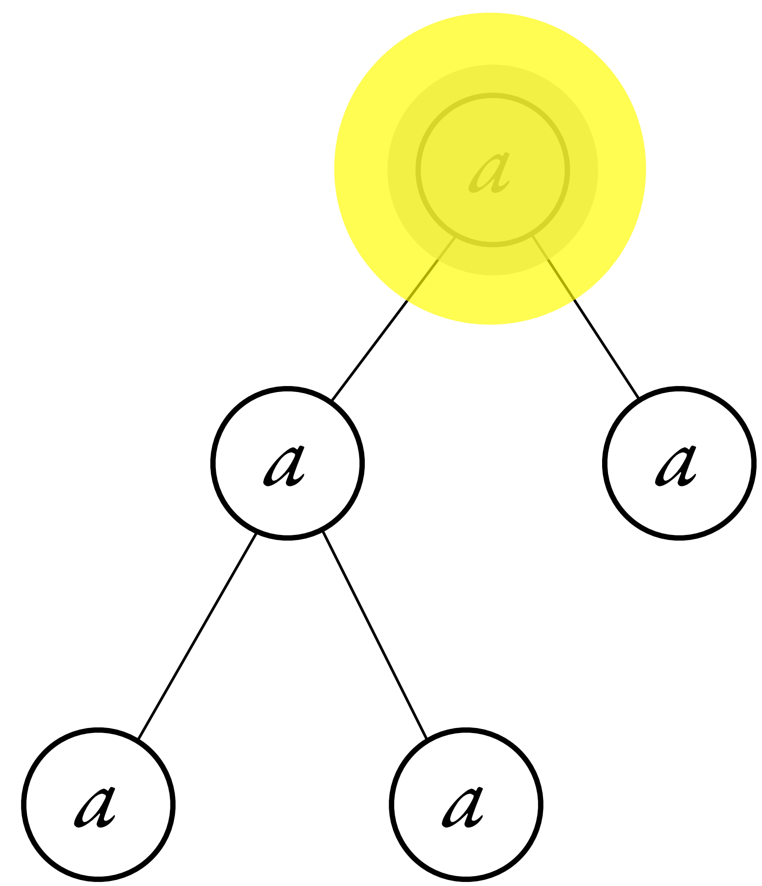

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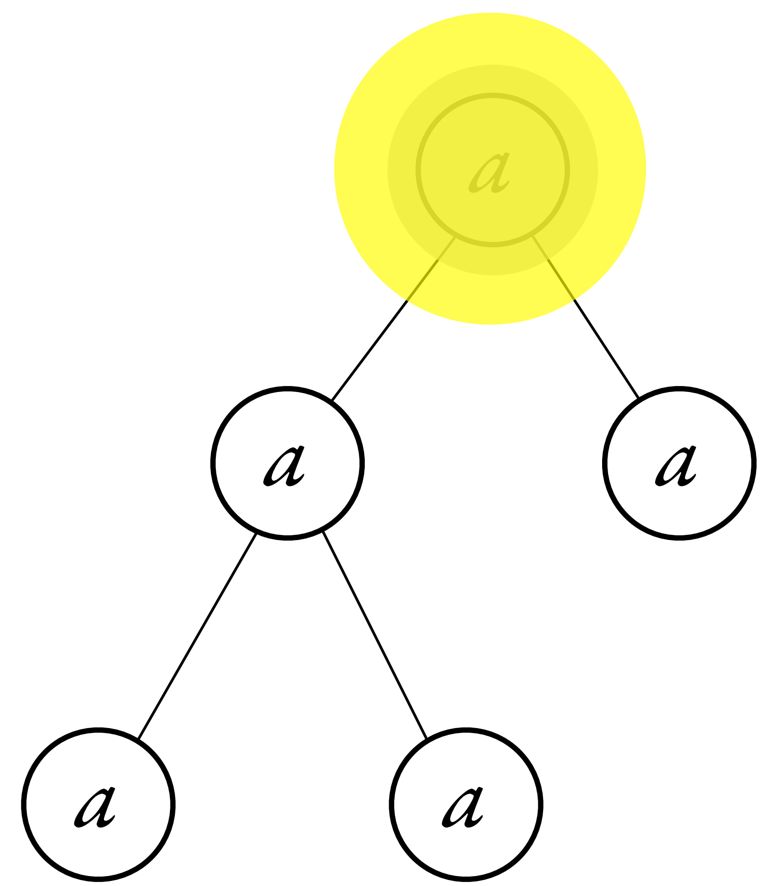

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Open problem:
Are nondeterministic
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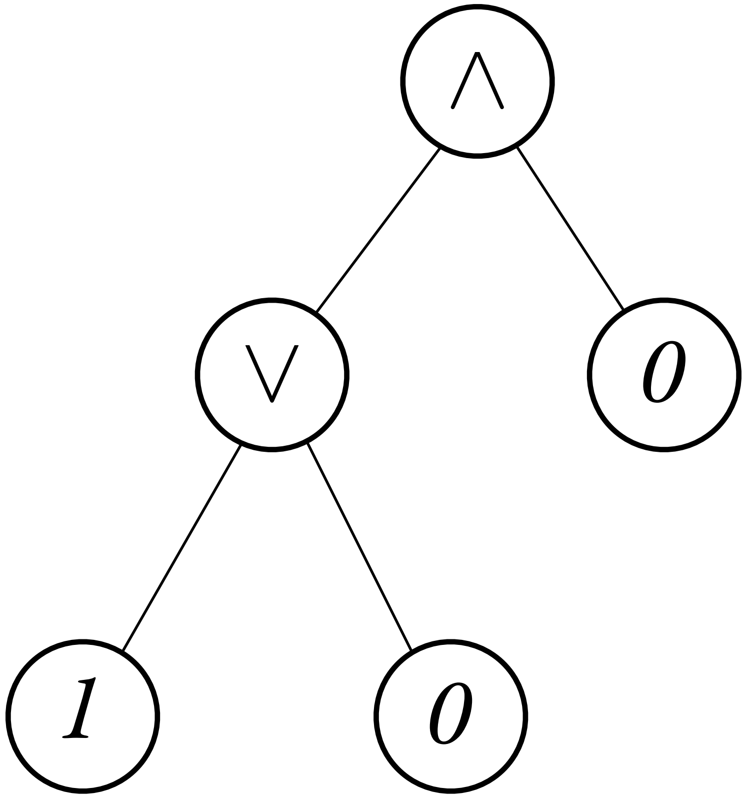

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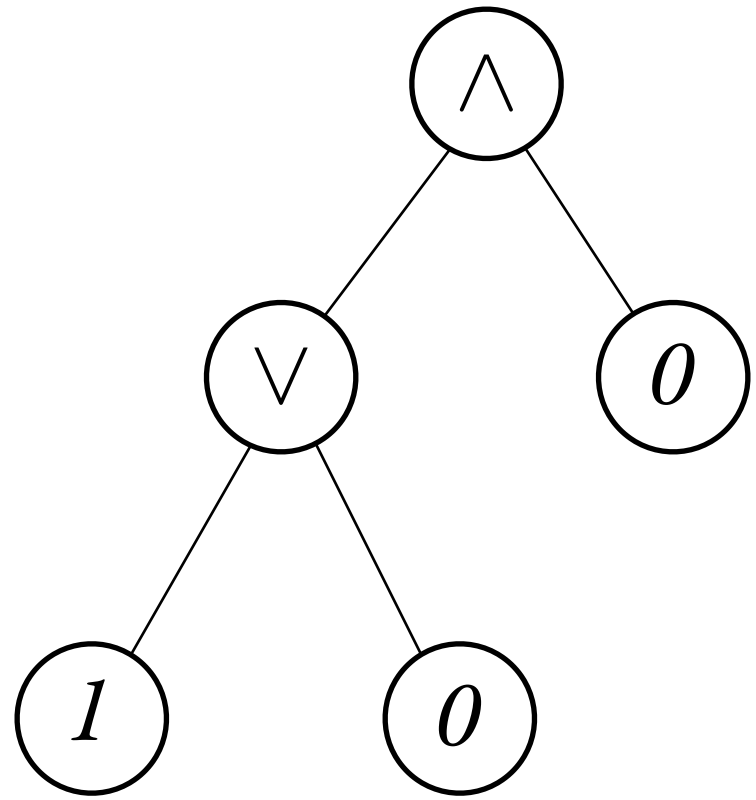

States: $\underbrace{\{q\}}_{\text{first time}} \cup \left(\underbrace{\{left, right\} \times \{0,1\}}_{\text{just evaluated evaluated left/right subtree to 0/1}}\right)$

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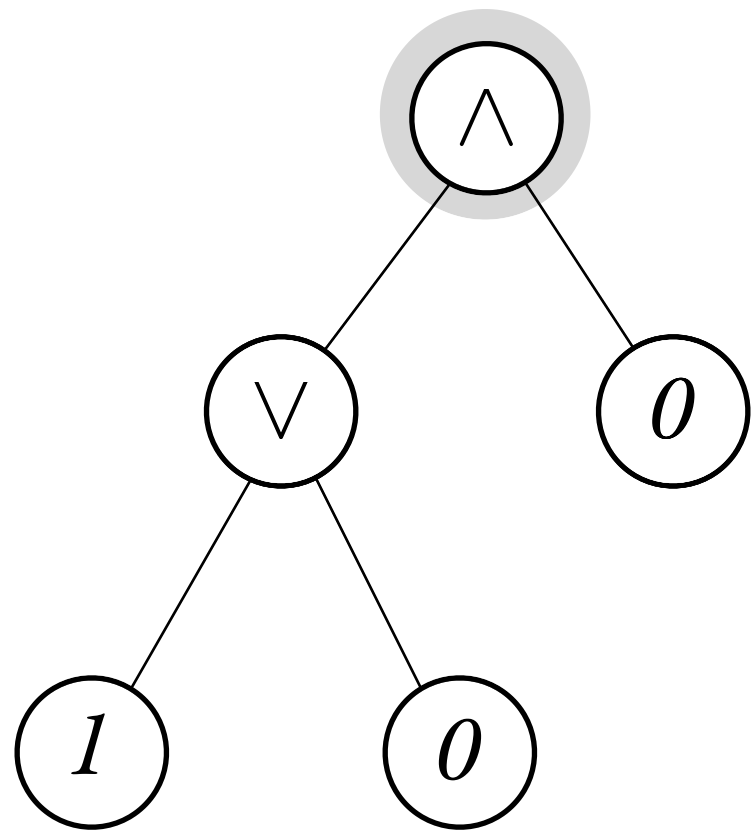

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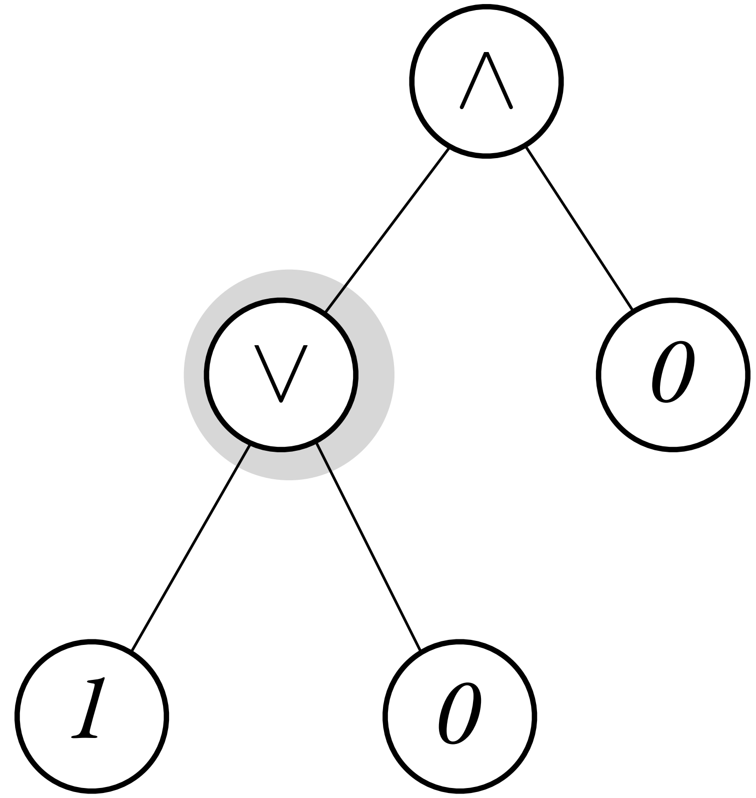

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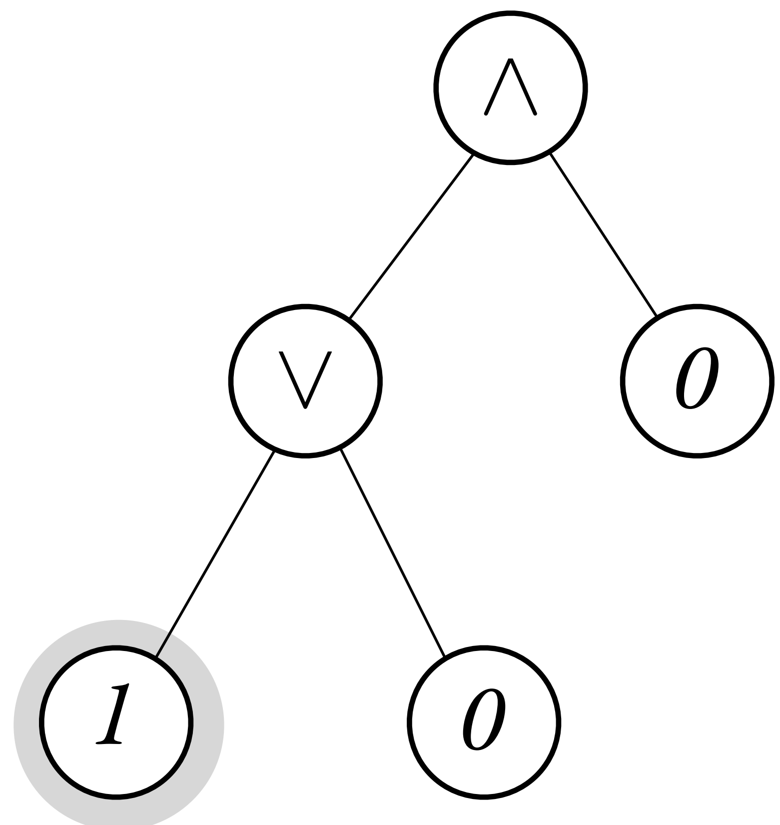

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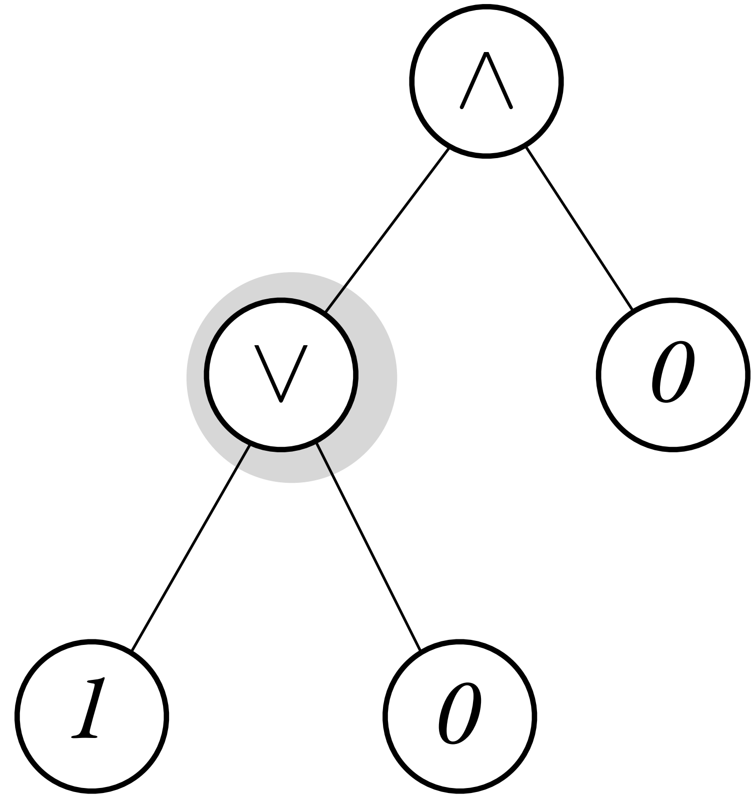

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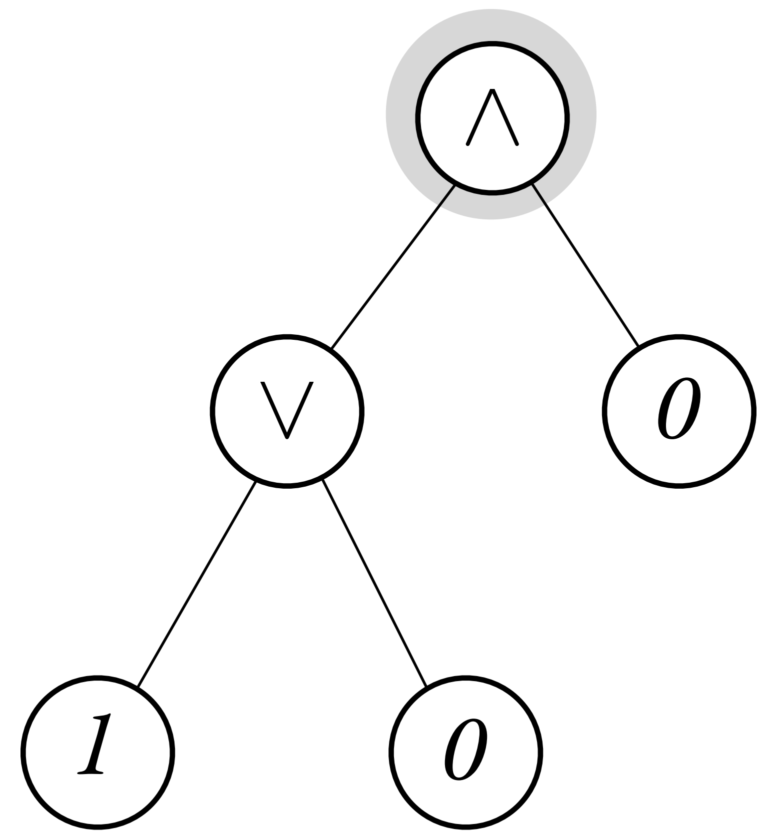

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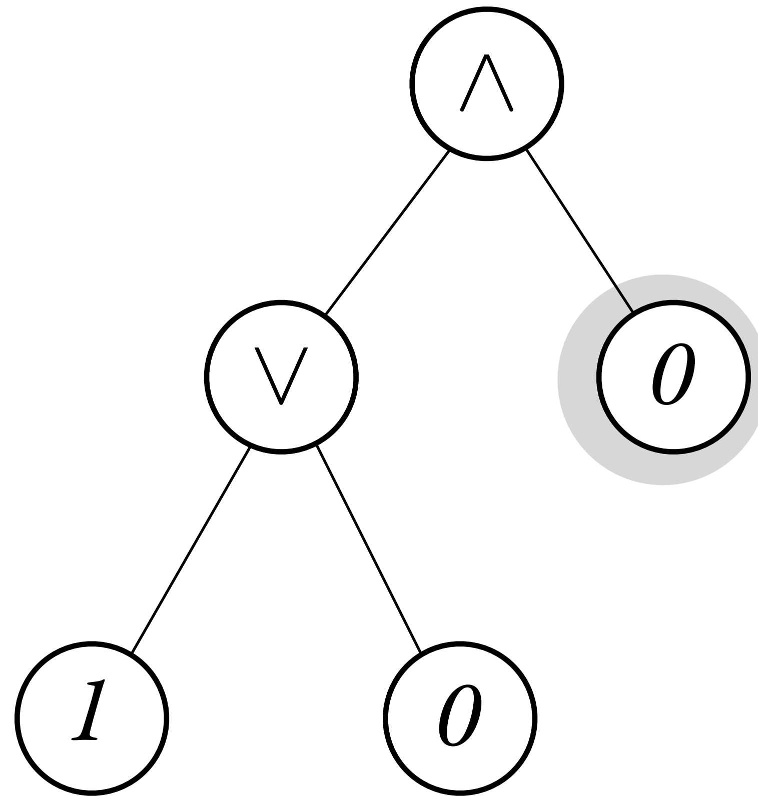

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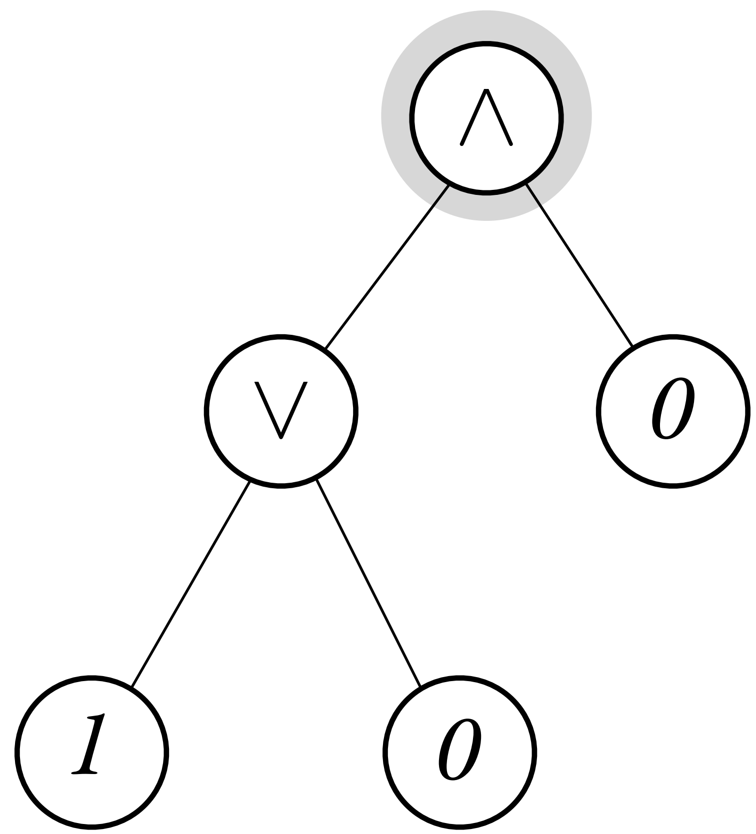

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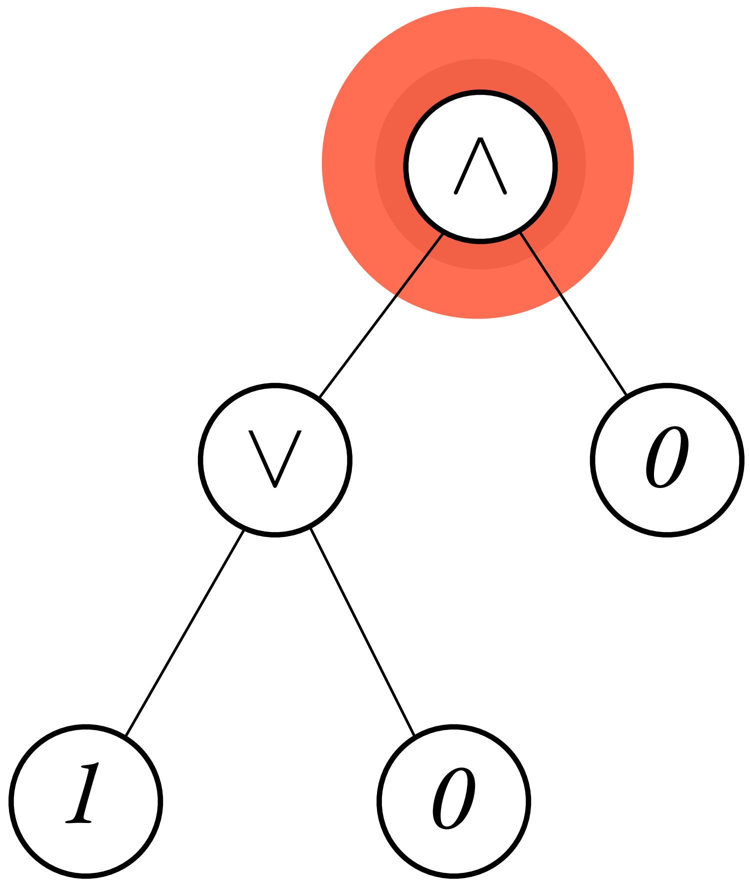

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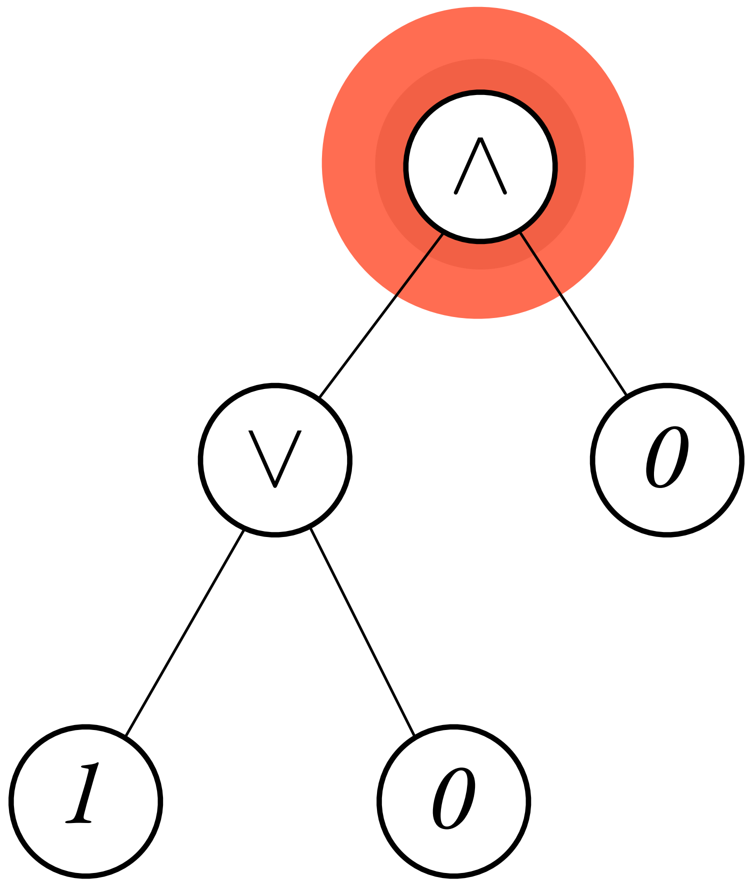
still works with negation, but what about XOR?



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Theorem. [Muscholl, Samuelides, Segoufin]
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Lemma.
Every deterministic tree-walking automaton is equivalent to one that ends every run with a *reject* or *accept* command.

Plan

Tree-Walking Automata

- definition
- some examples
- problems

Expressive Power

- comparison with tree automata
- complexity
- determinization

Pebble Automata

- definition
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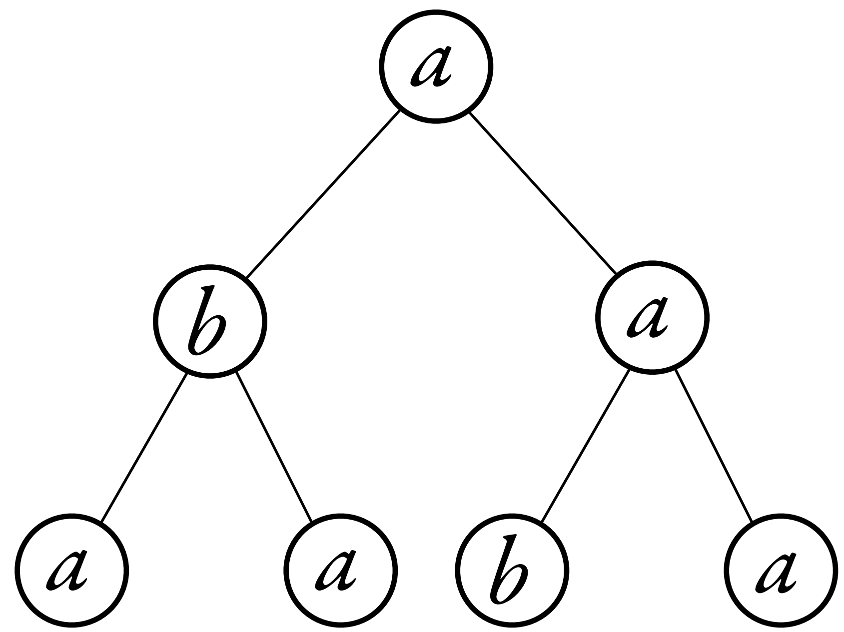
How do tree-walking automata relate to “real” tree automata?



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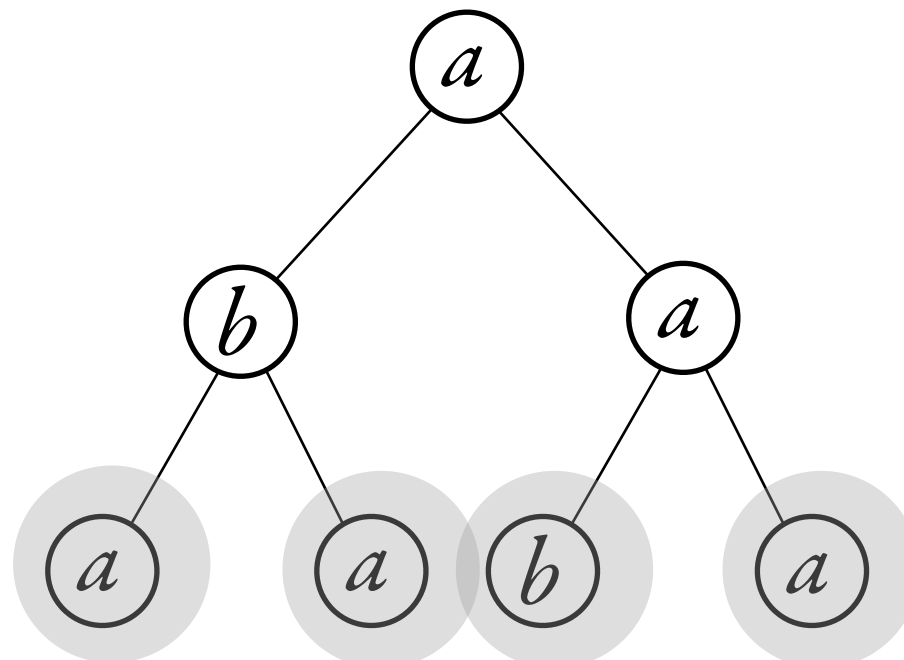

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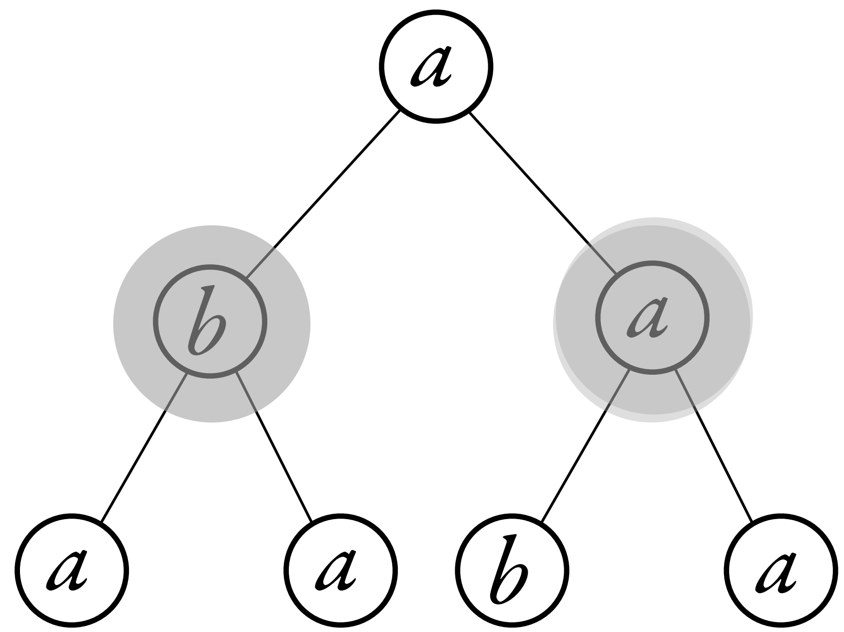

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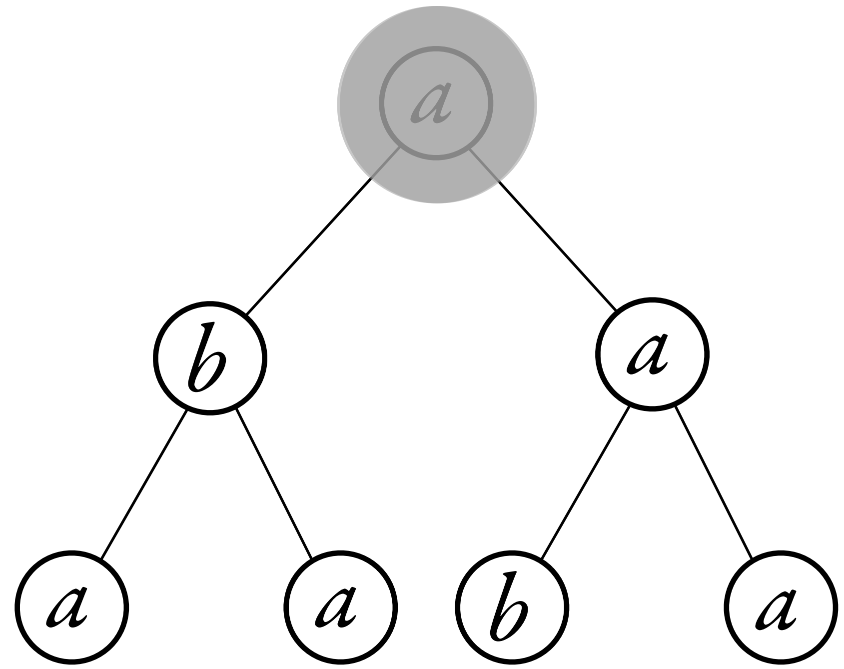

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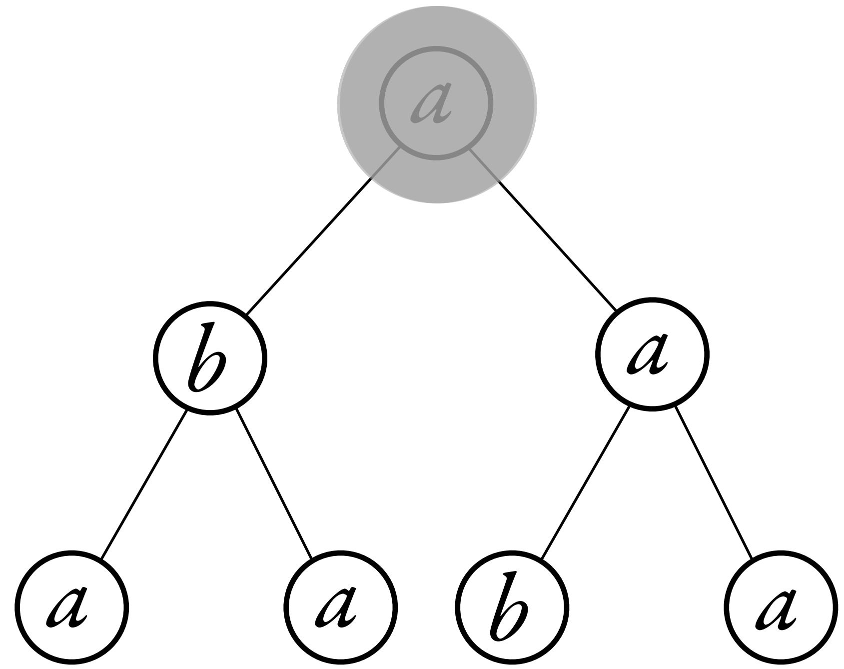

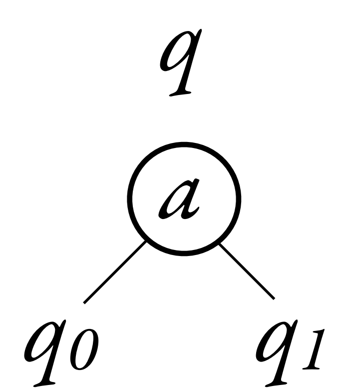

If the root label is a, the left subtree has value q_0, and the right subtree has value q_1, then the whole tree has value q.

Question: how do tree-walking automata relate to regular languages?

$$\mathrm{TWA} \subseteq \mathrm{REG}$$

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$$\langle Q, q_I, \Sigma, \delta \rangle$$

we associate a branching automaton that accepts the same trees.

States $P(Q \times Q)$

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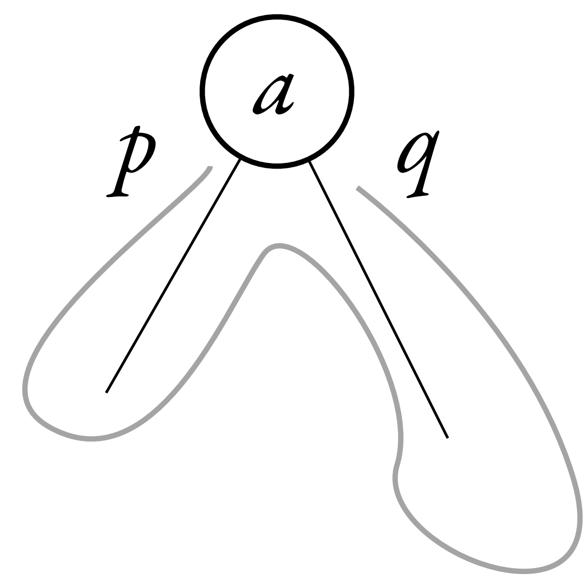

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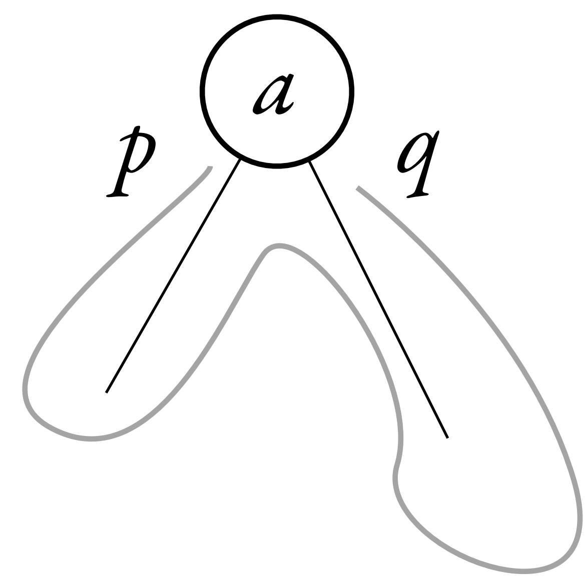

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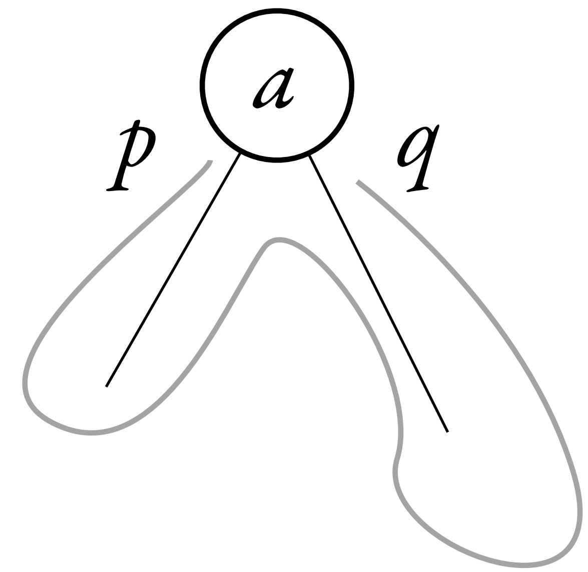

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Corollary. Emptiness for tree-walking automata is in EXPTIME.

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Hardness. Reduction from APSPACE.

For an alternating Turing machine that uses n memory cells, we write a tree-walking automaton with equivalent emptiness and $O(n)$ states.

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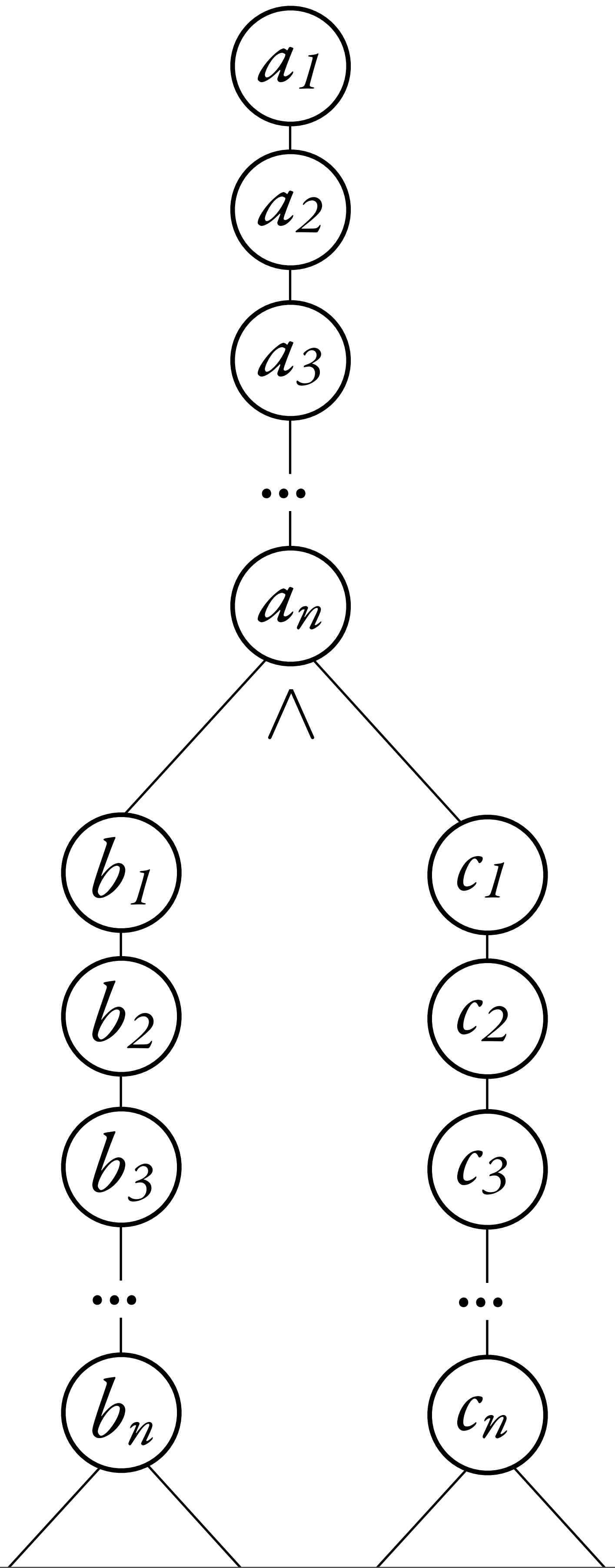

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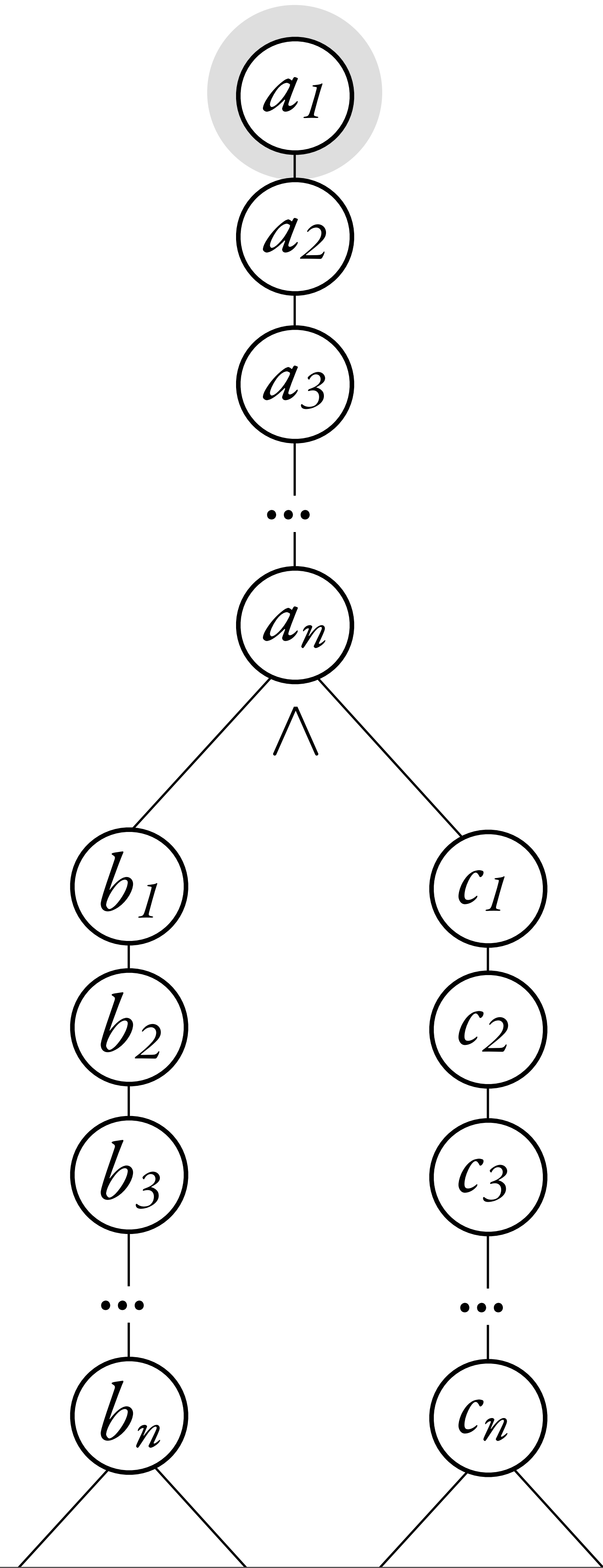

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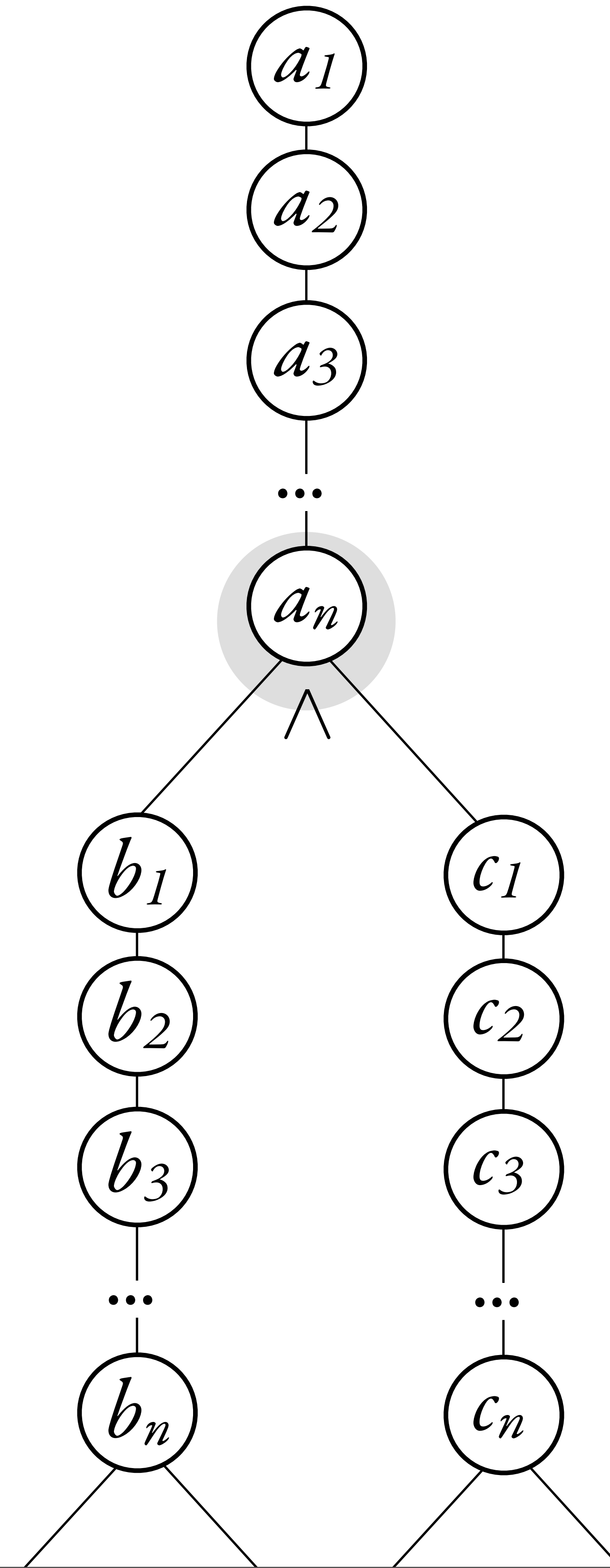

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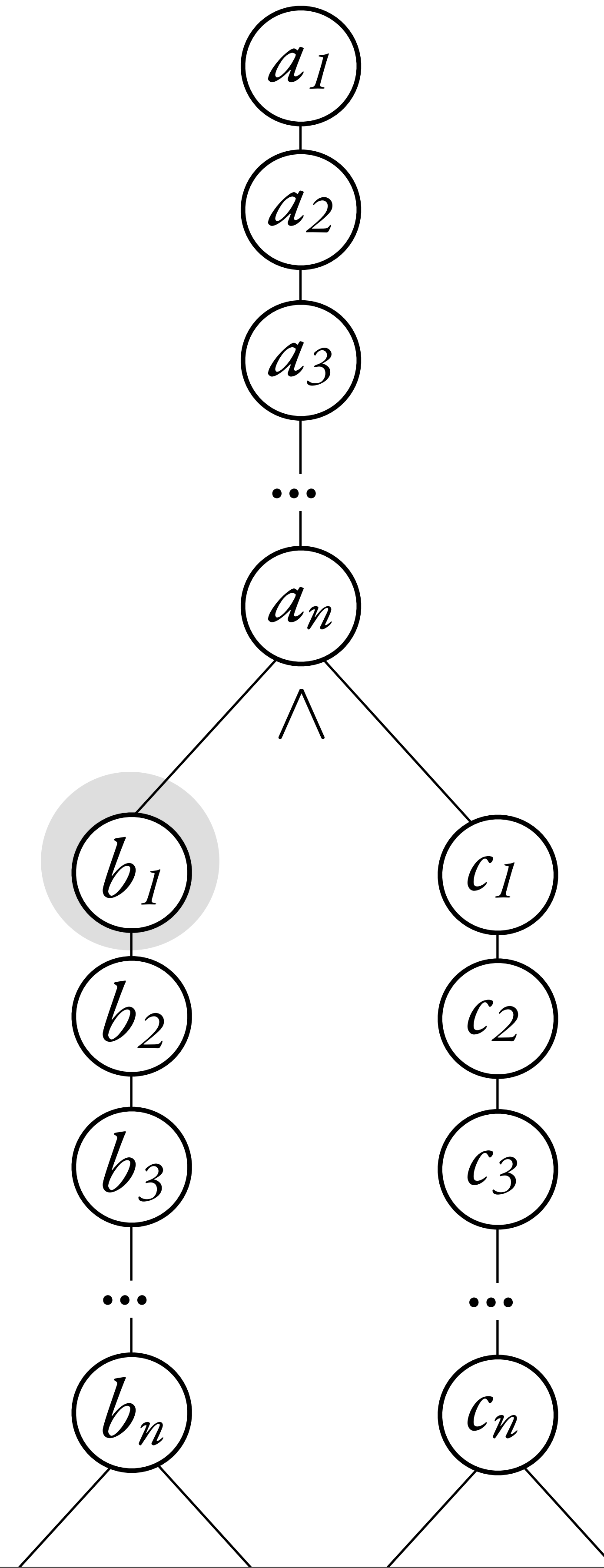

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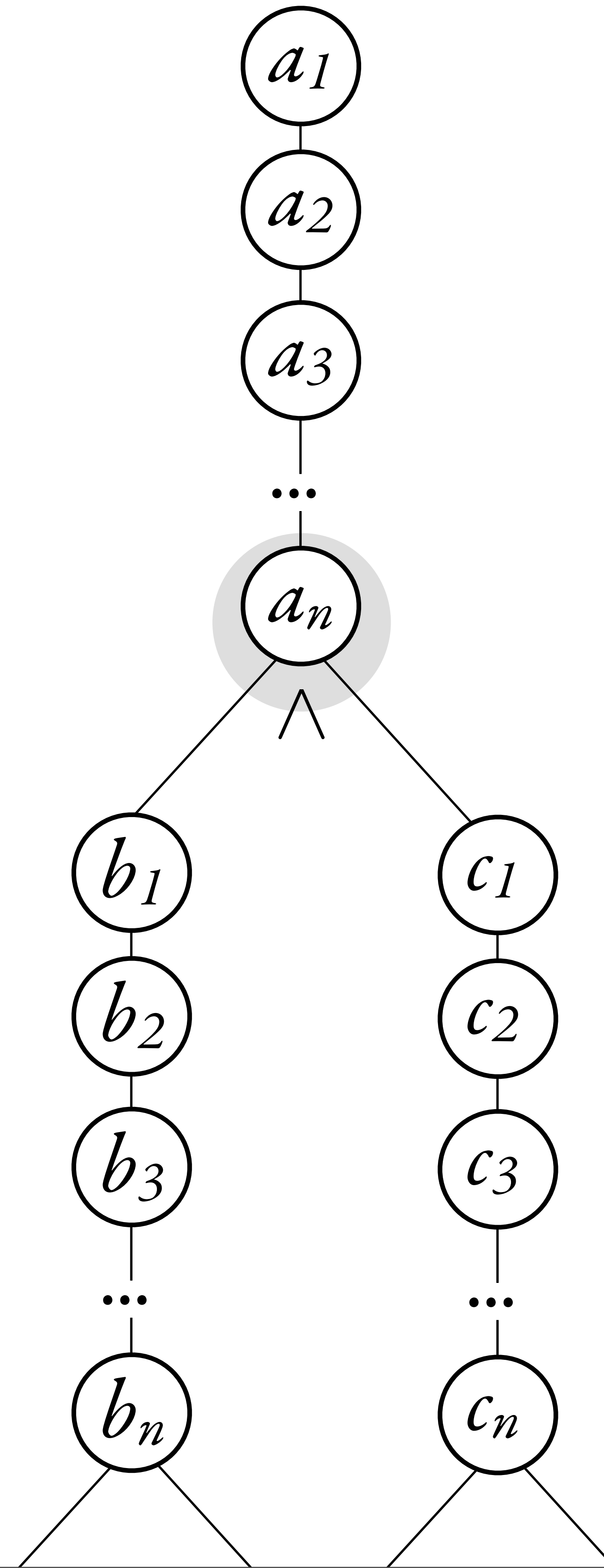

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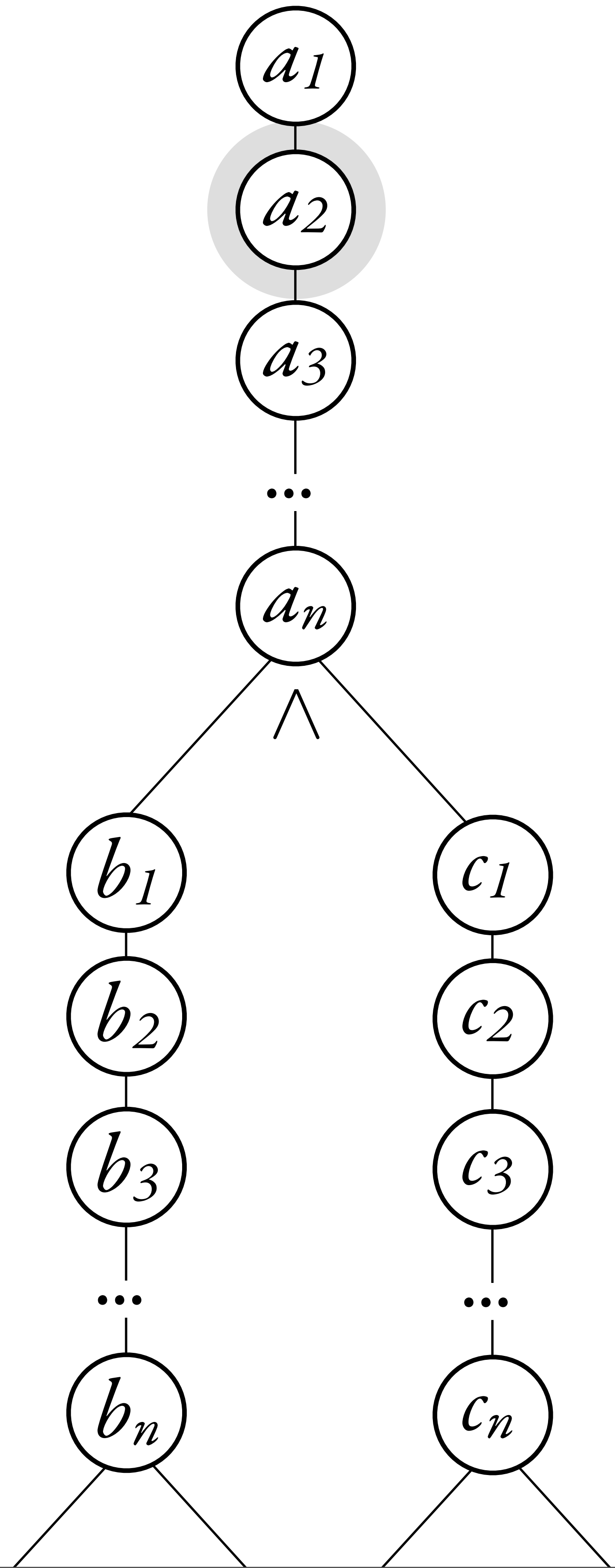

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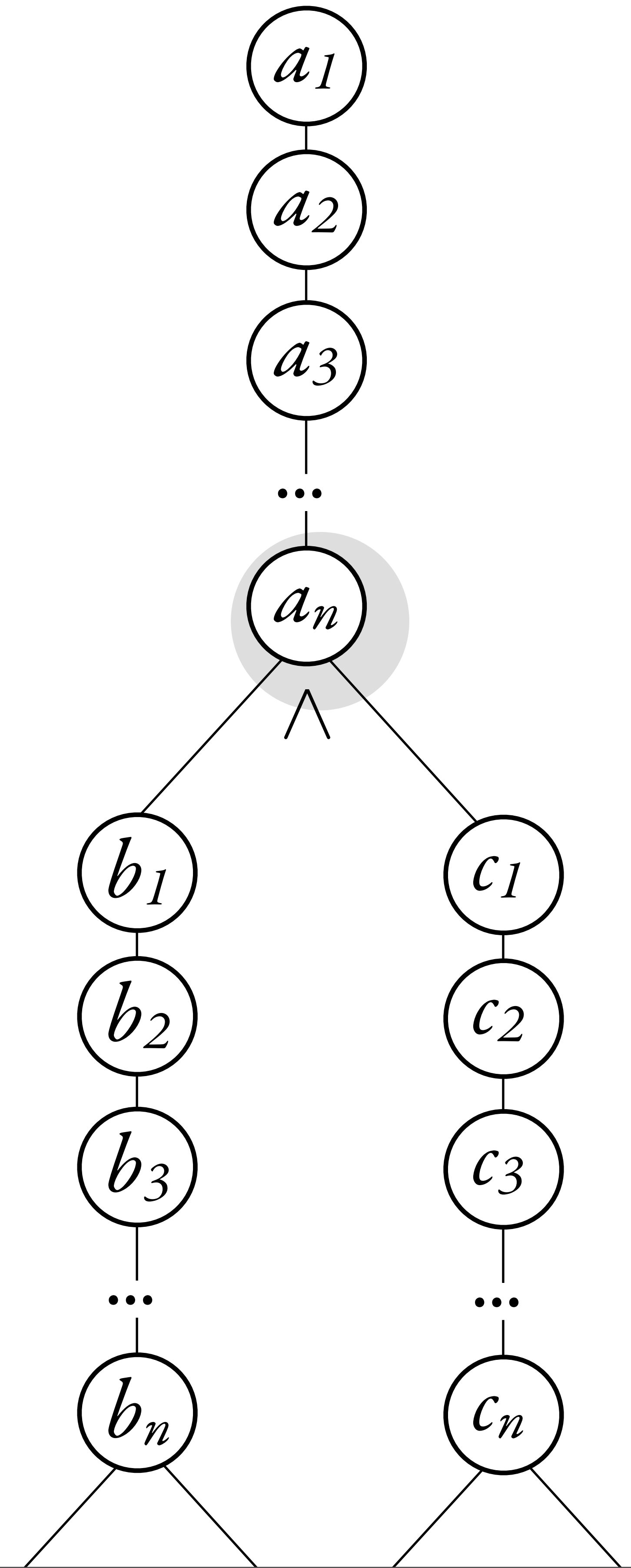

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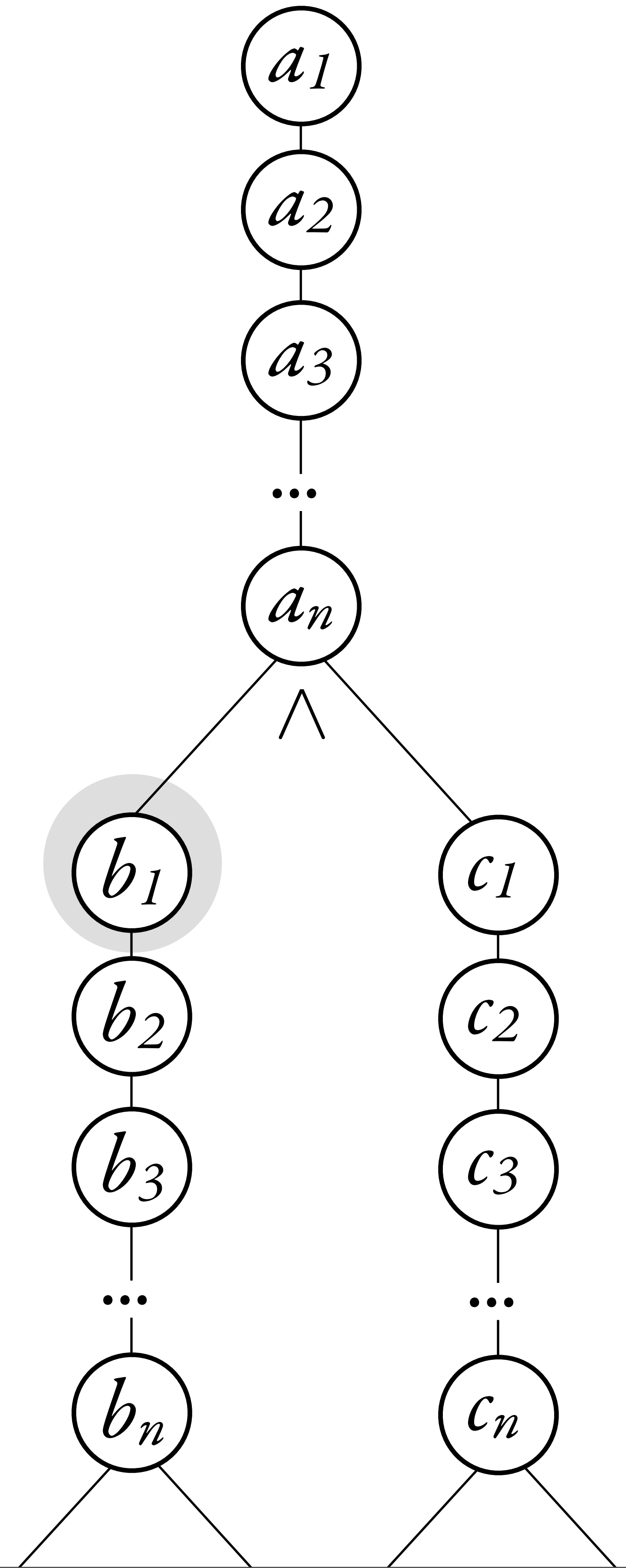

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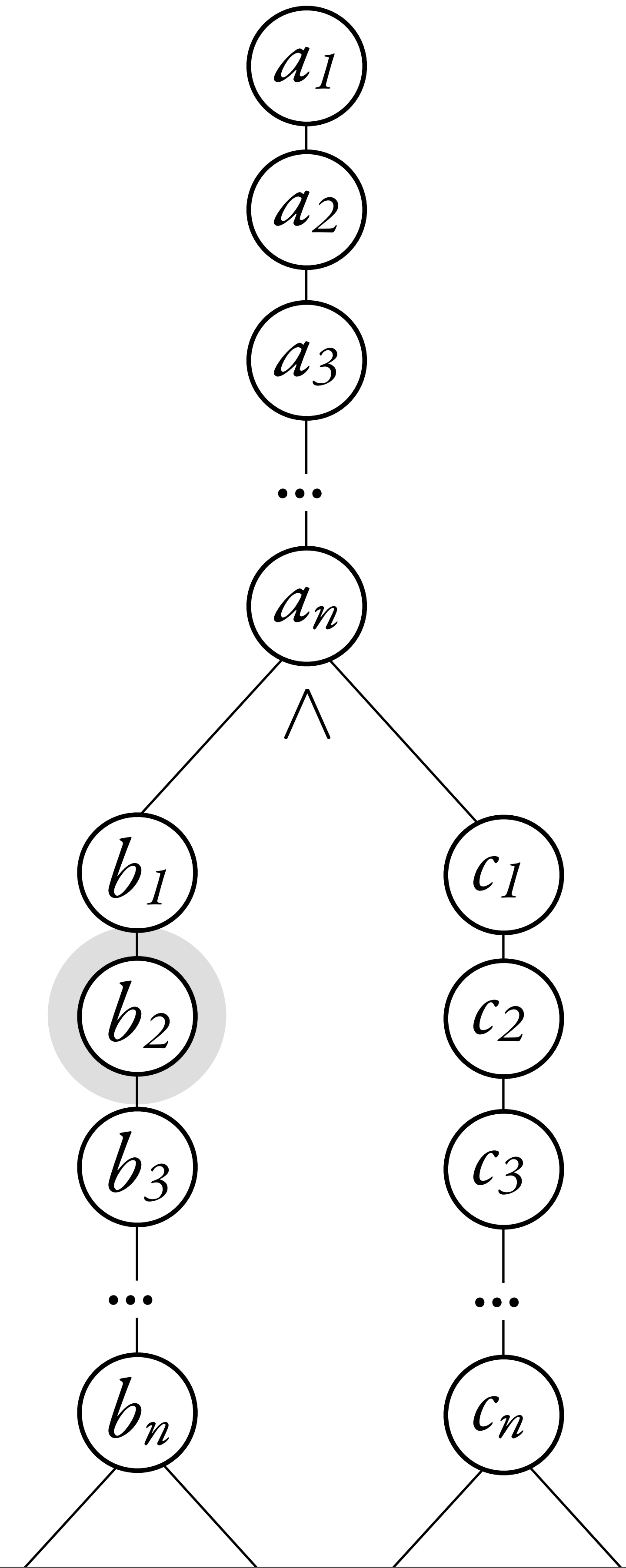

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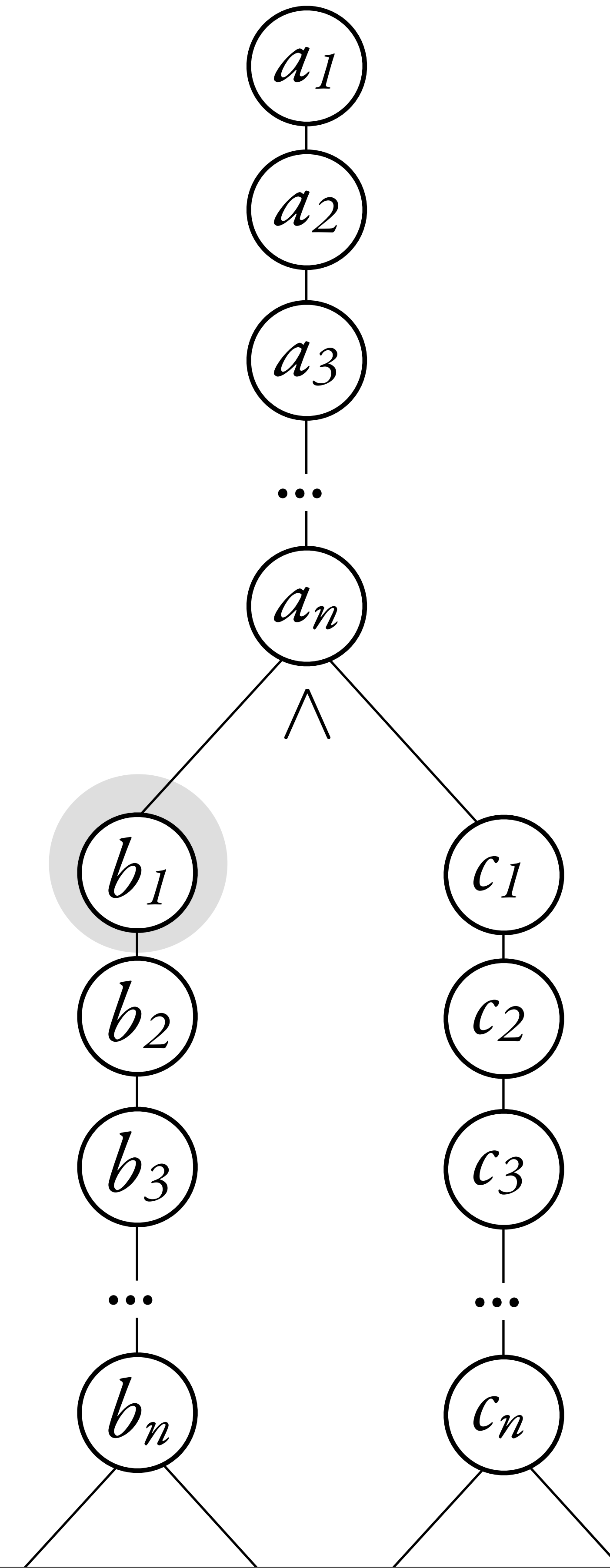

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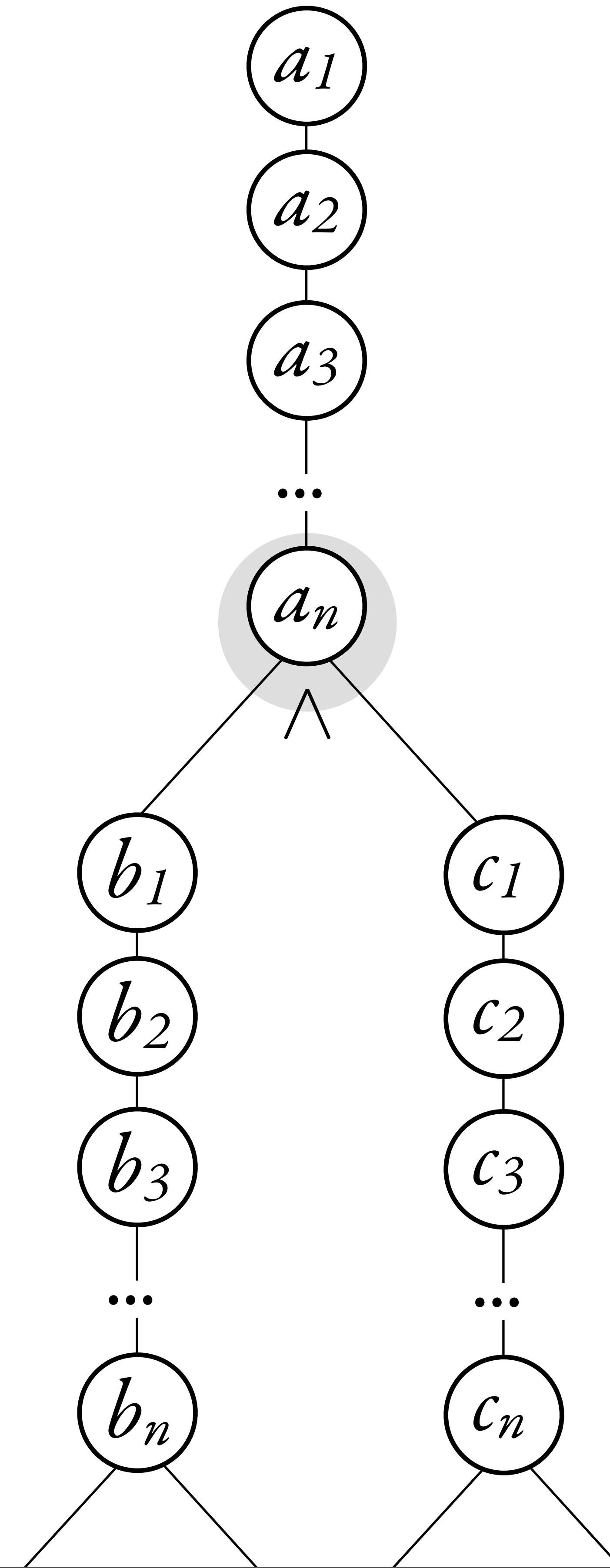

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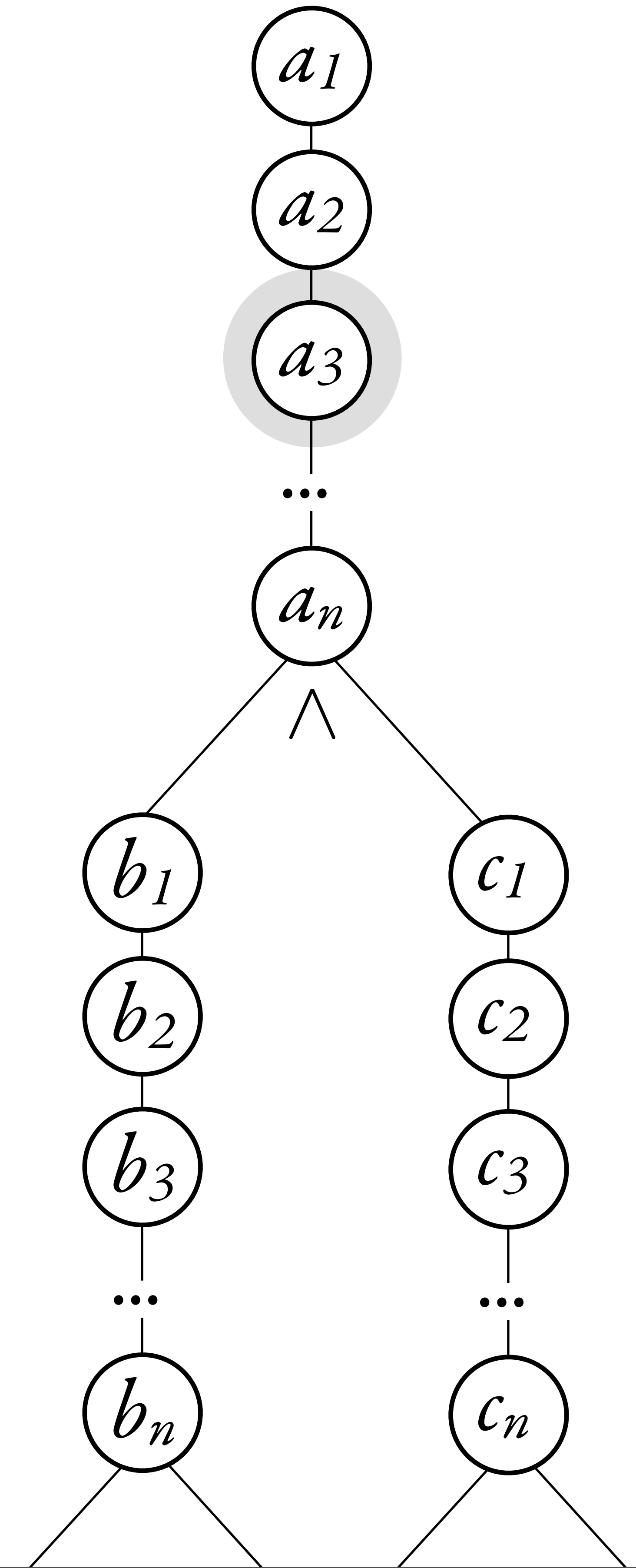

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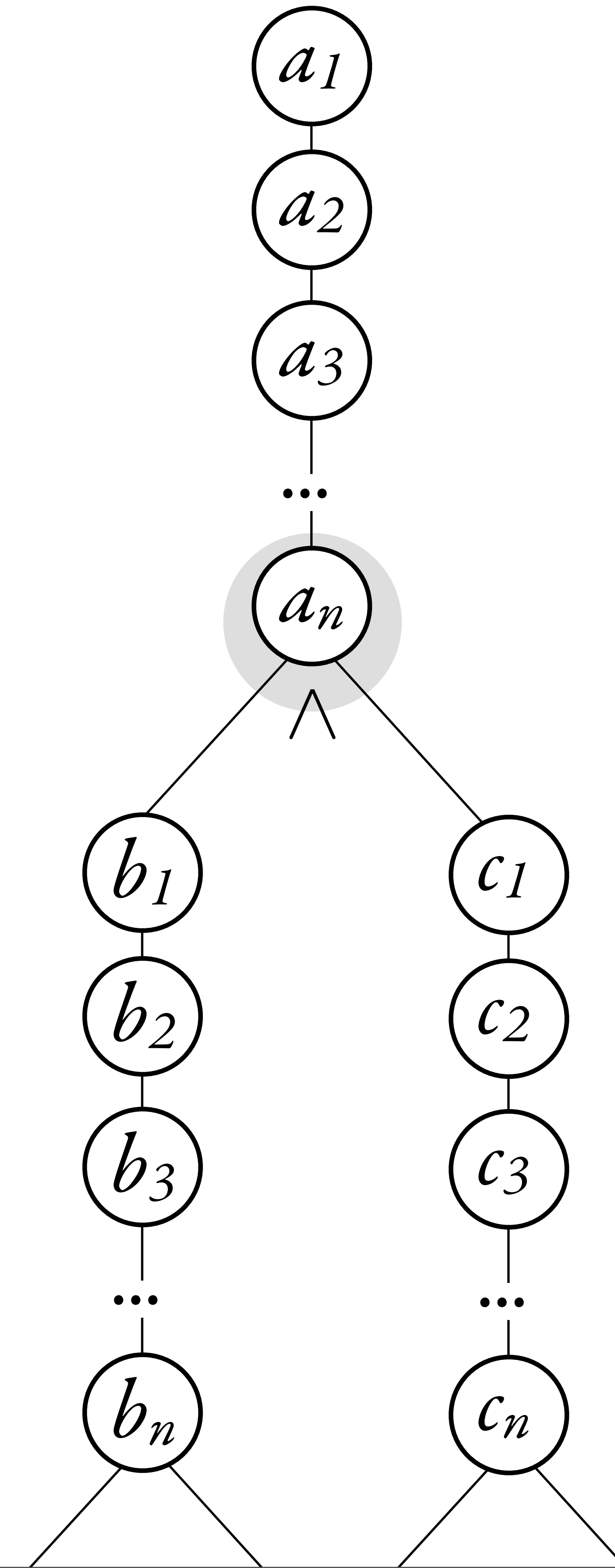

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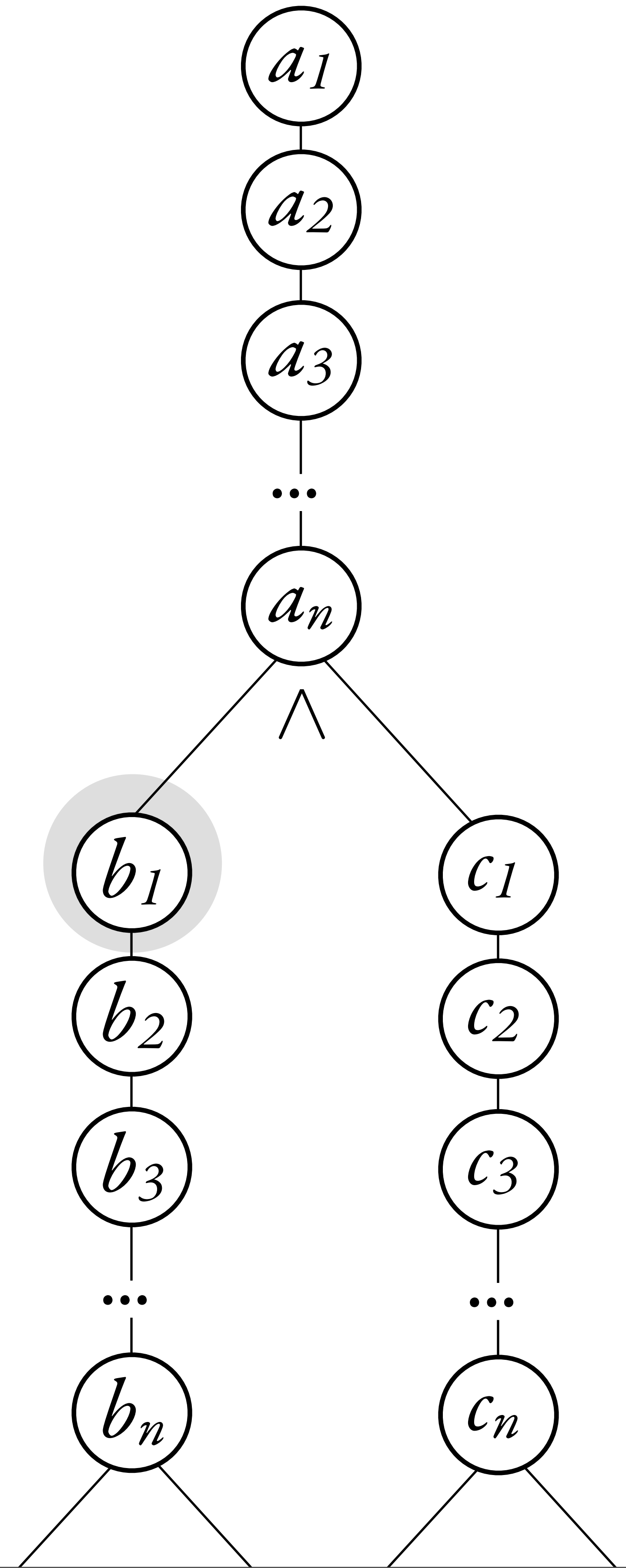

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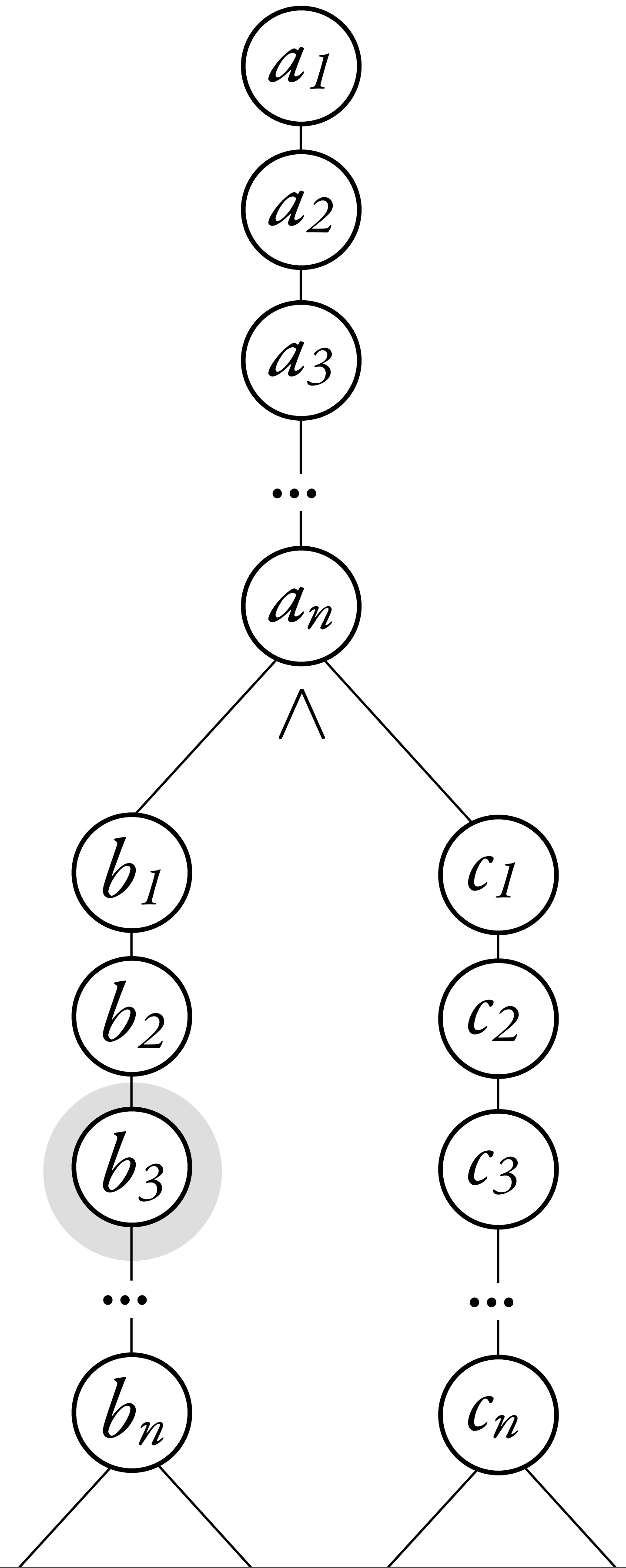

TWA $\subseteq$ REG

Is the inclusion strict?

Theorem (B., Colcombet '05) The inclusion is strict.

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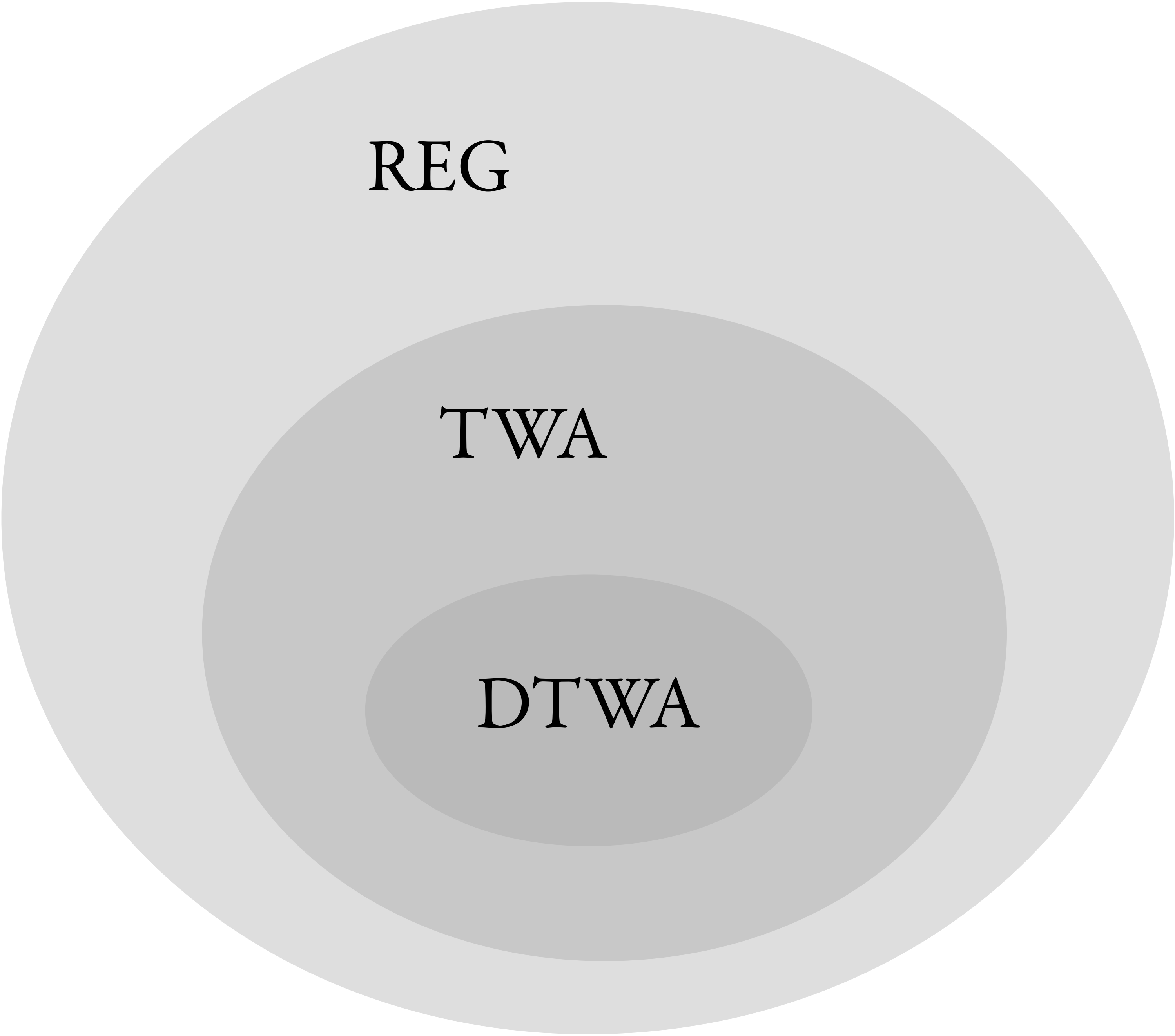
Theorem (B., Colcombet '04)
Tree-walking automata cannot be determinized.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What if the automaton could mark nodes with pebbles?

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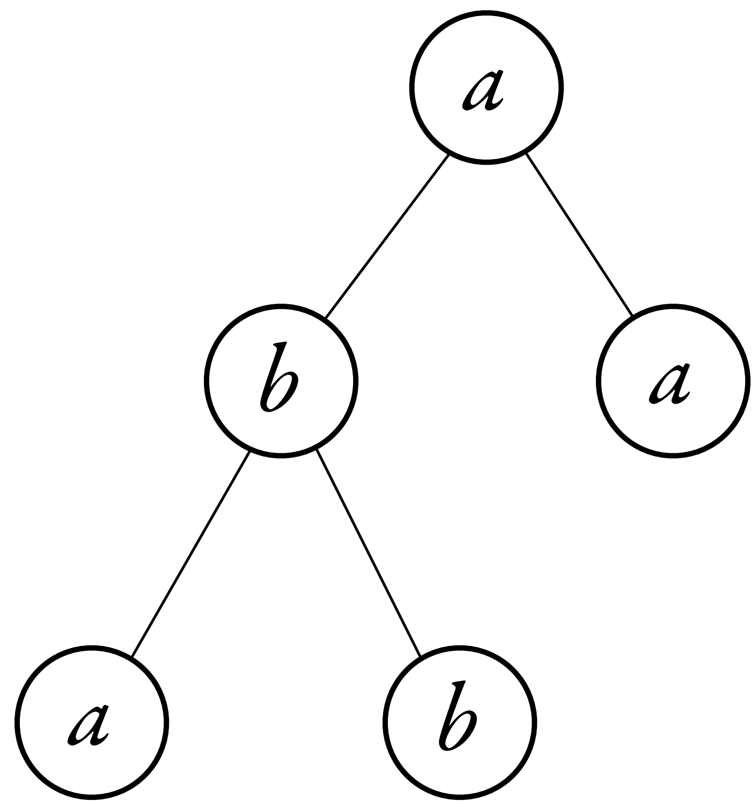

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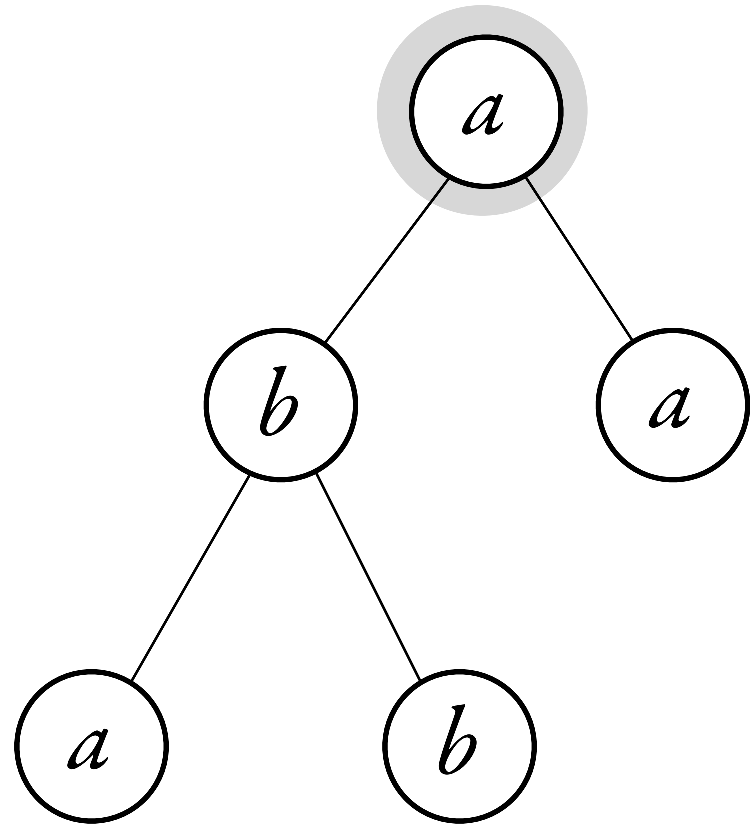

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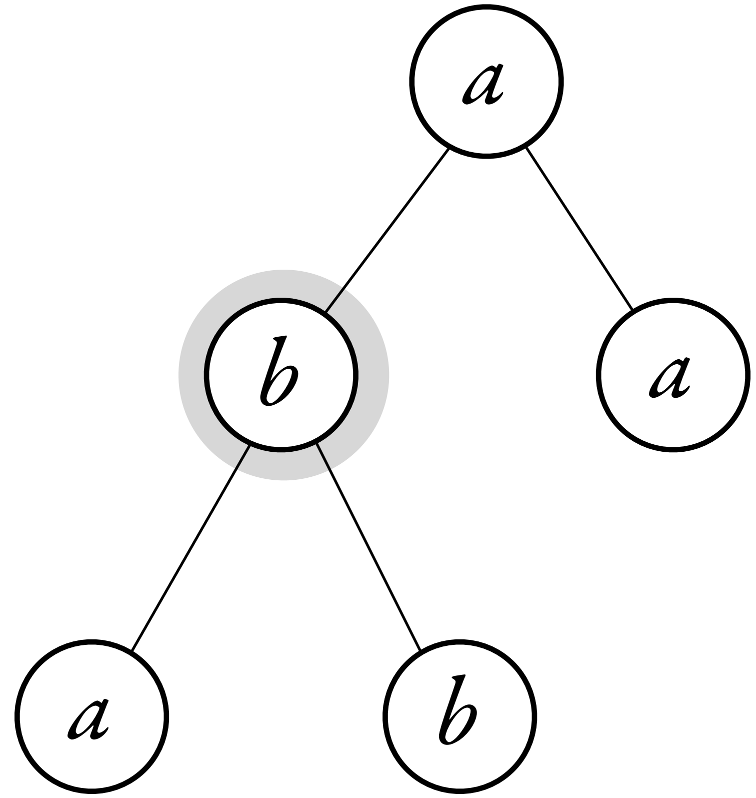

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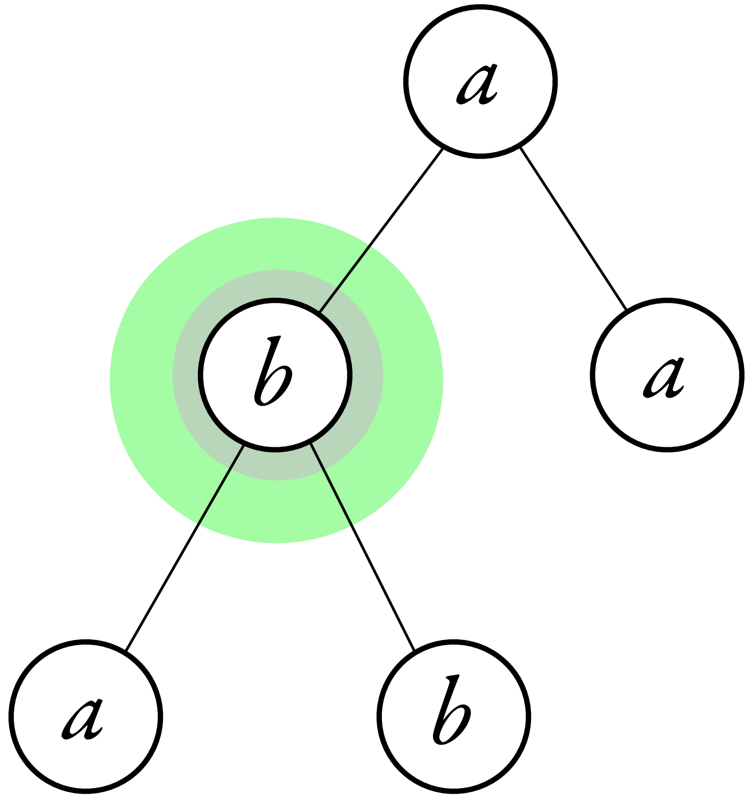

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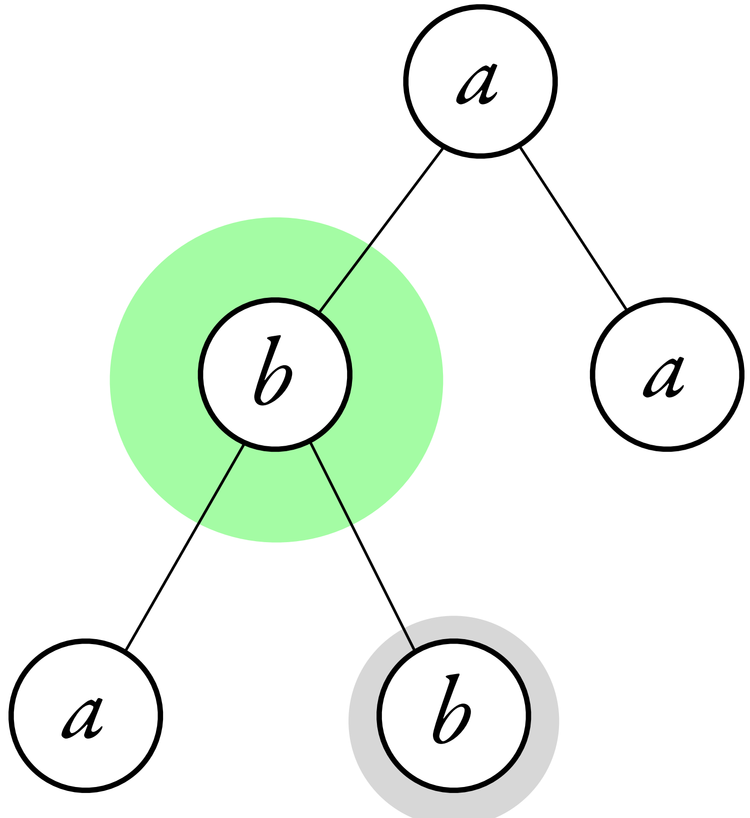

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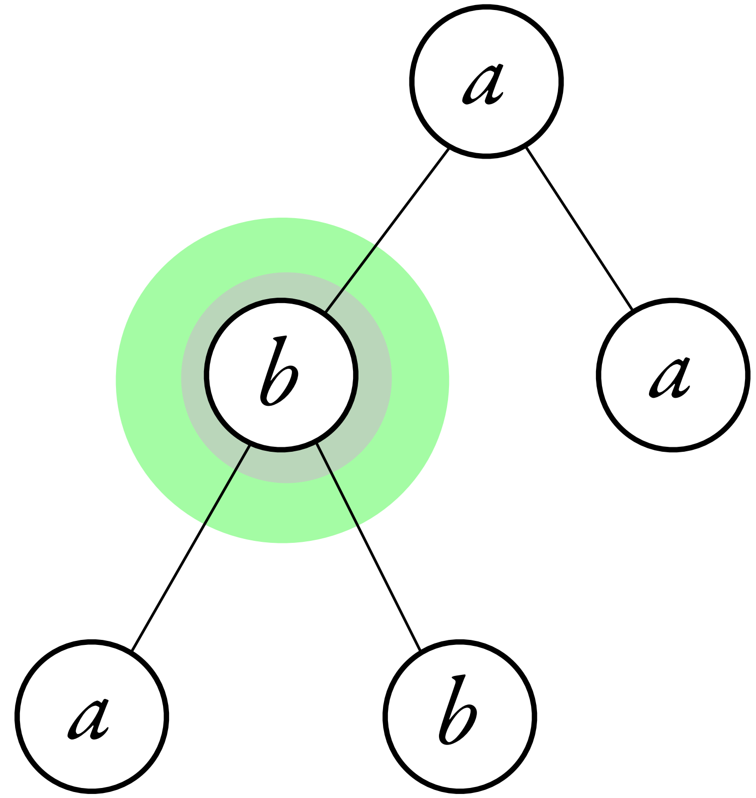

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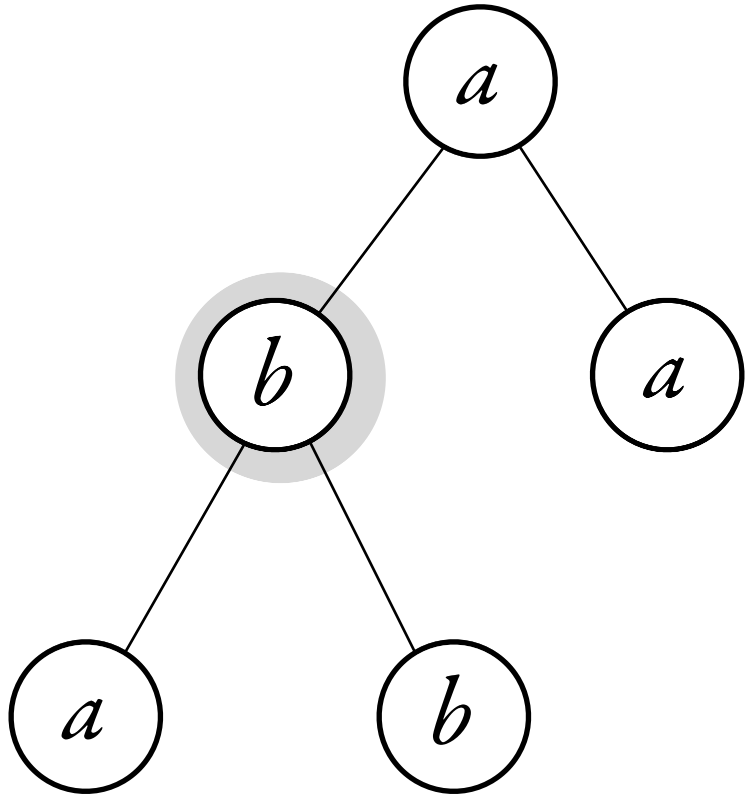

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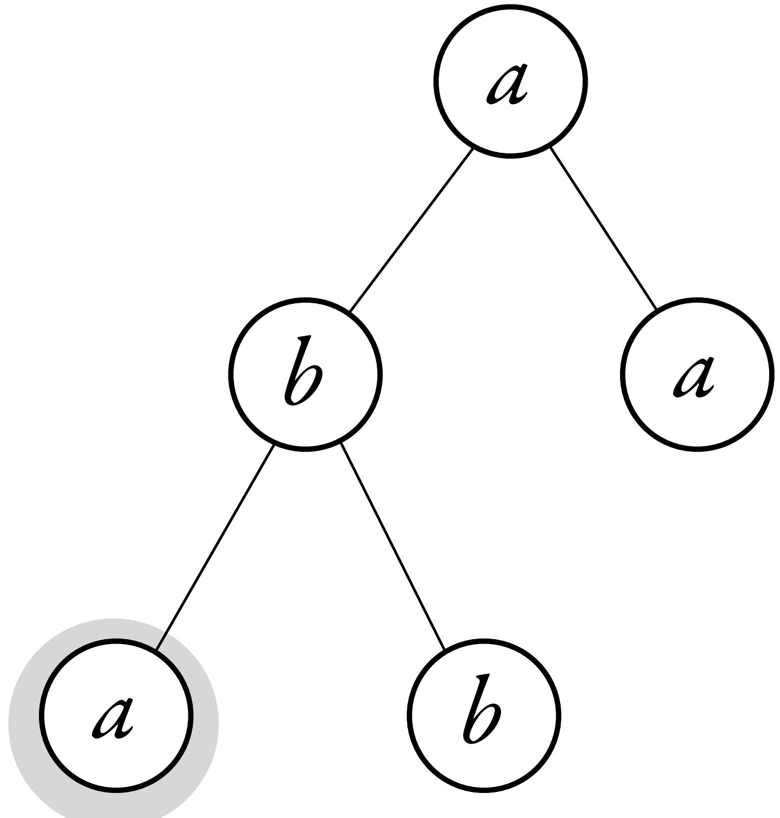

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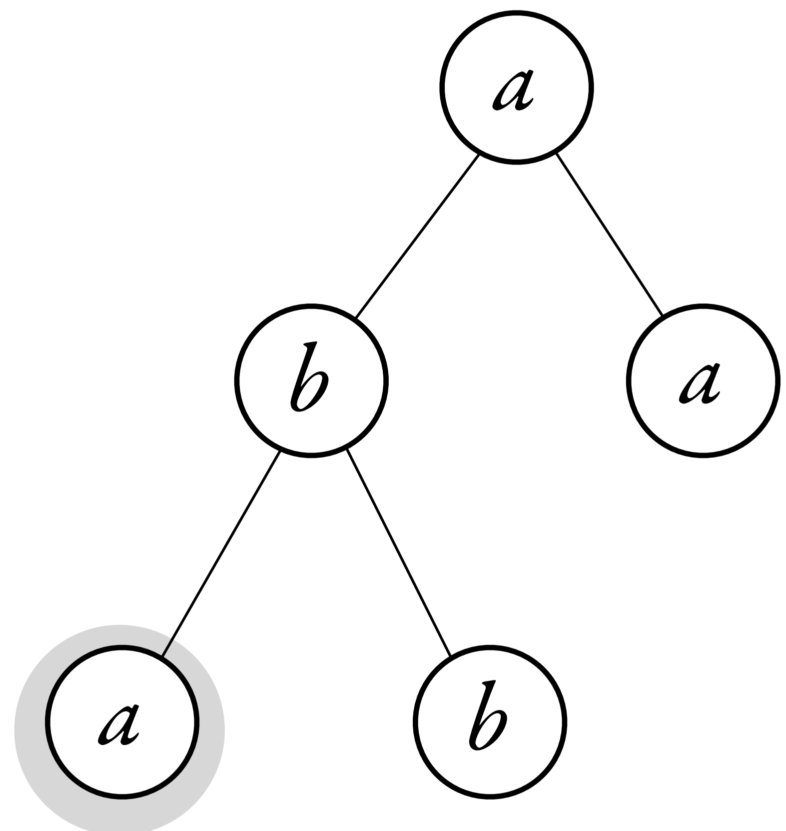
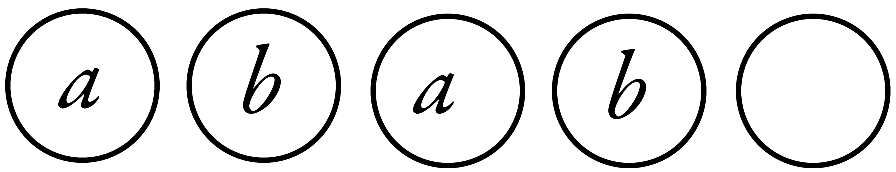
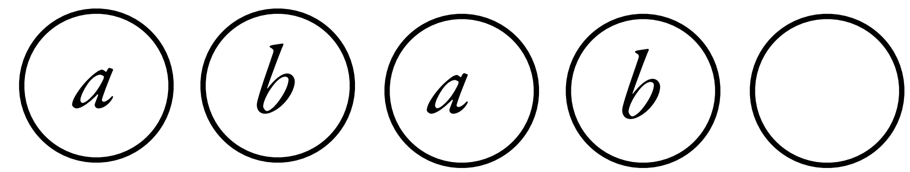


An *n-pebble automaton* has pebbles *1,...,n*.
New tests: "is pebble *i* on the current node?"
New commands: "place pebble *i* on the current node"
"lift pebble *i* from the current node".

Bad news: emptiness is undecidable for pebble automata, even on words and with two pebbles.





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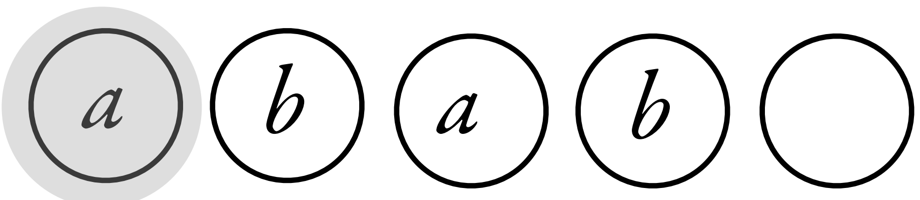

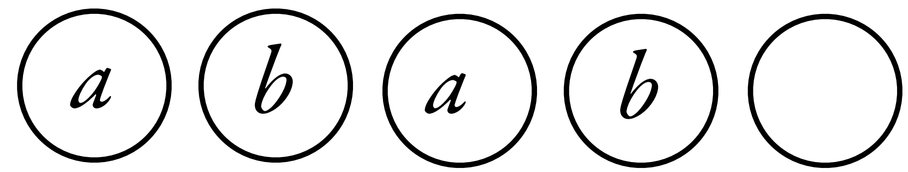

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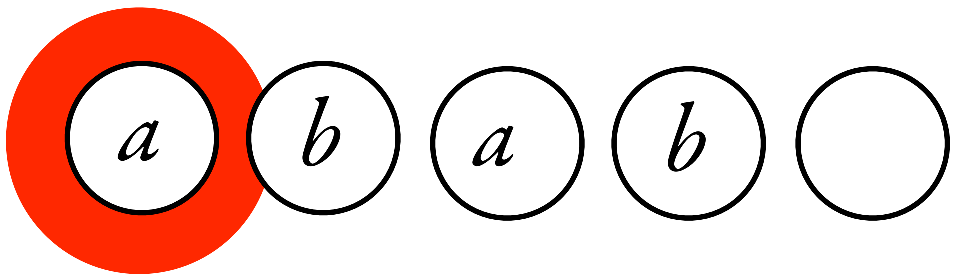

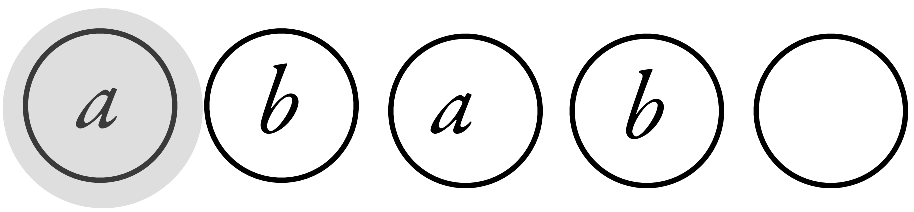

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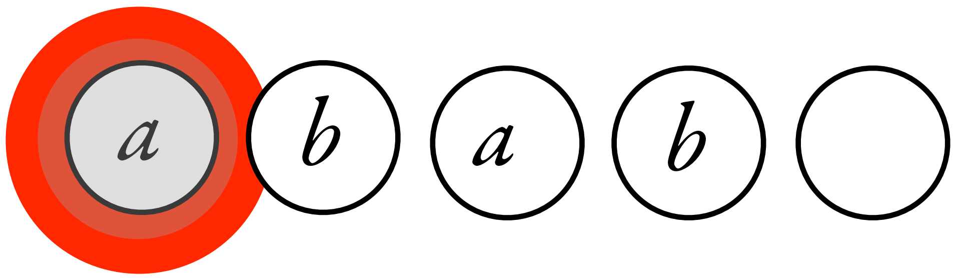

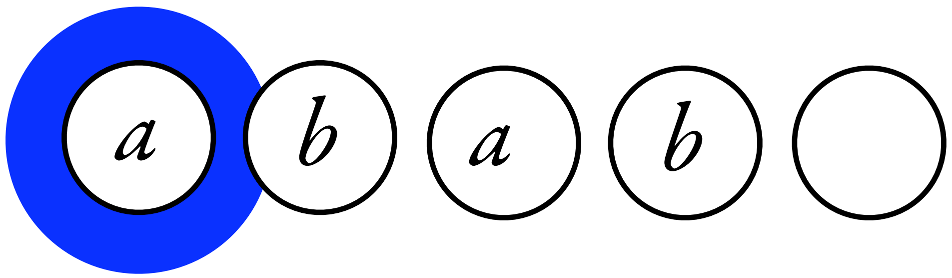

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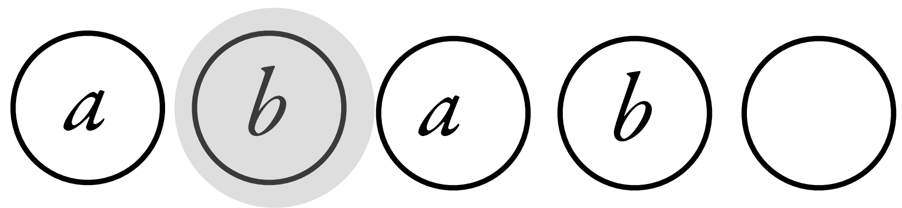

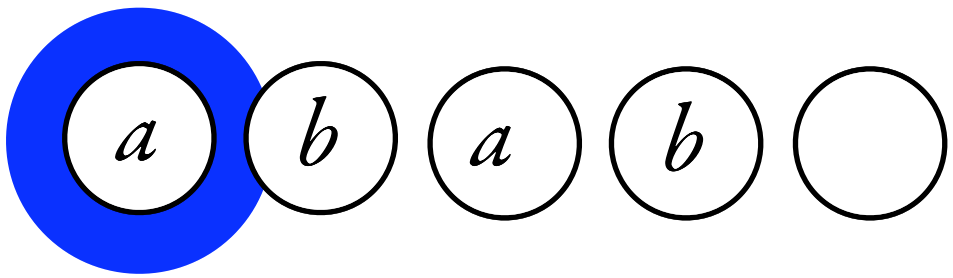

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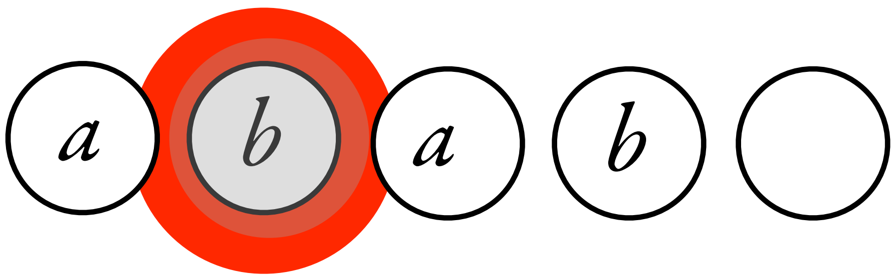

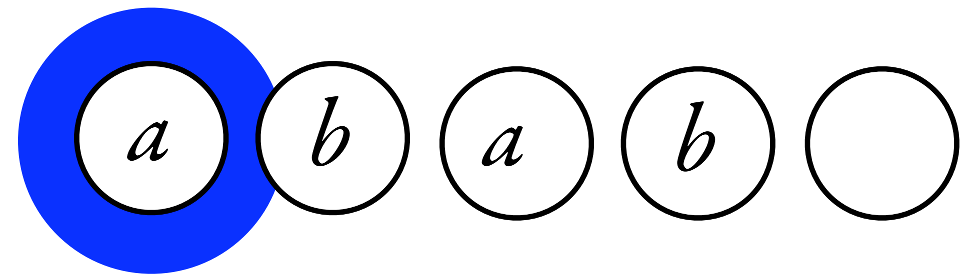

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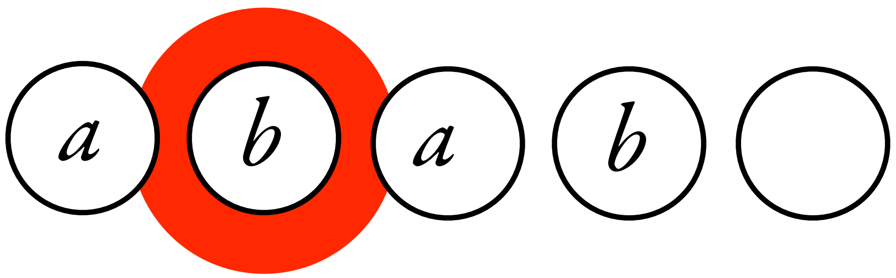

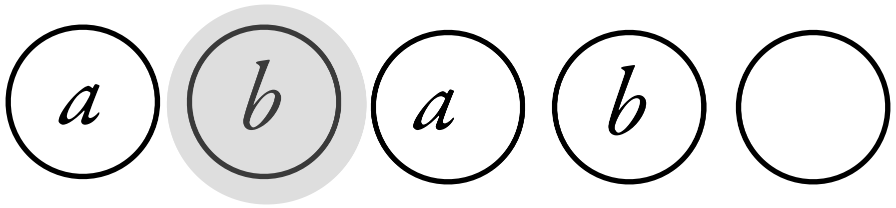

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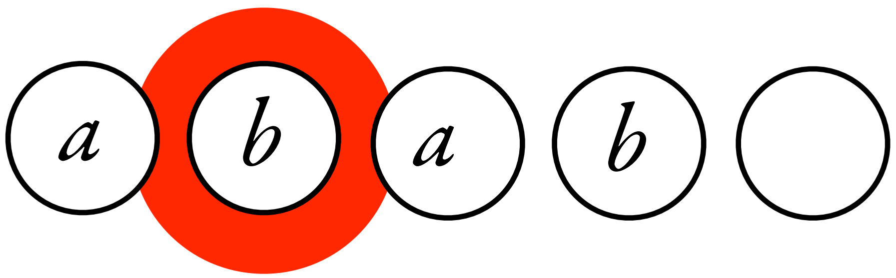

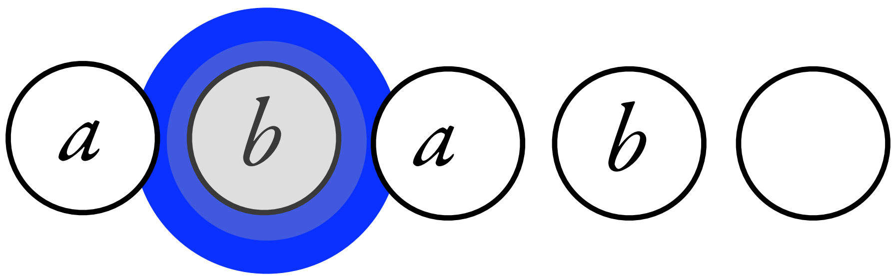

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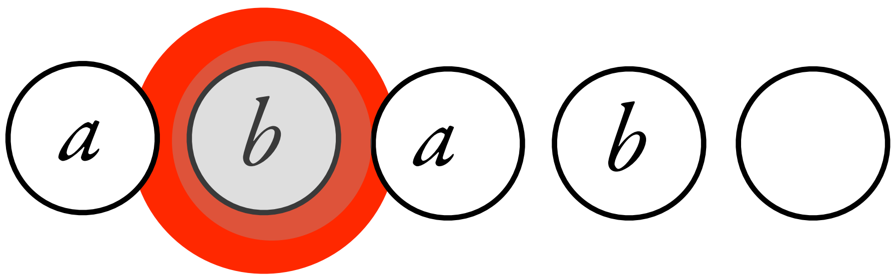

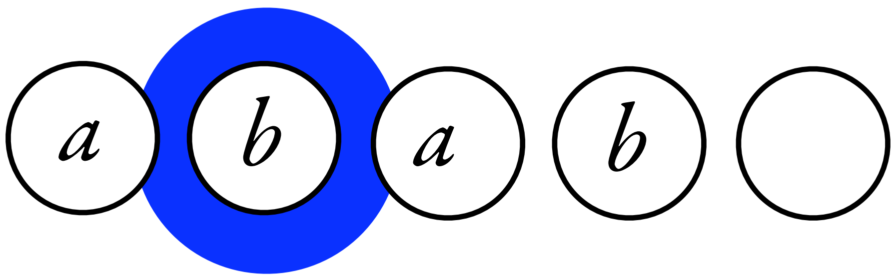

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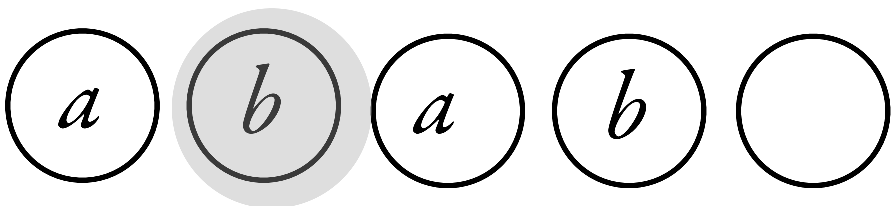

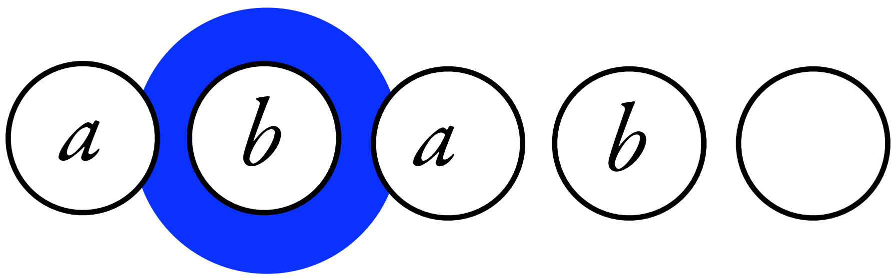

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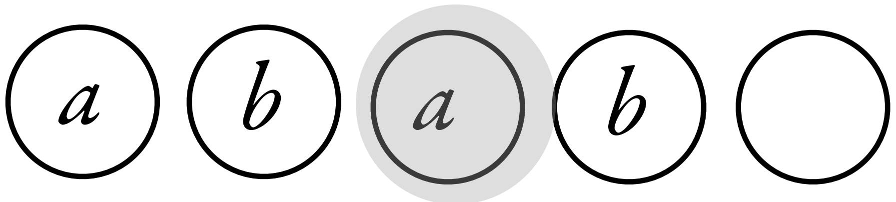

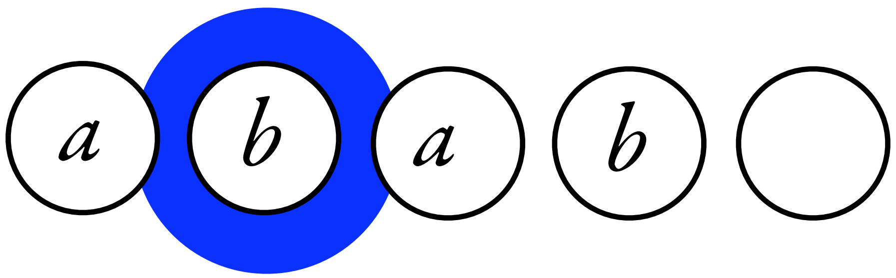

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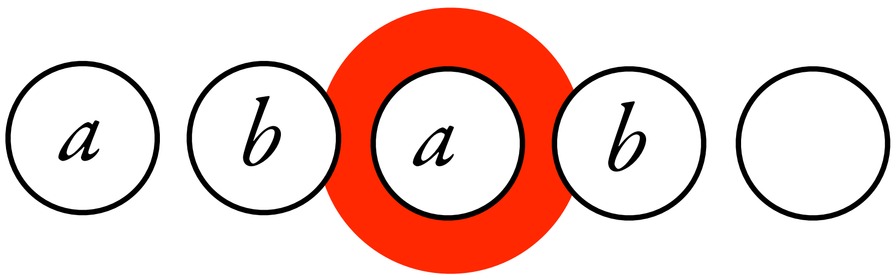

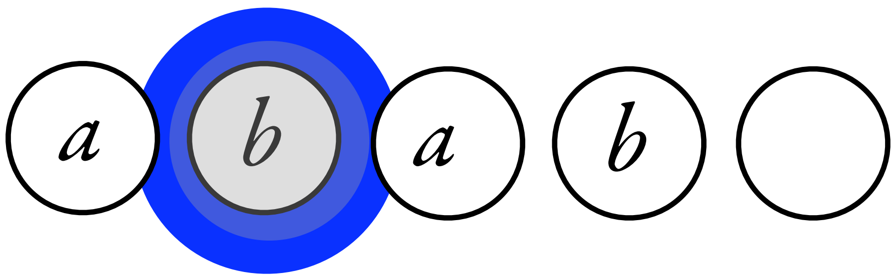

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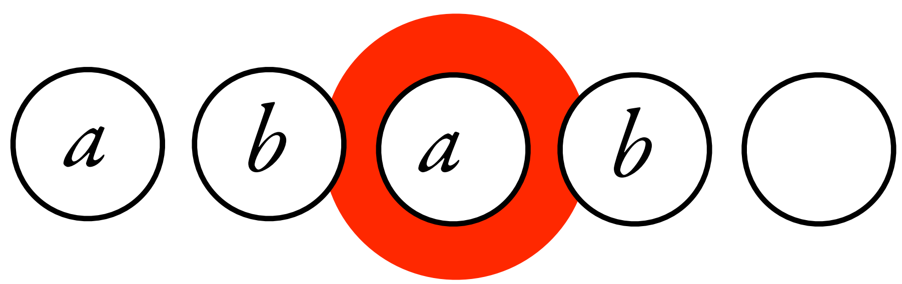

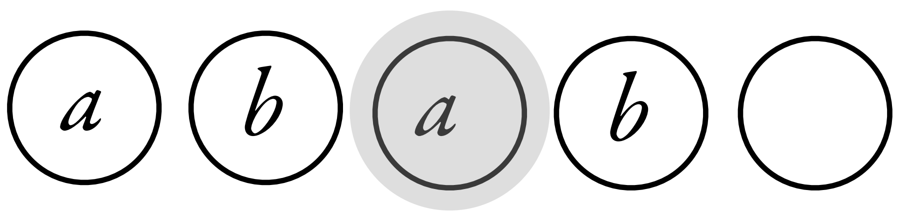

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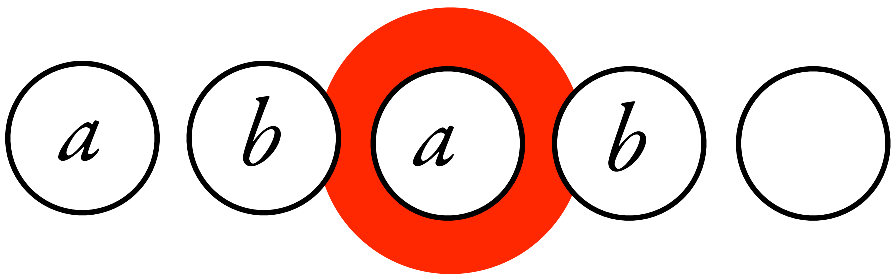

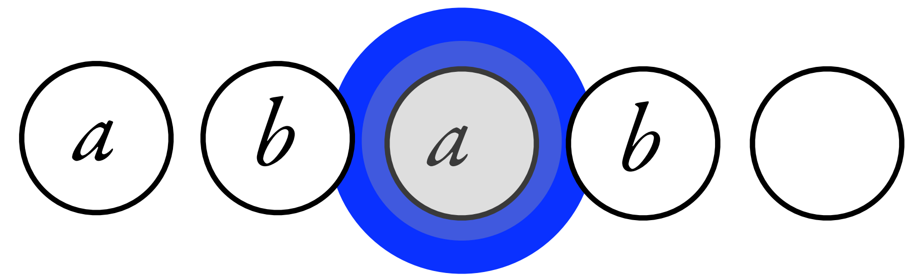

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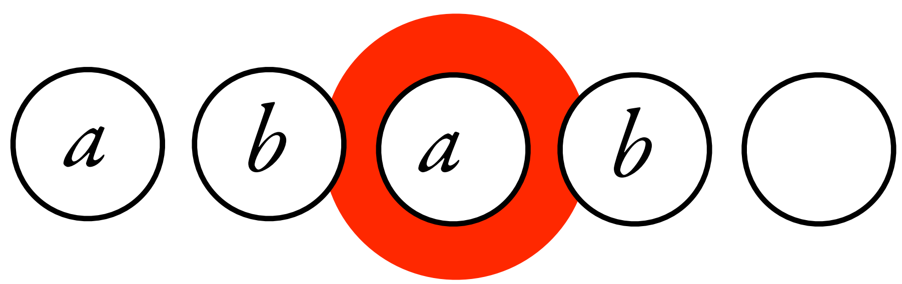

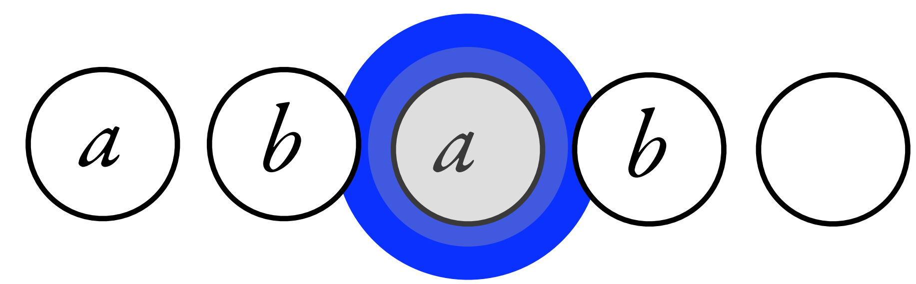

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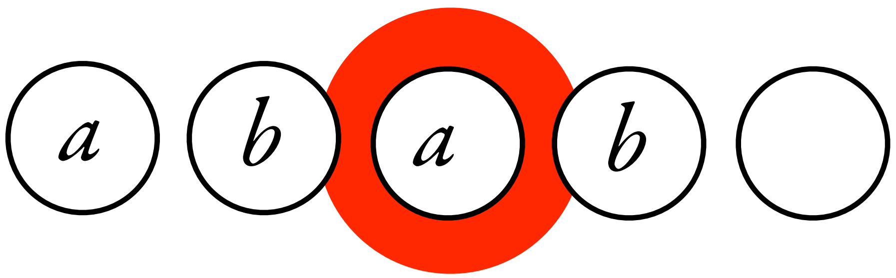

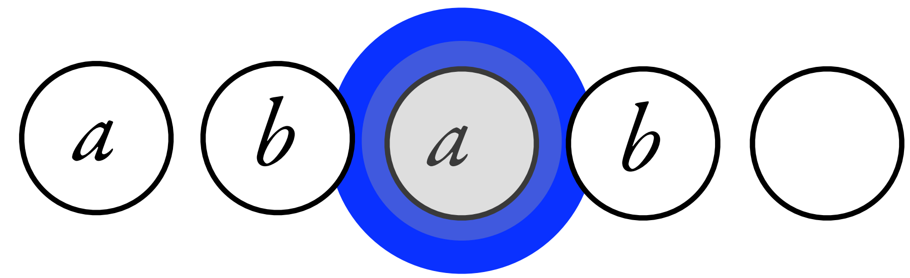

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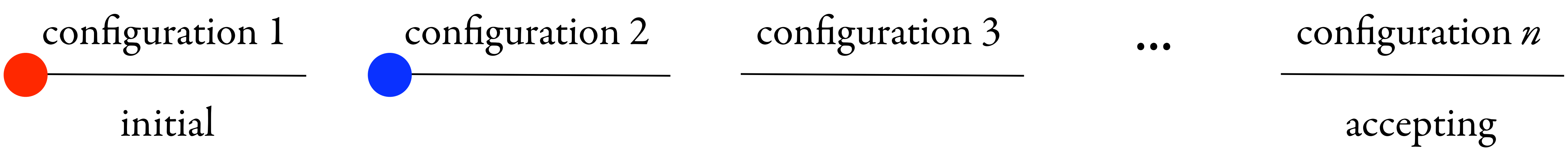

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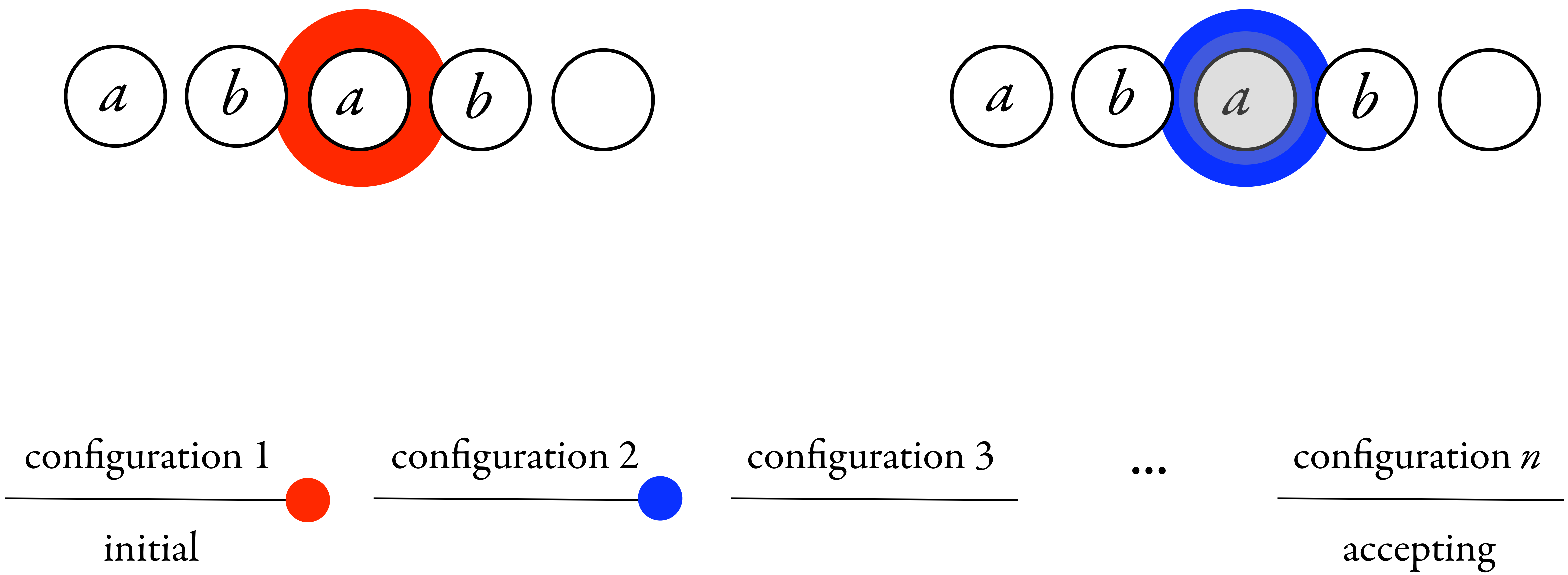

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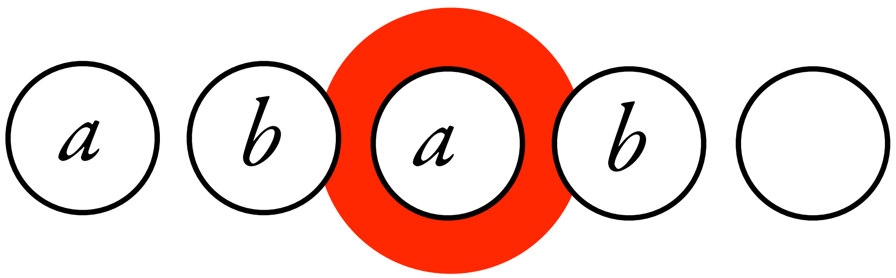

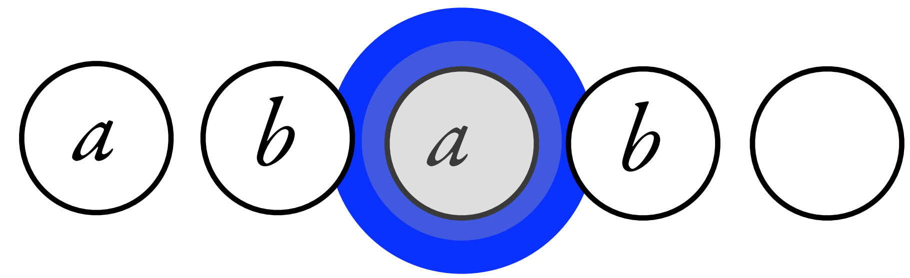

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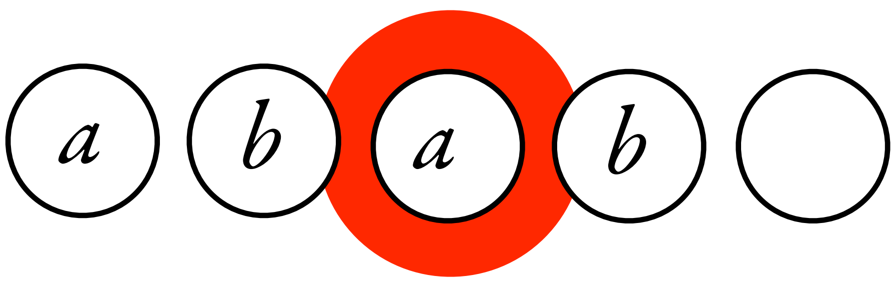

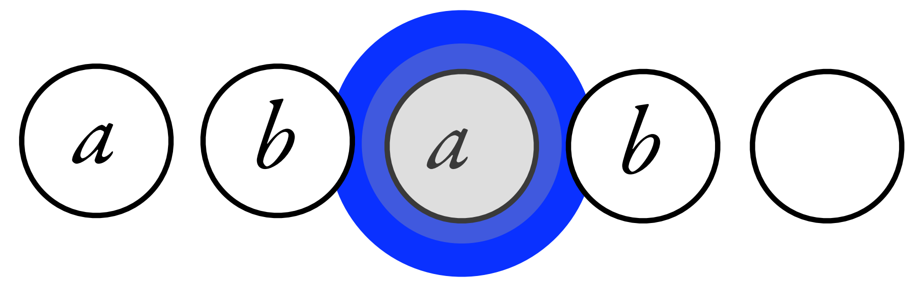

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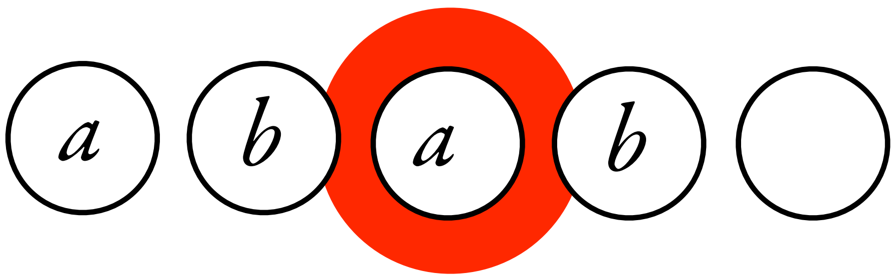

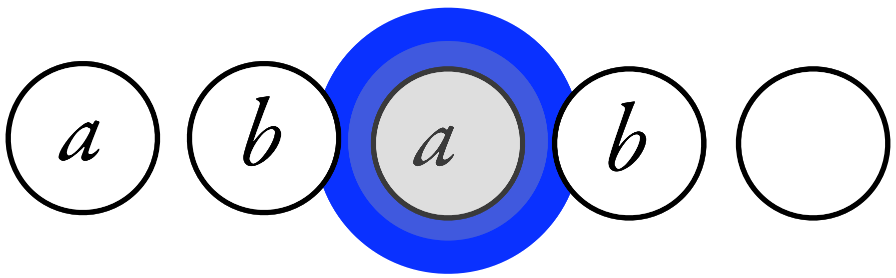

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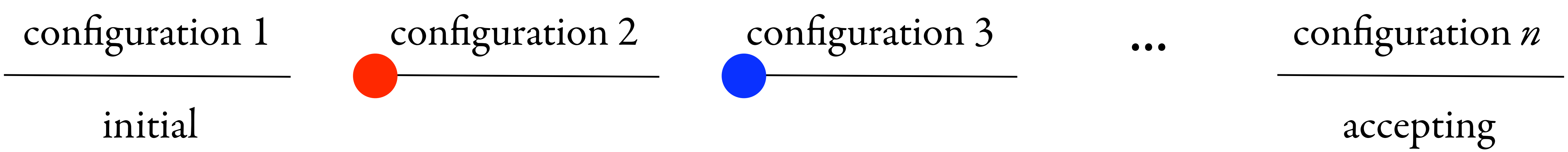

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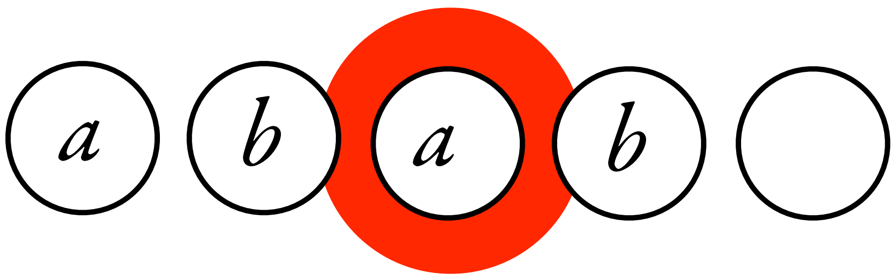

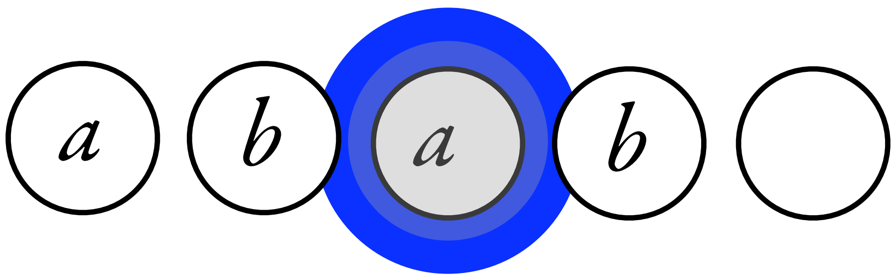

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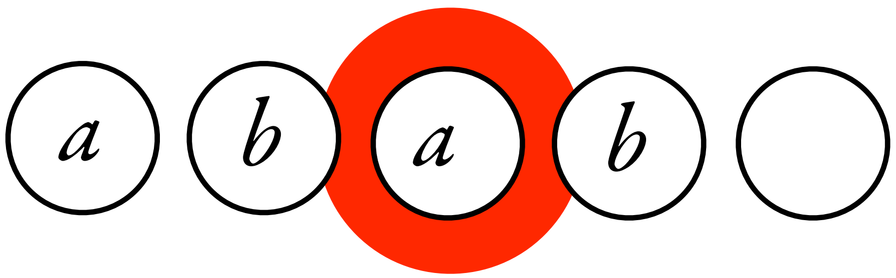

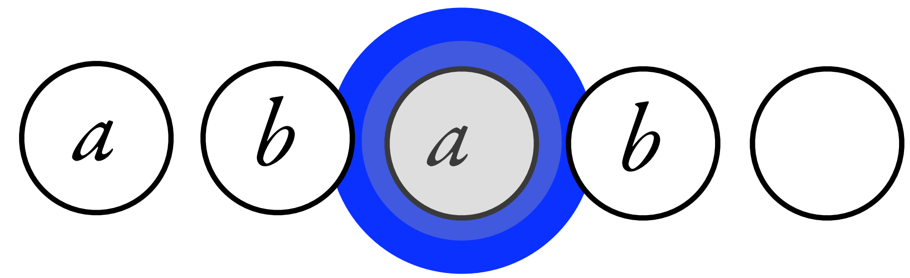

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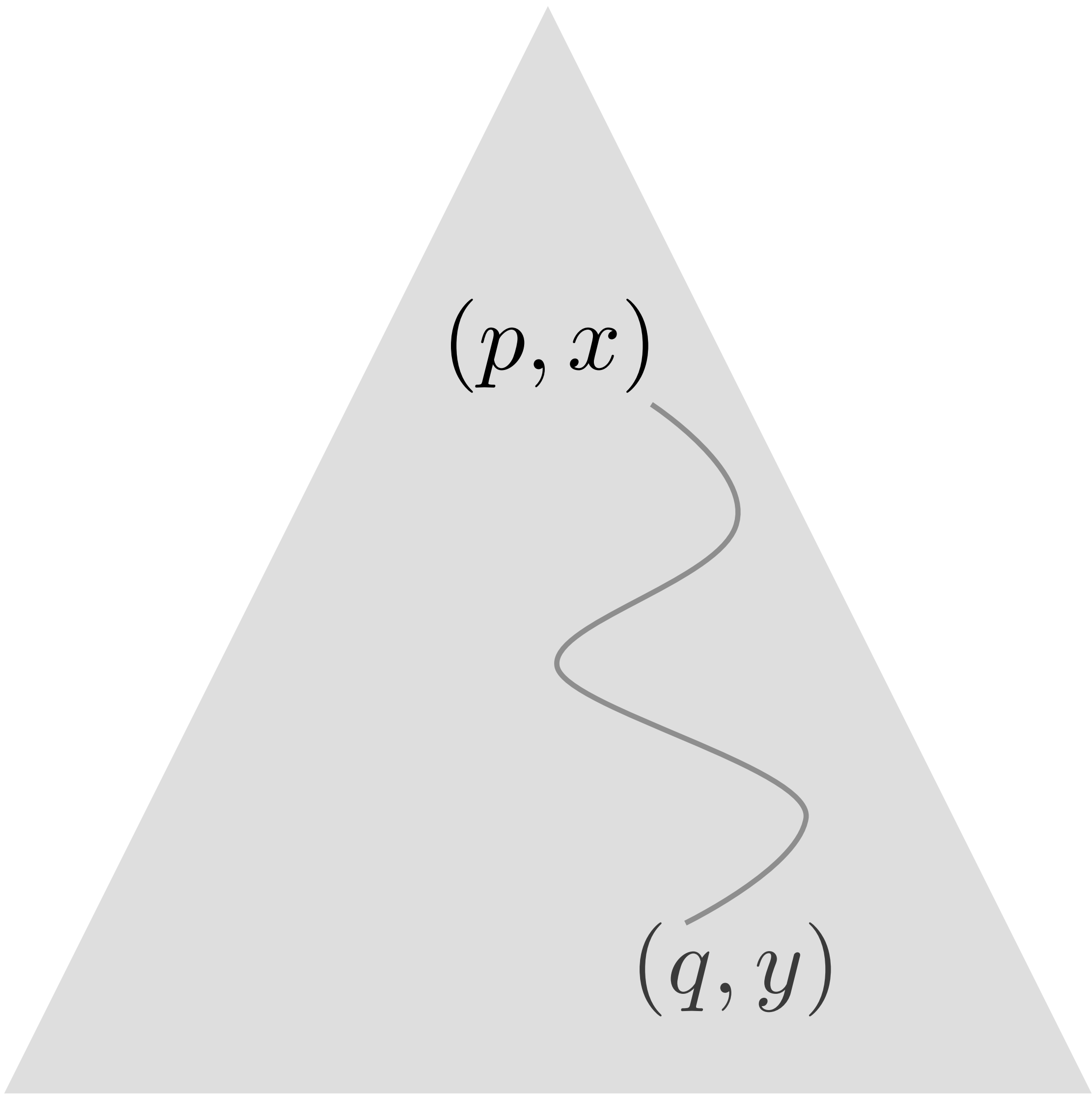
If the pebble automaton has *n* pebbles, the tree automaton may have

$$\overbrace{2^{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}}^{n \text{ times}}$$

states. Likewise, emptiness is non-elementary.

$$\varphi_{p,q}(x, y, x_1, \ldots, x_i)$$

There is a run that begins in (p, x) and ends in (q, y)
The pebbles at the beginning and end are in $x_1, \ldots, x_i$
During the run, pebble x_i is not lifted, but pebbles can be added.



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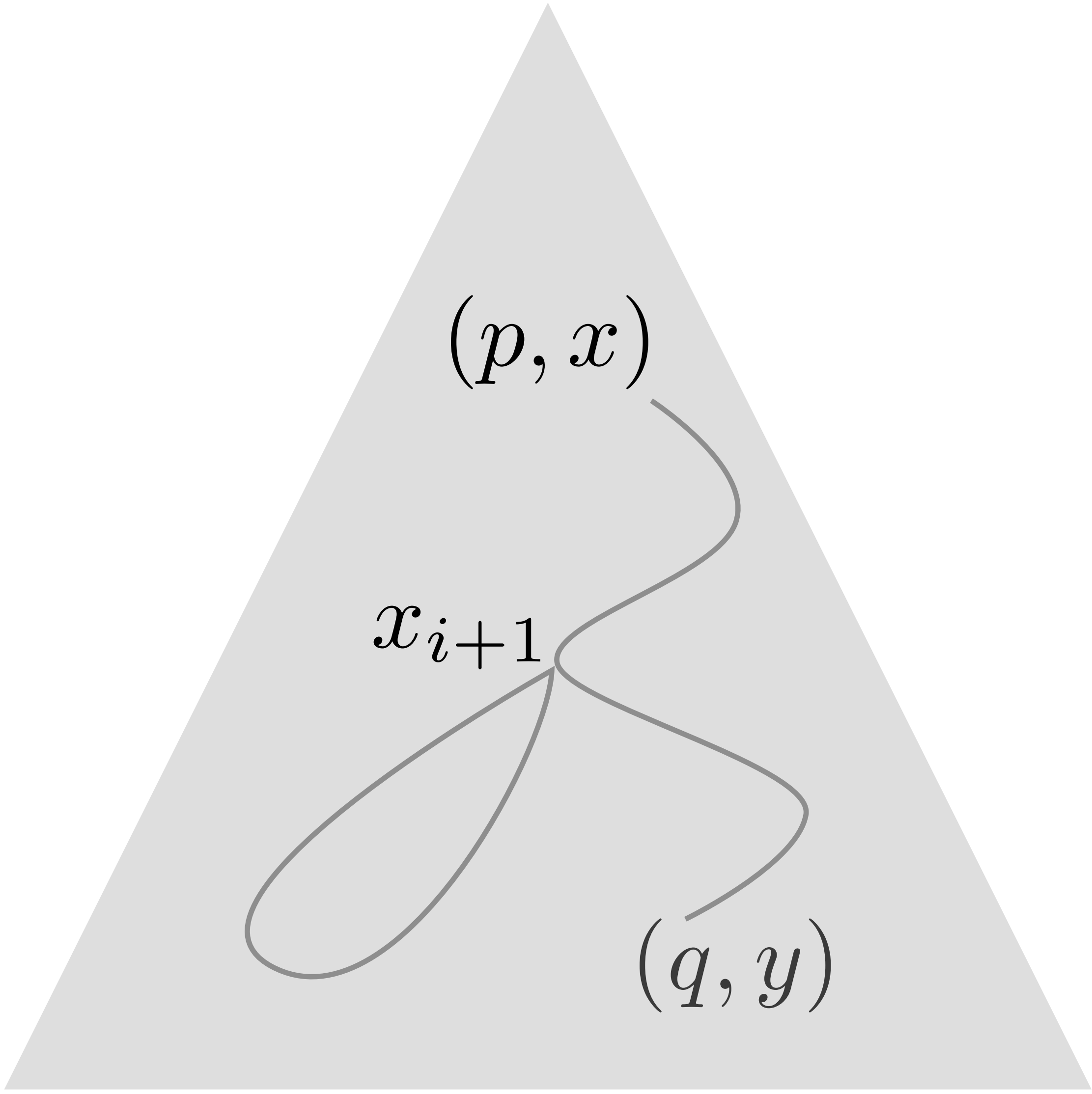

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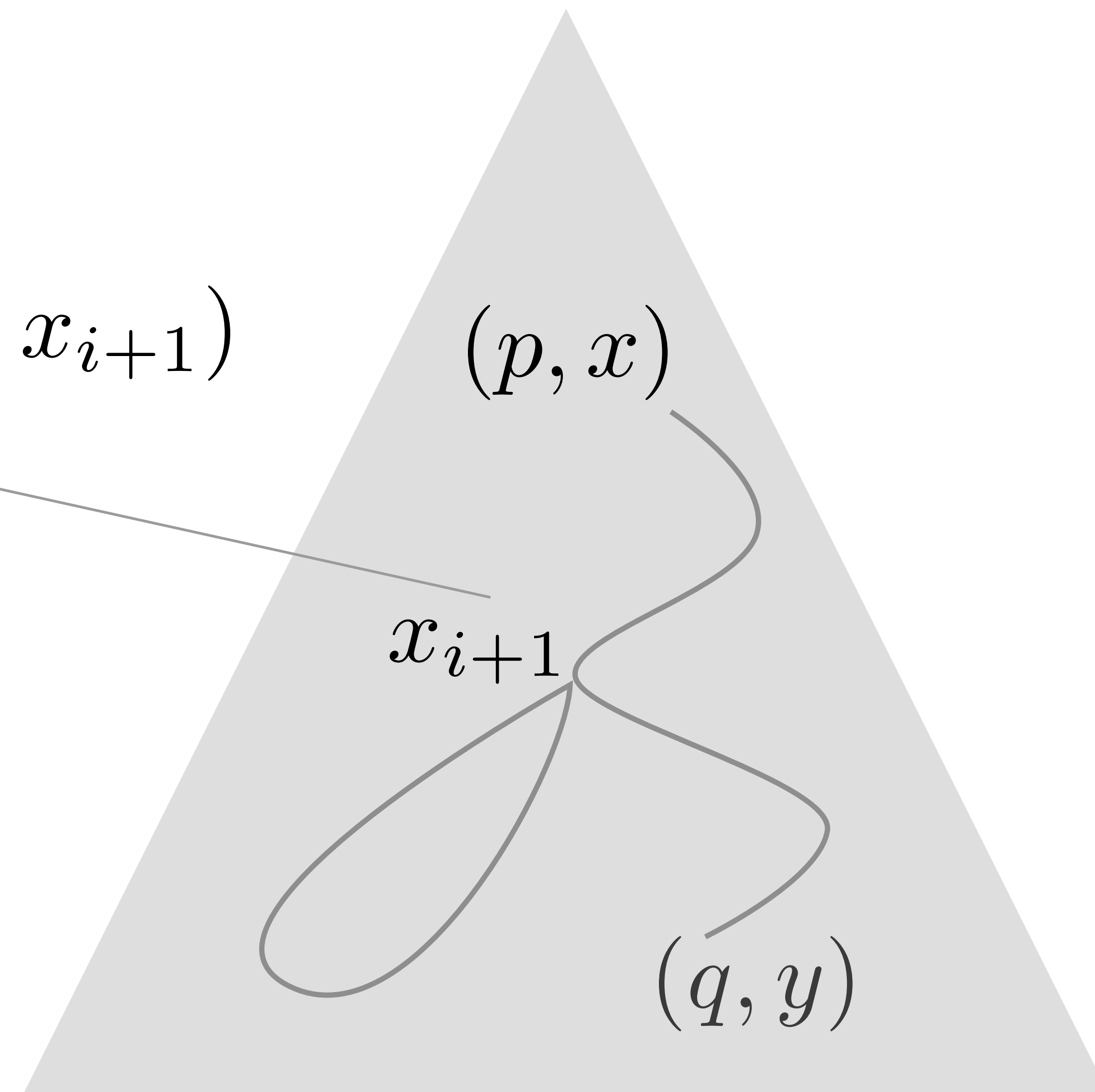
$\varphi_{r,s}(x, y, x_1, \ldots, x_i, x_{i+1})$

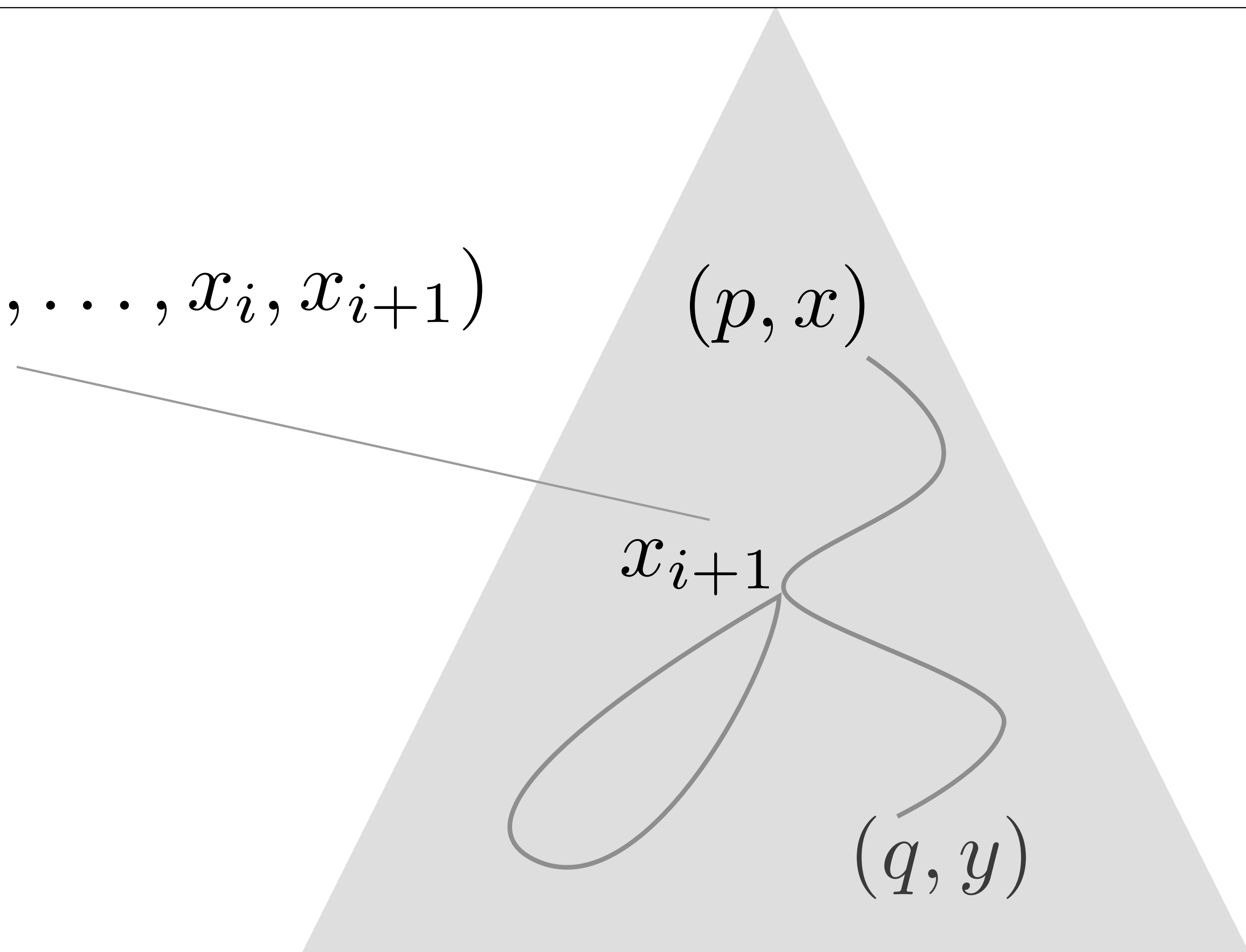


$$\varphi_{p,q}(x, y, x_1, \ldots, x_i)$$

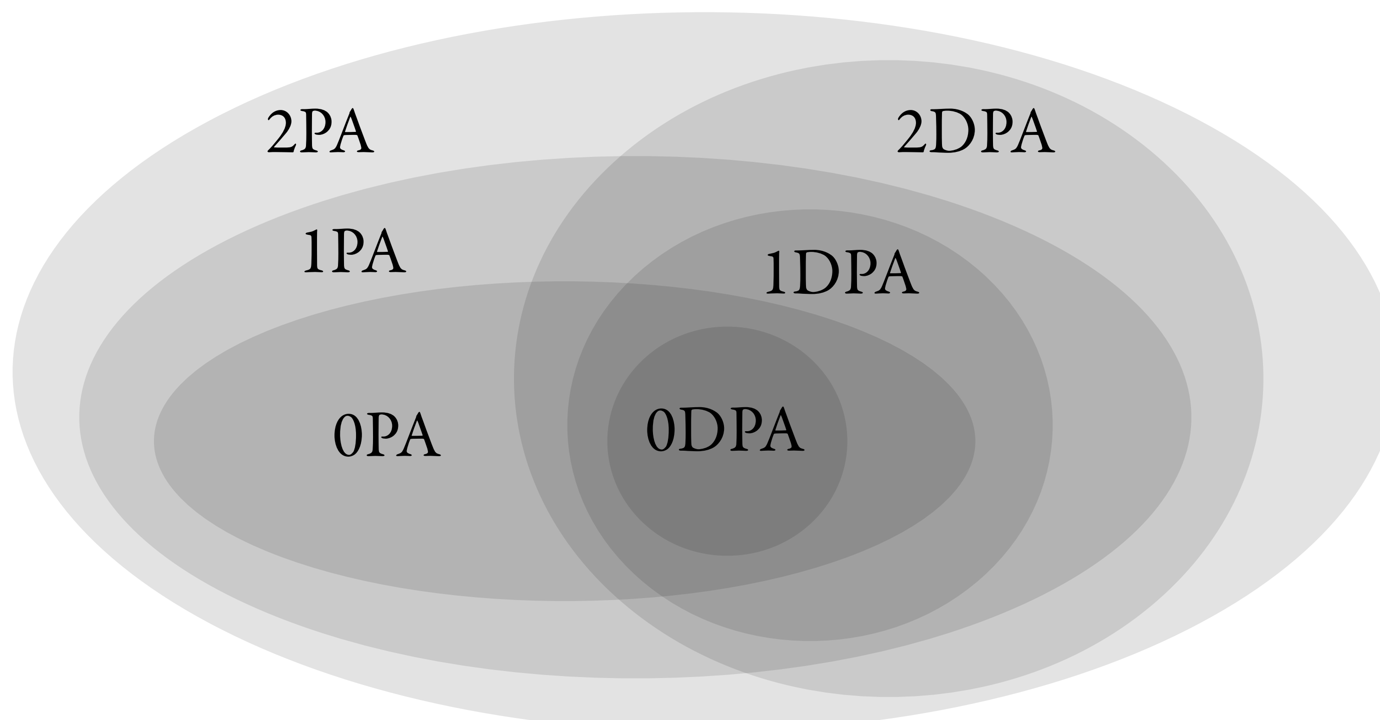
What logic? Monadic second-order logic is good enough.

Pebble automata = first-order logic with positive transitive closure.

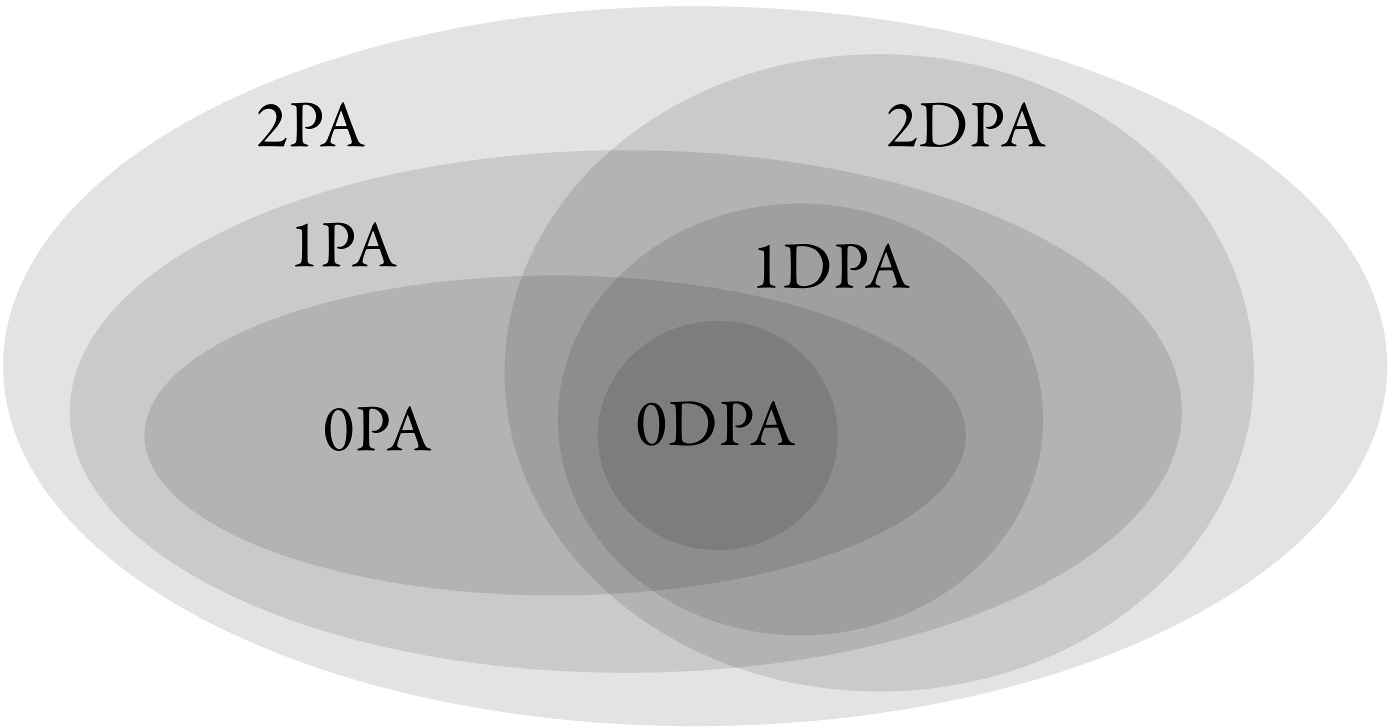
$$\varphi_{r,s}(x, y, x_1, \ldots, x_i, x_{i+1})$$



Theorem. [B., Samuelides, Schwentick, Segoufin 06]
-Pebble automata do not recognize all regular languages.
-Deterministic *n* pebbles are weaker than nondeterminstic *n* pebbles.
-*n* pebbles are weaker than *n+1* pebbles, both in det and nondet.



Theorem. [B., Samuelides, Schwentick, Segoufin 06]
-Pebble automata do not recognize all regular languages.
-Deterministic *n* pebbles are weaker than nondeterminstic *n* pebbles.
-*n* pebbles are weaker than *n+1* pebbles, both in det and nondet.



Open question:

$$\bigcup_i i\mathrm{PA} = \bigcup_i i\mathrm{DPA}$$

Known:

$$\forall i \quad 0\mathrm{PA} \not\subseteq i\mathrm{DPA}$$

Pebble automata = first-order logic with positive transitive closure.

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First-order logic.

$$\forall x \forall y \; a(x) \wedge child(x,y) \Rightarrow b(y)$$

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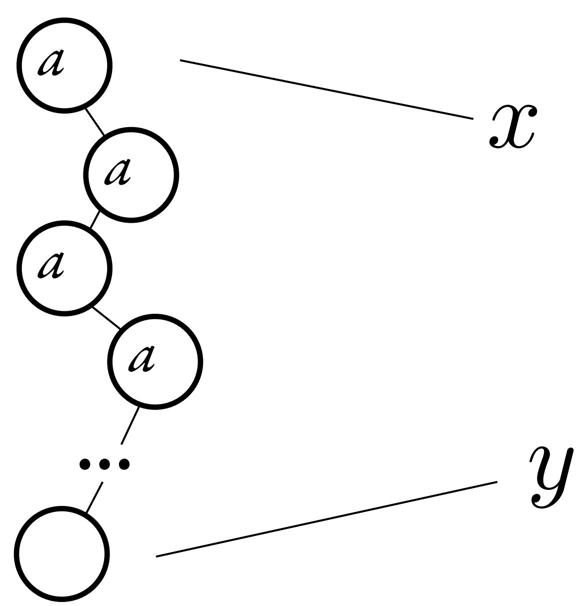
First-order logic with transitive closure.

$$TC\big(child(x,y) \wedge a(x)\big)(x,y)$$



For words,
first-order logic with transitive closure = regular languages.

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What about trees?
first-order logic with positive transitive closure = pebble automata

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What about trees?
first-order logic with positive transitive closure = pebble automata

Theorem. [ten Cate, Segoufin '08]
For trees, not all regular languages are captured by first-order logic with transitive closure.

Conclusion

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What did we miss?

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-caterpillar expressions

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Open questions:

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Open questions:
-complementation

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Open questions:
-complementation
-detereminization of pebble automata

Conclusion

What did we miss?
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Open questions:
-complementation
-detereminization of pebble automata
-better understanding