XPath Evaluation in Linear Time

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Goal: find the nodes in an XML document d that satisfy an XPath unary query q.

We consider a fragment of XPath called FOXPath.

Previous algorithms:

– exponential in the document size
– quadratic in the document size (Benedikt, Koch)

We give two algorithms:

– linear in the document size: $O(\ 2^{|q|} \cdot |d|\ )$
– good combined complexity: $O(|q| \cdot |d| \cdot log(|d|))$

XML Document

```
<document>

  <team name="Borussia">
    <player name="Kuba"></player>
    <player name="Frei"></player>
  </team>

  <team name="Schalke">
    <player name="Kuranyi">
  </team>

  <team name="Poland">
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document node,
i.e. opening tag

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XPath query: "select teams that share a player with another team"

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```
child[player]@name = sibling[team]/child[player]@name
```

XPath query: "select teams that share a player with another team"

Programs - select node pairs.

- `child`, `parent`, `next-sibling`, `prev-sibling`, `descendant`, etc.
- any regular expression on programs is a program, e.g. `child`*
- if *t* is a test, then *[t]* is a program that selects *(x,x)* if node *x* satisfies *t*

Tests - select single nodes.

- any tag name `a` is a test that selects nodes with this tag.
- boolean operations: *or, and, not*
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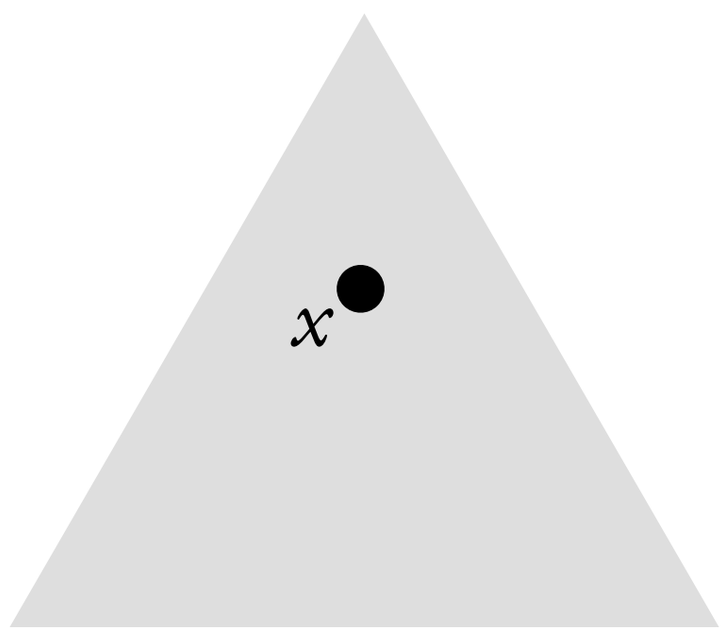

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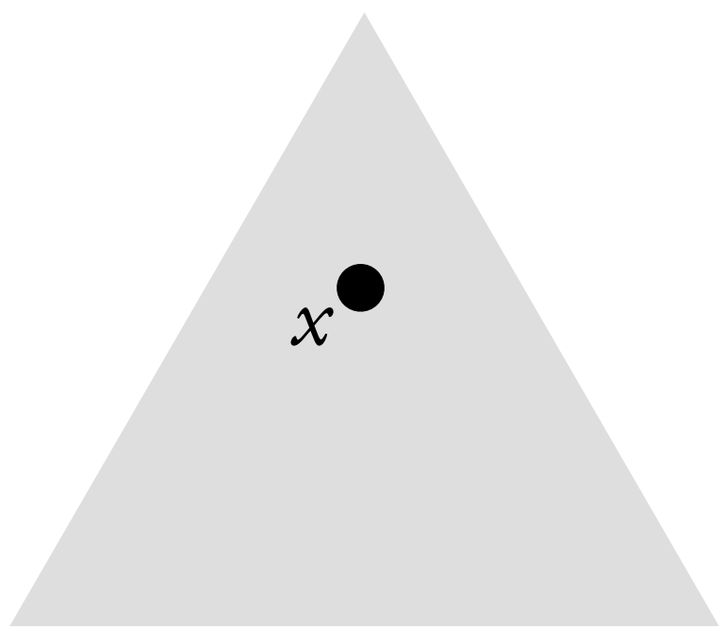

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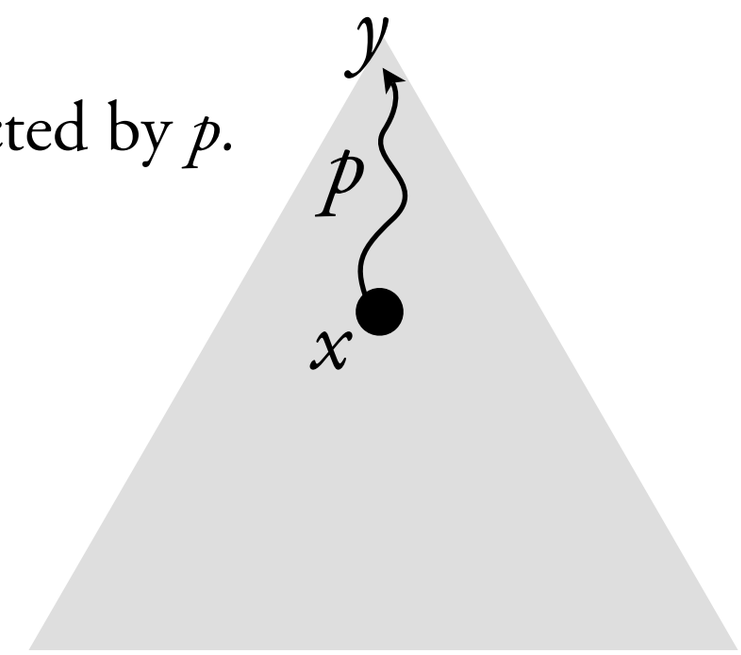

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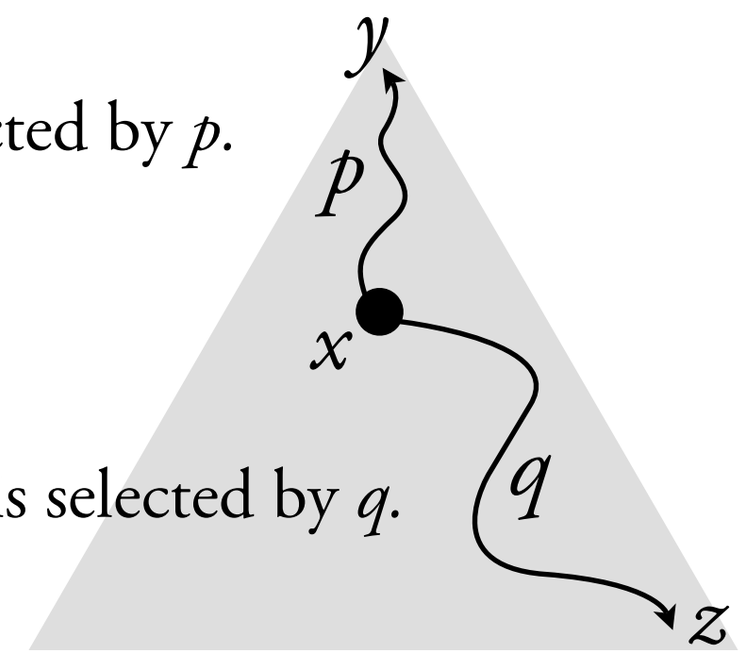

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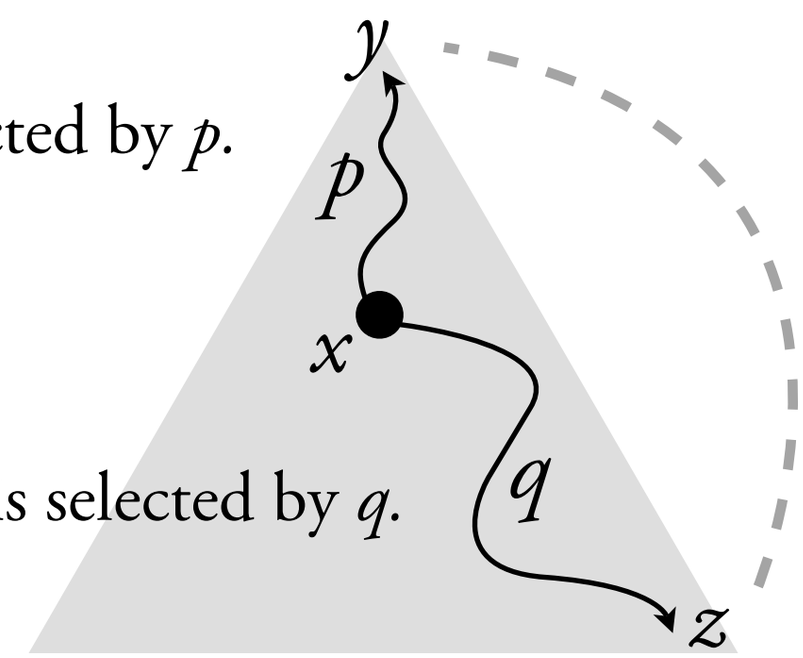

the attribute values
y@`a` and *z*@`b` are the same.

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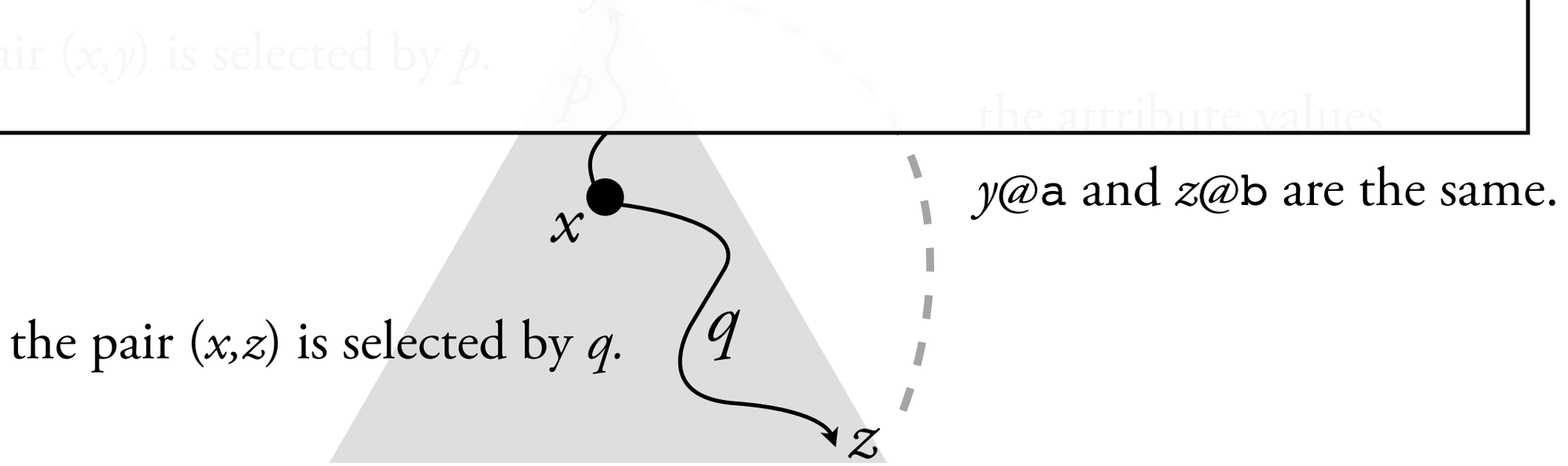
Thm.

Let *t* be an FOXPath test and *d* an XML document.

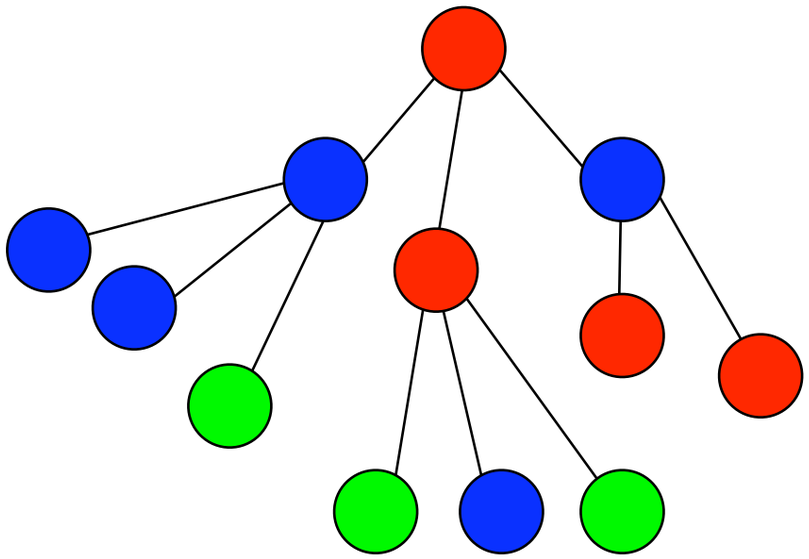
The set of nodes in *d* selected by *t* can be computed in time $O(|d|2^{|t|})$ as well as in time $O(|d|\log(|d|)|t|^2)$



High level overview

find nodes that satisfy `p@a = q@a`

1. decompose trees into classes
   (class = set of nodes with same value of atribute a)



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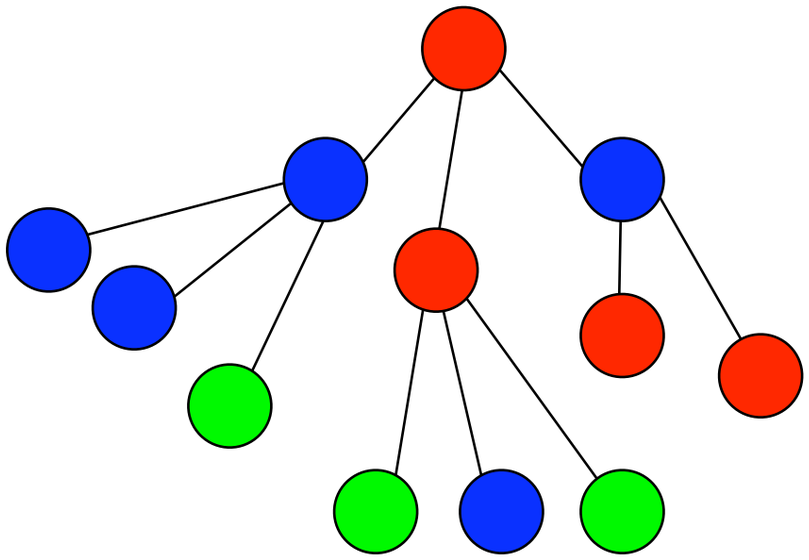

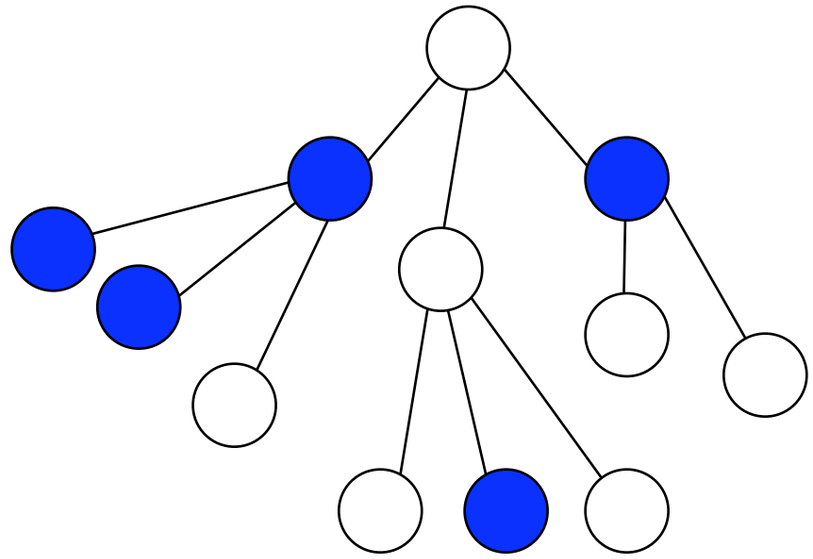

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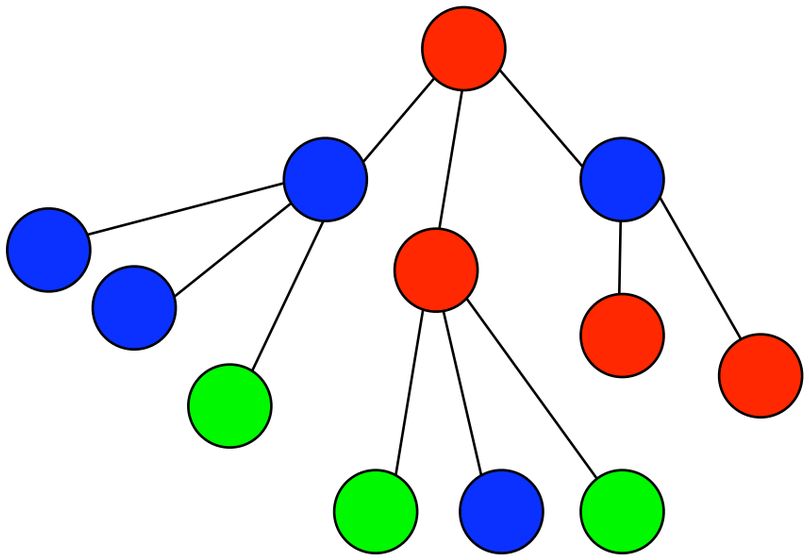

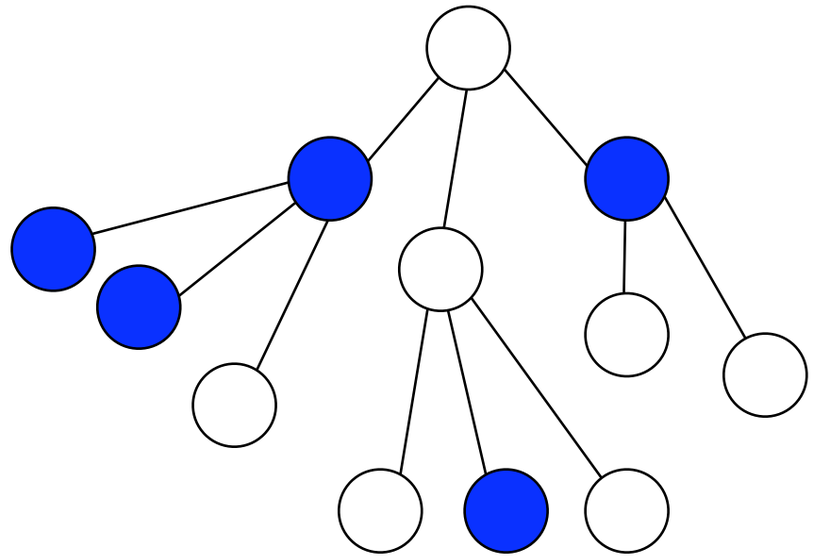

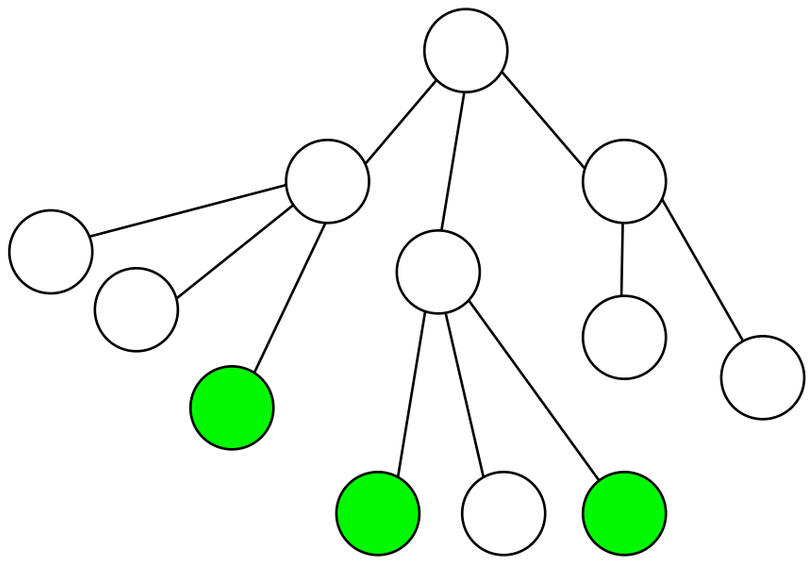

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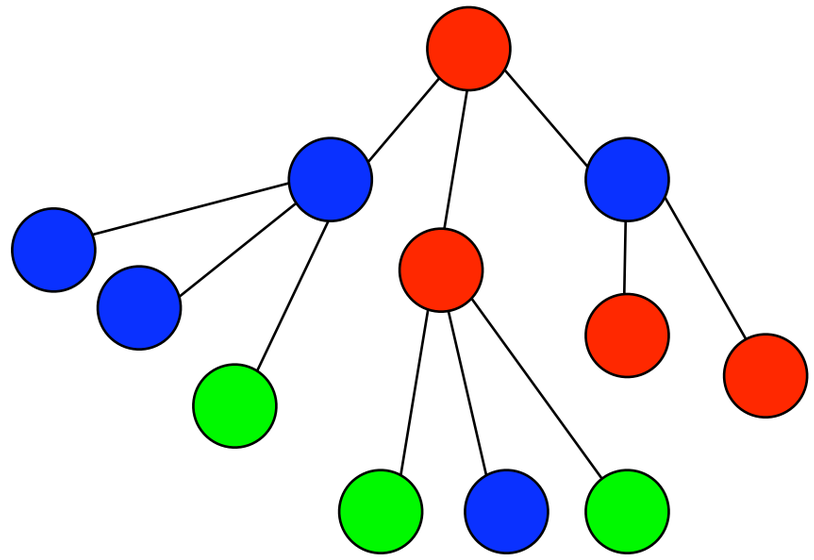

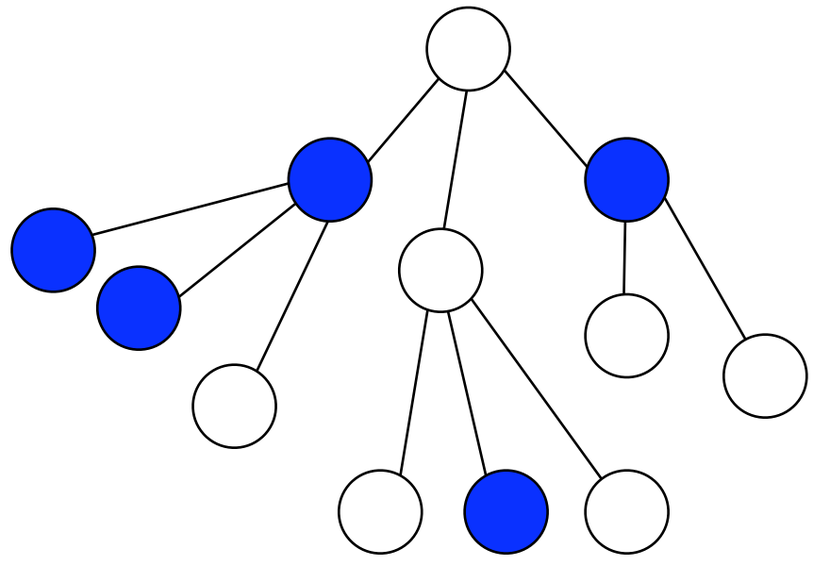

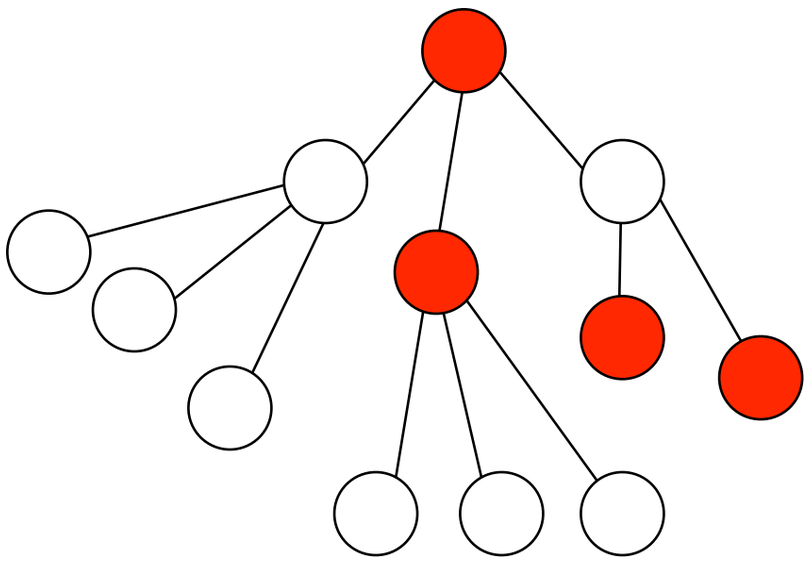

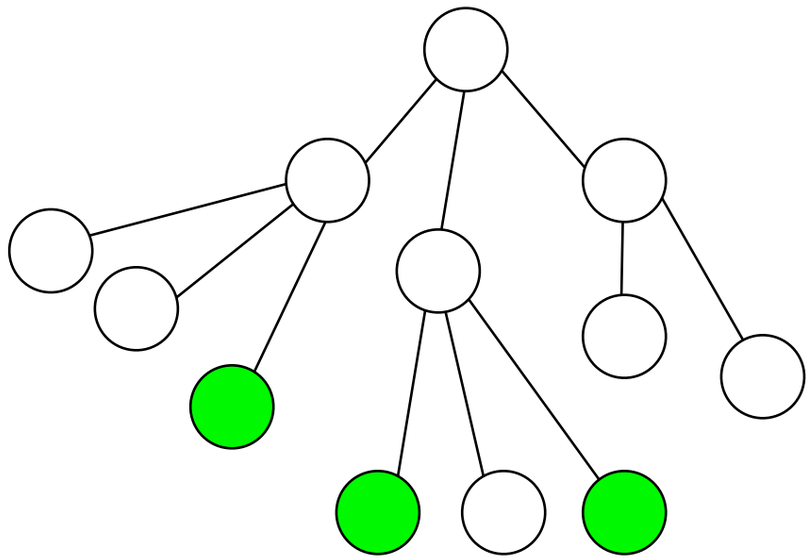

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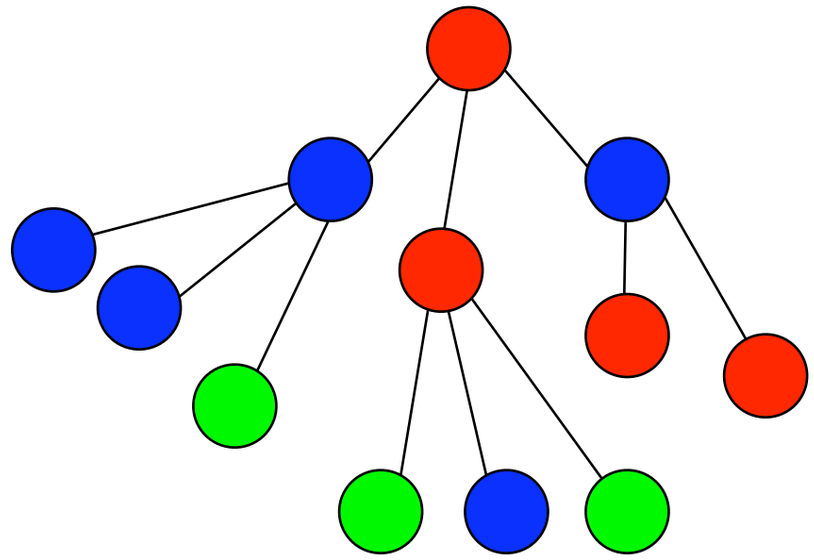

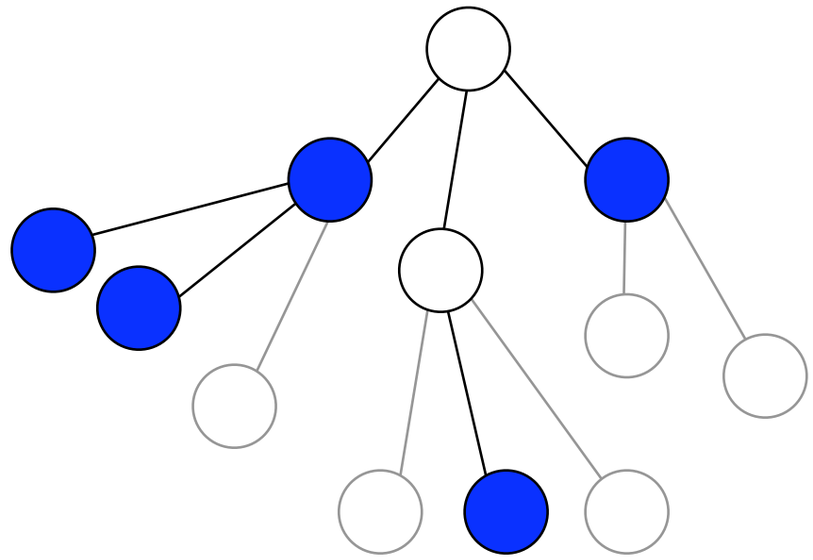

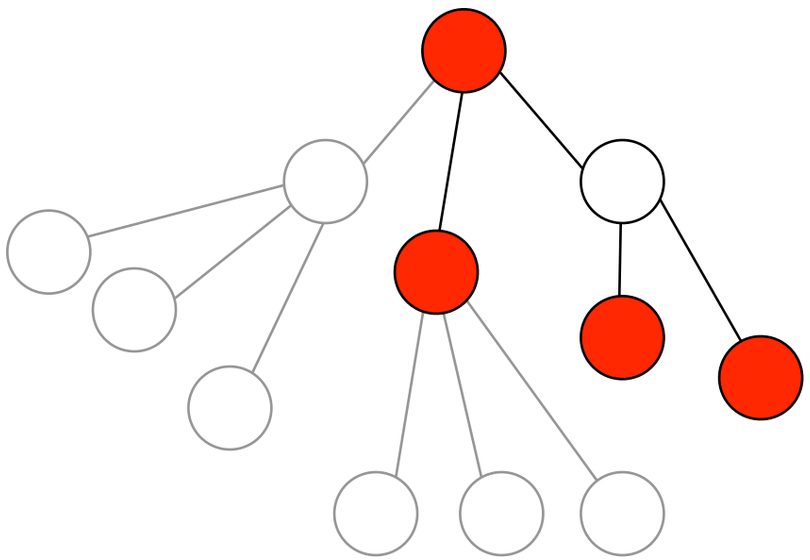

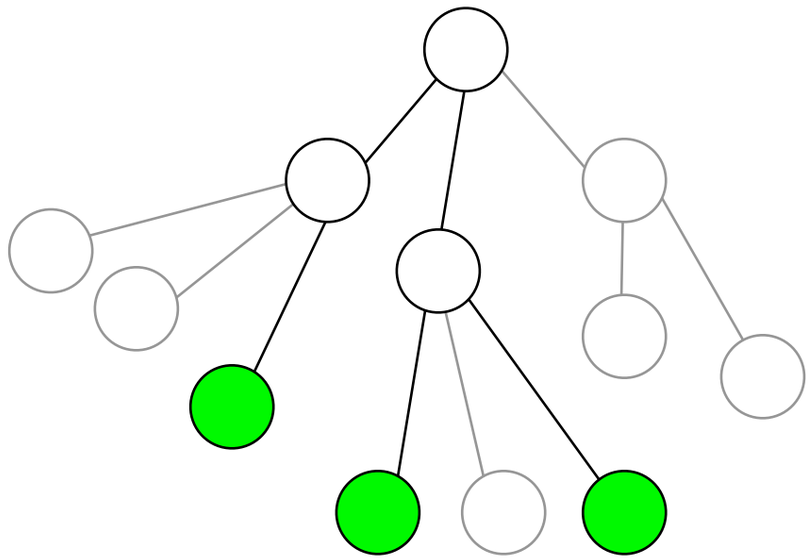

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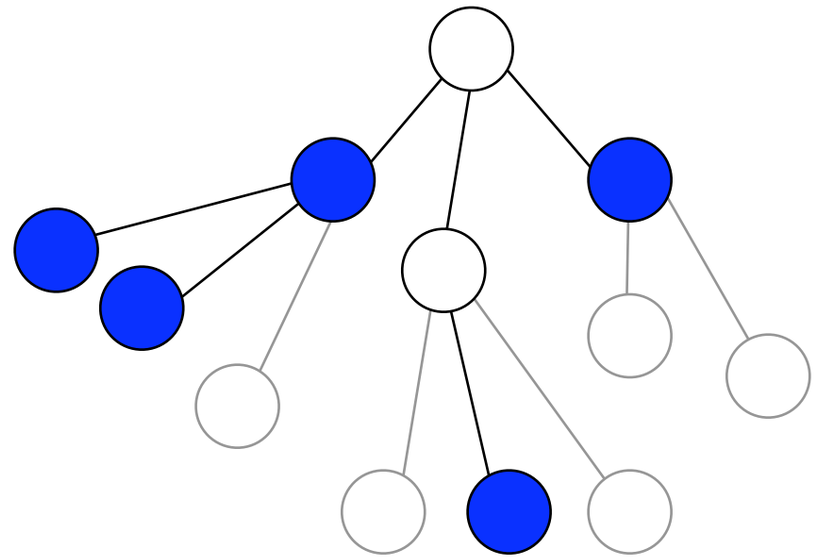

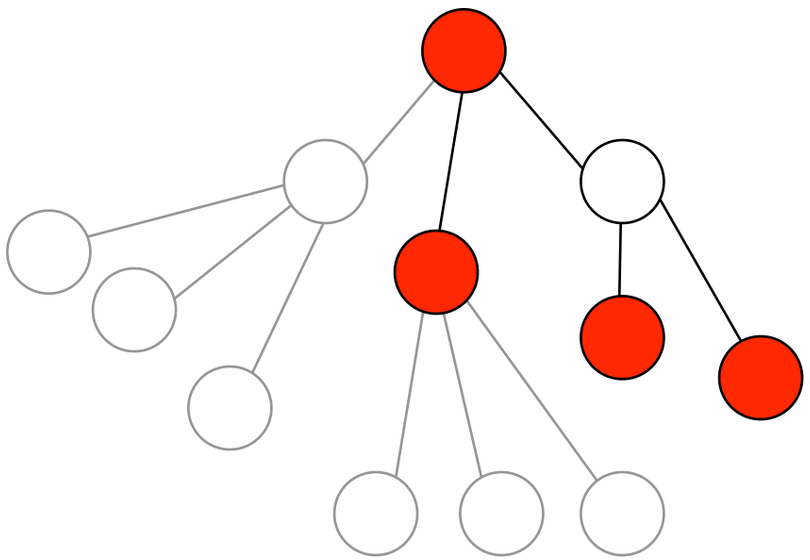

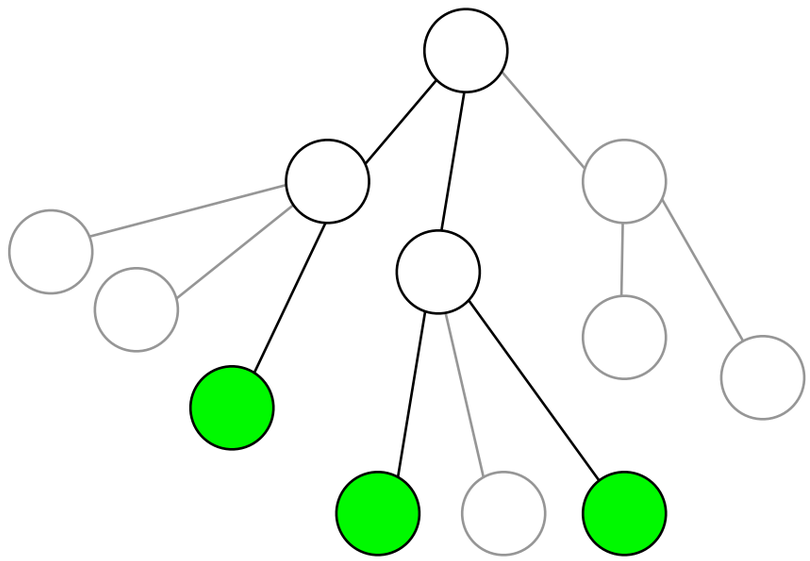

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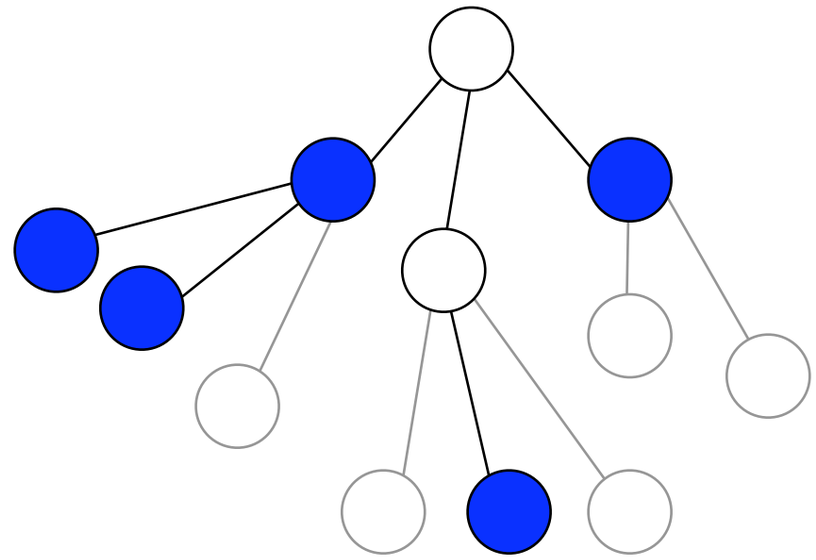

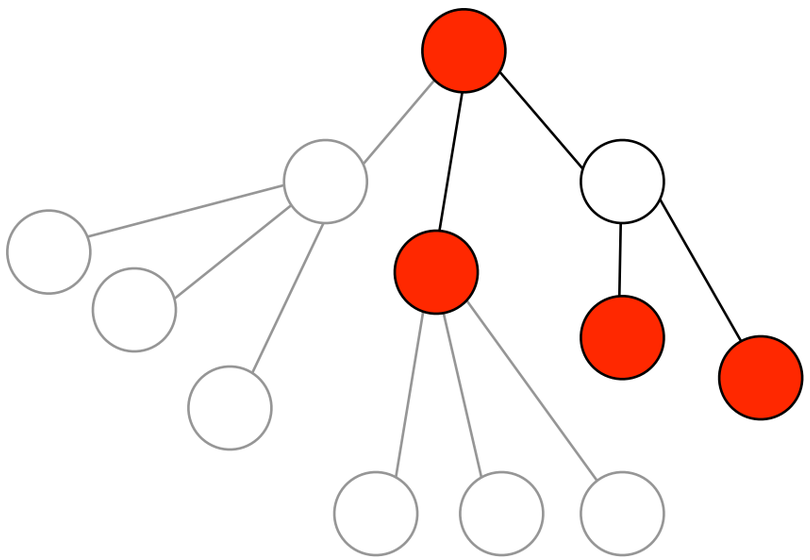

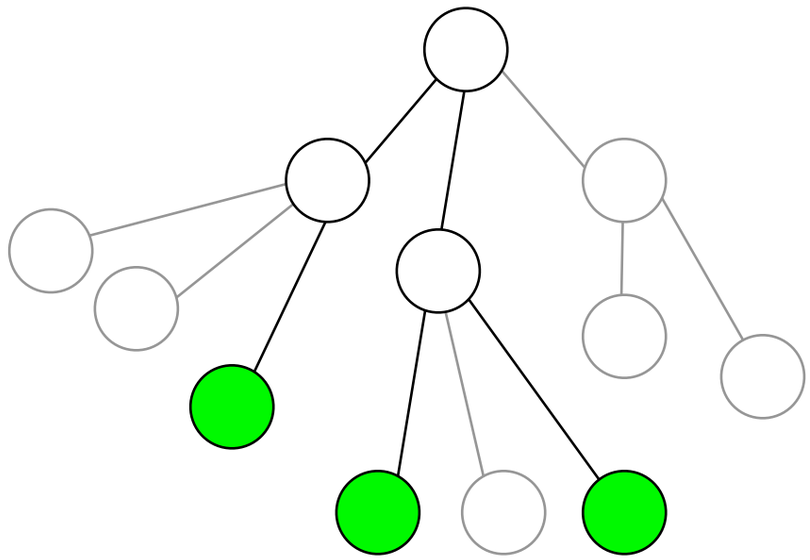

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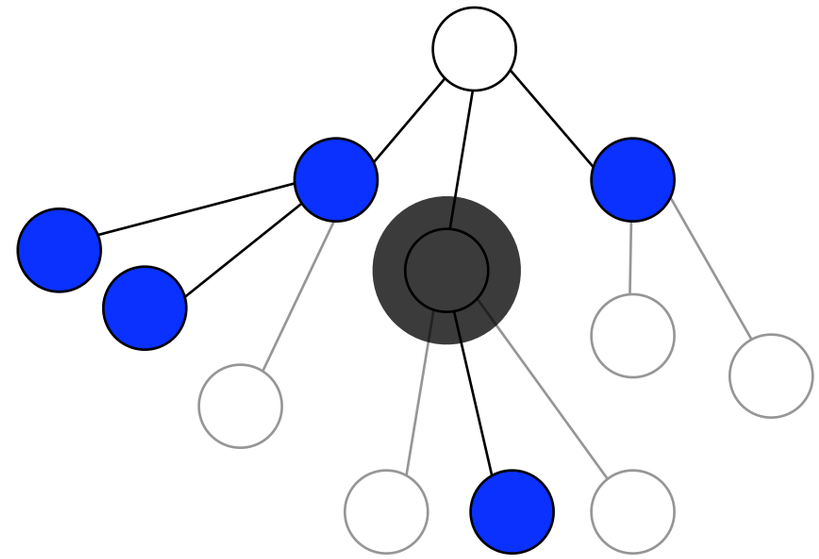

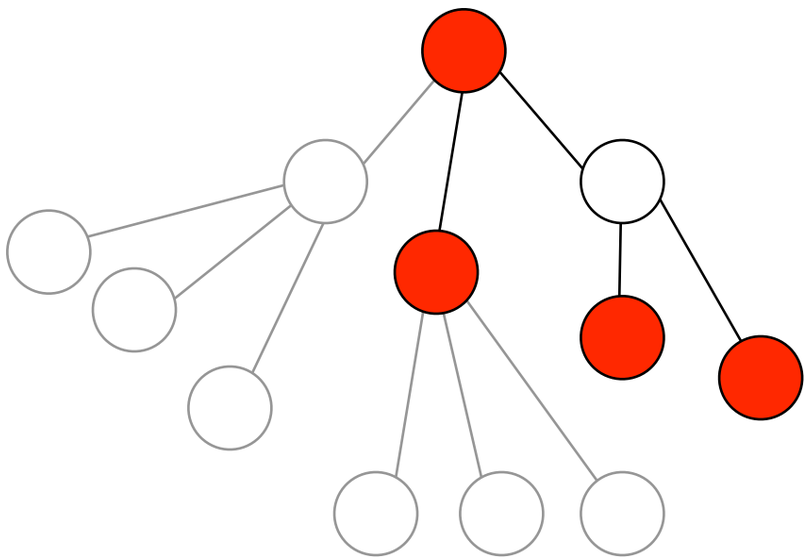

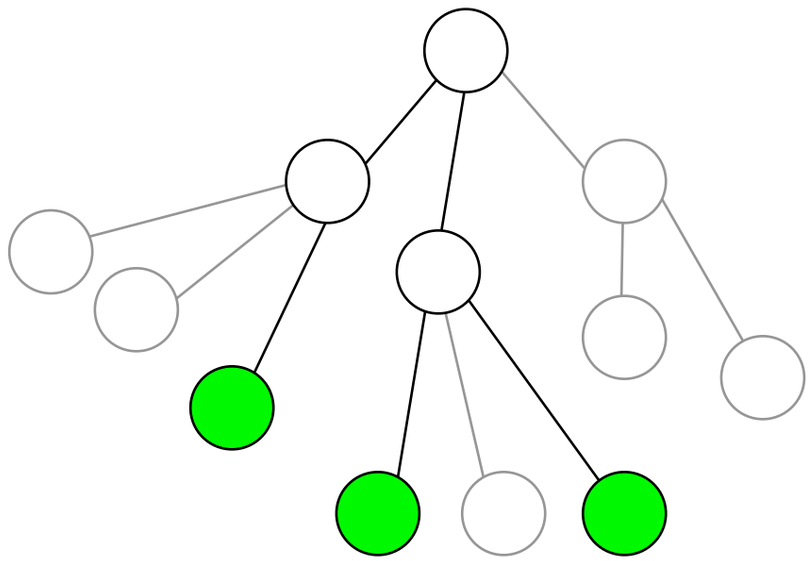

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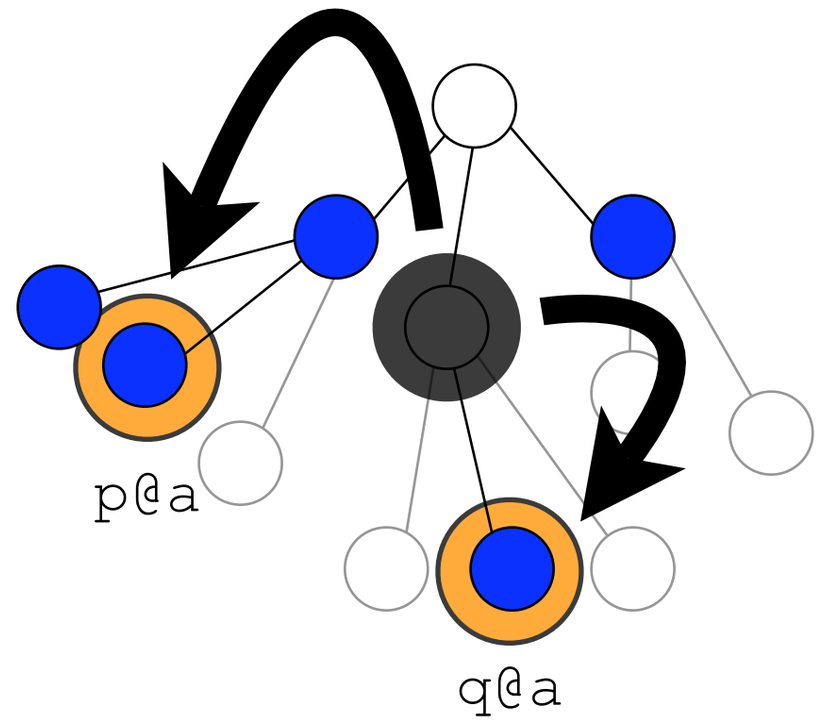

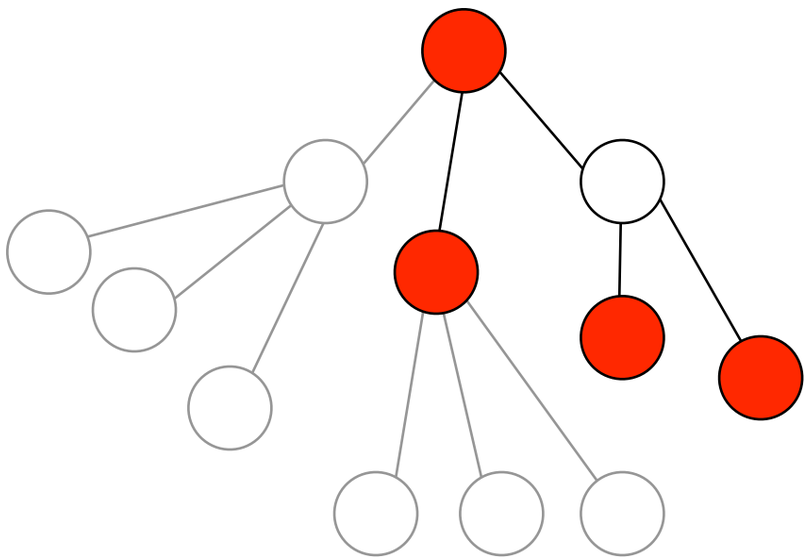

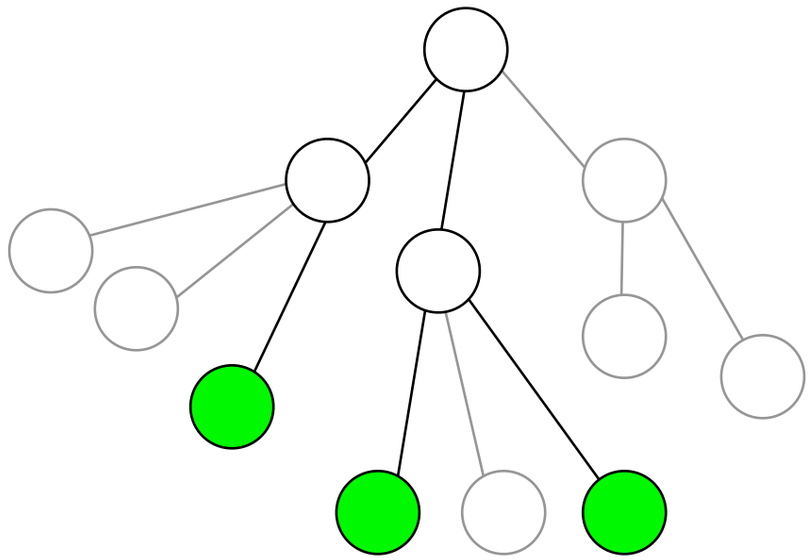

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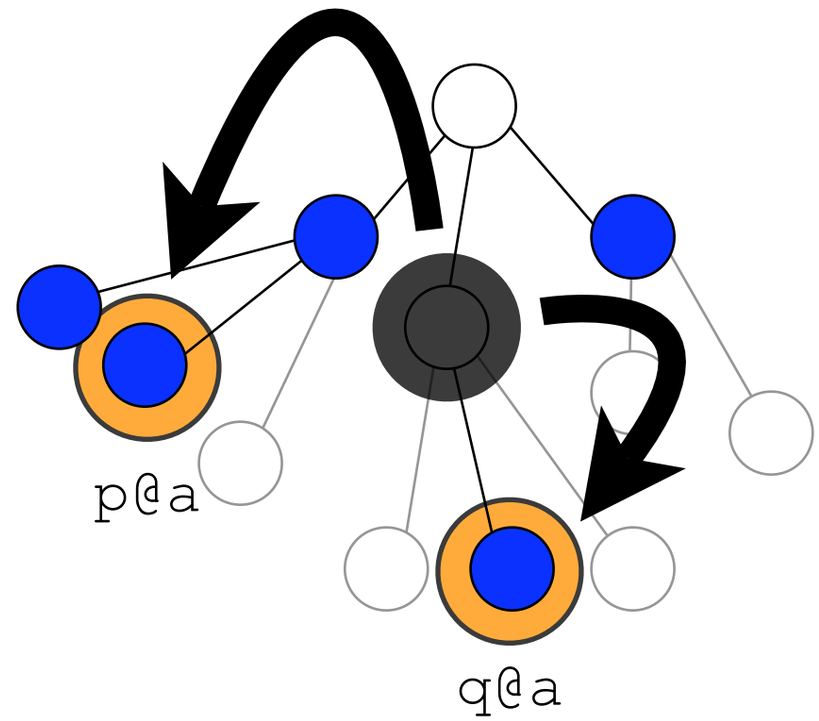

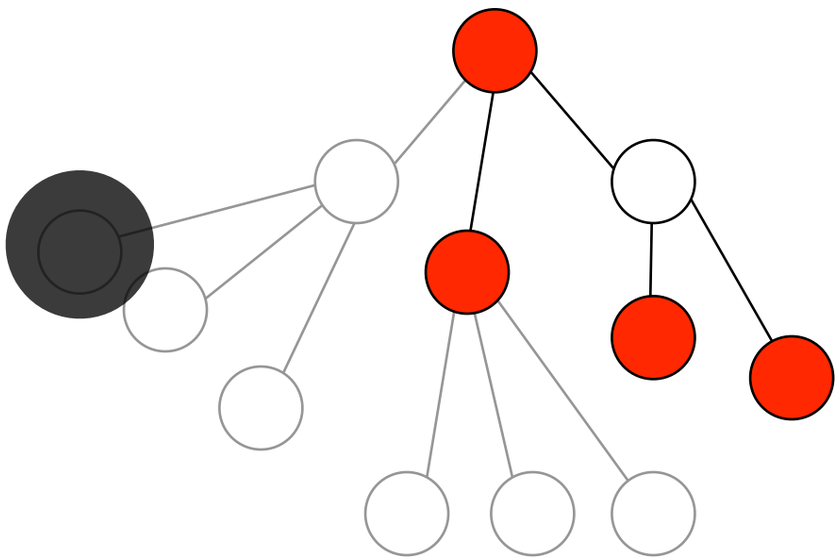

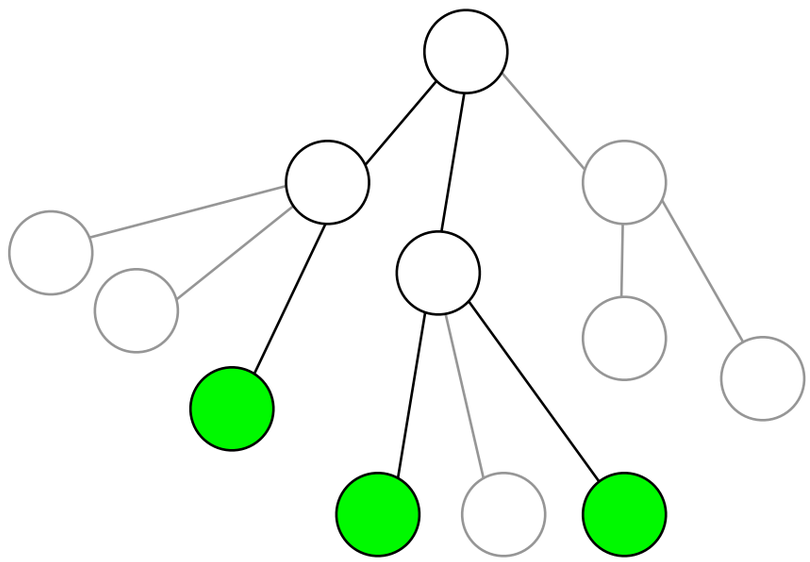

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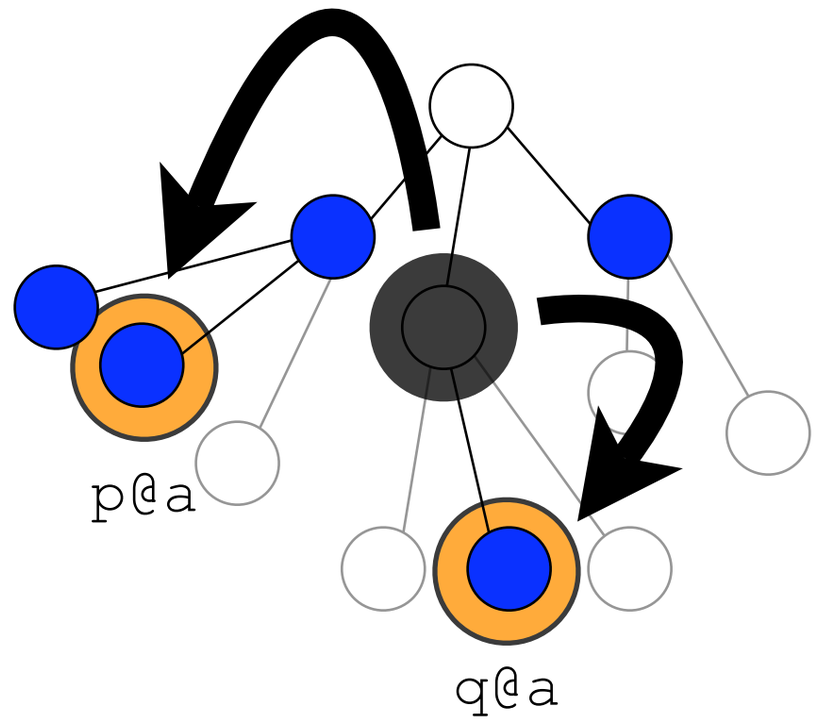

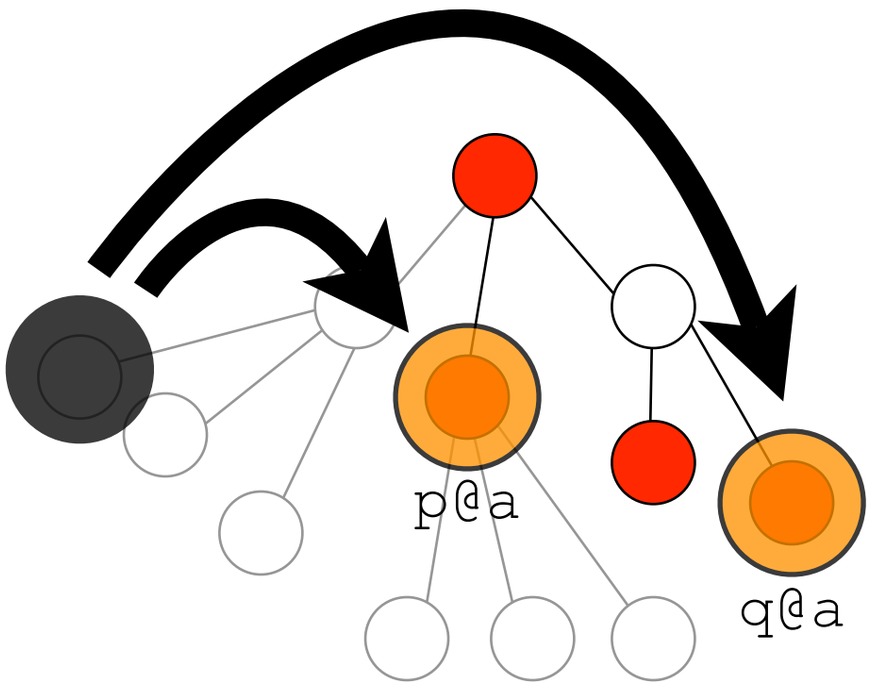

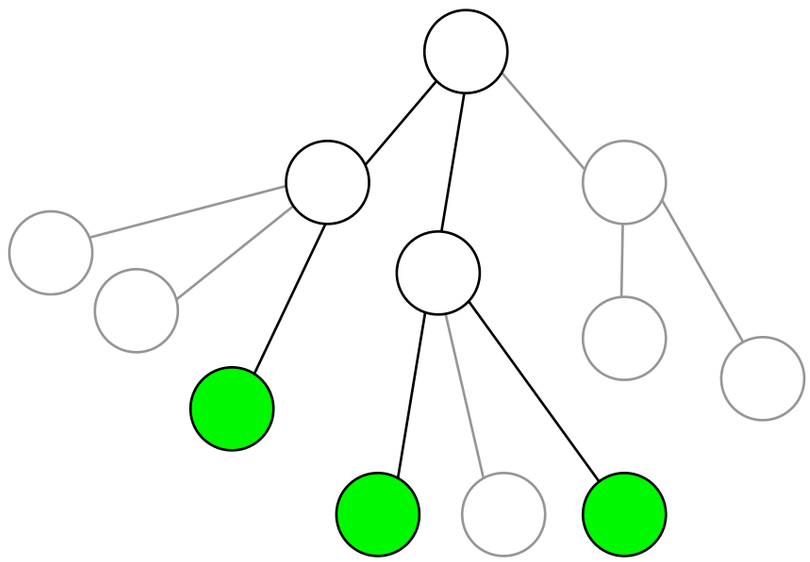

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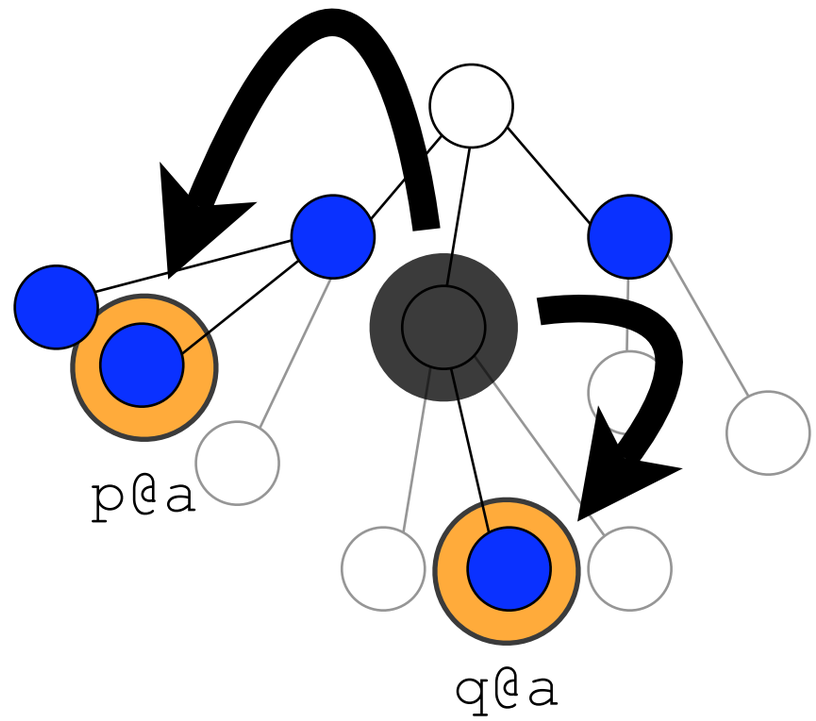

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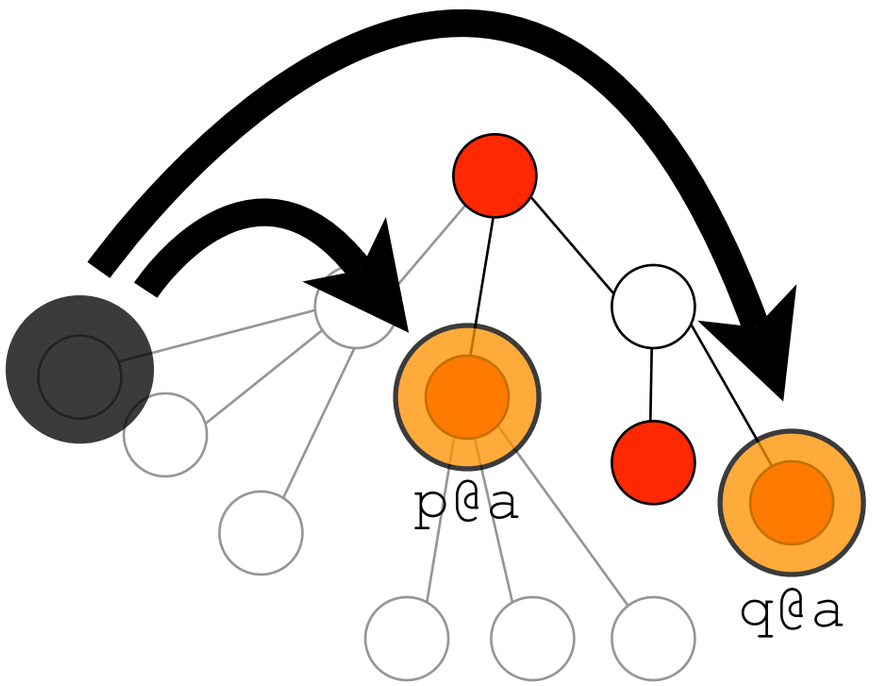

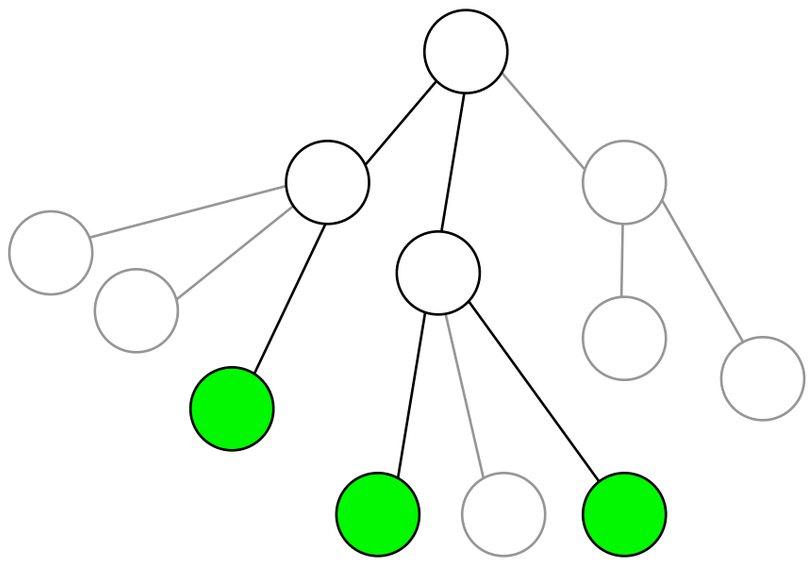

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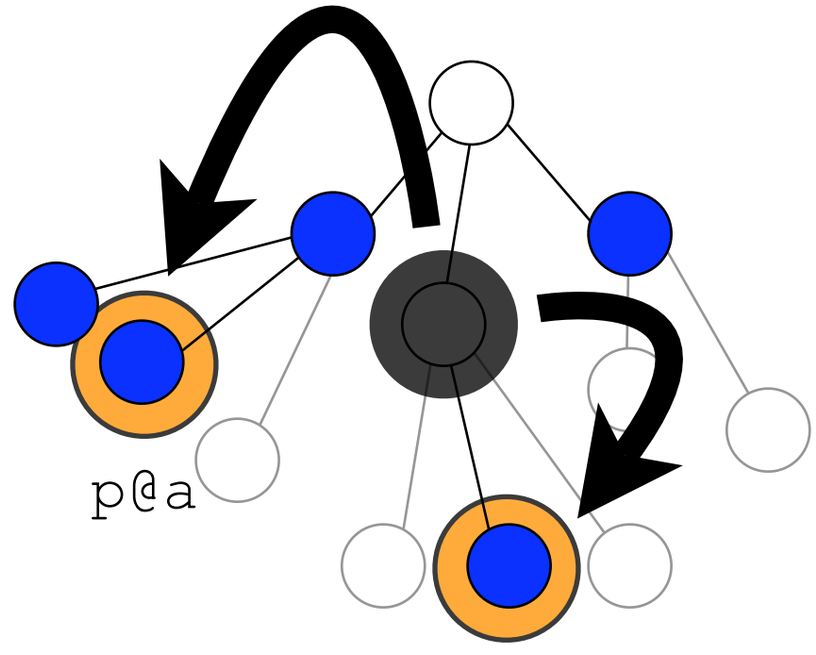

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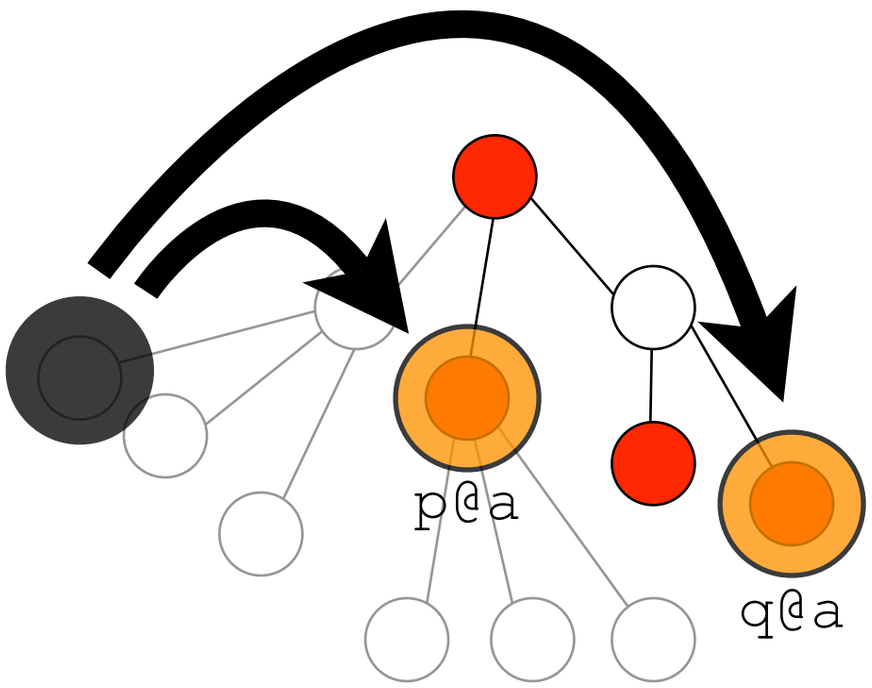
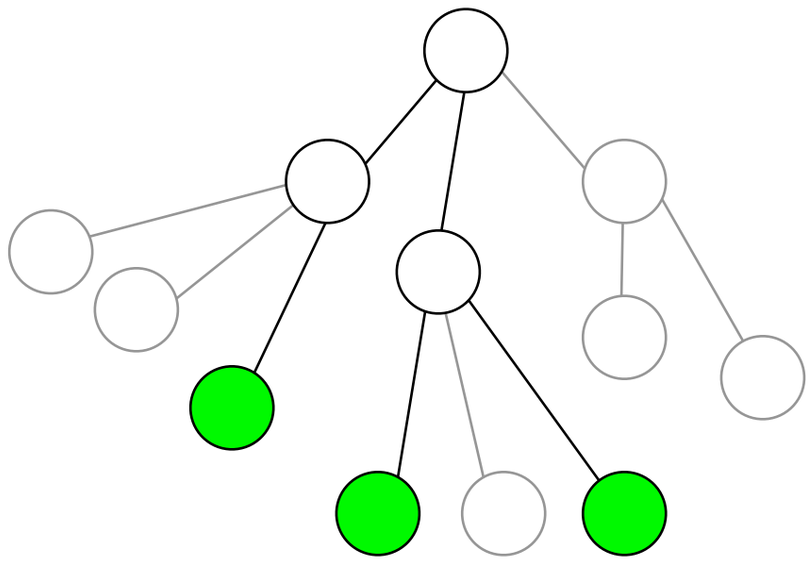
Using Simon decompositions,
a fancy algebraic result





What is the Simon decomposition?

L a regular word language.
Do a linear time precomputation on $w=a_1a_2\ldots a_n$
For any infix, membership $a_i\ldots a_j \in L$ can be computed in time $\log n$

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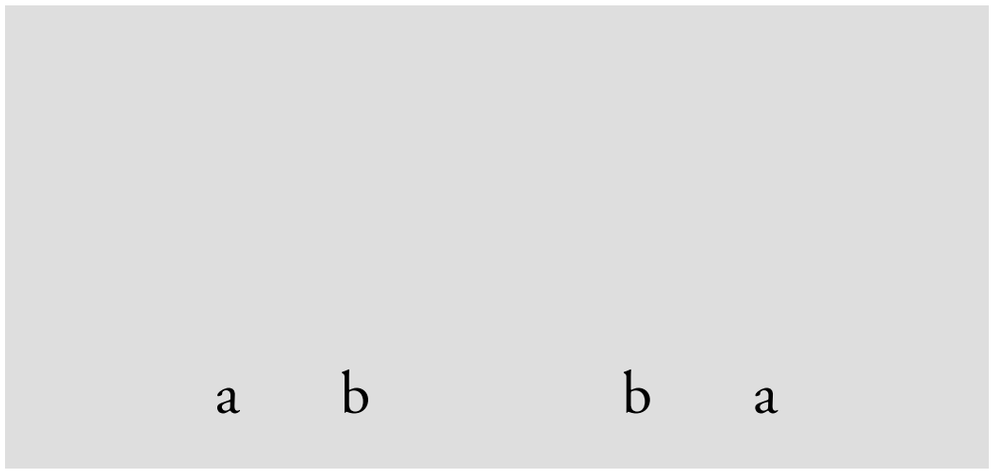

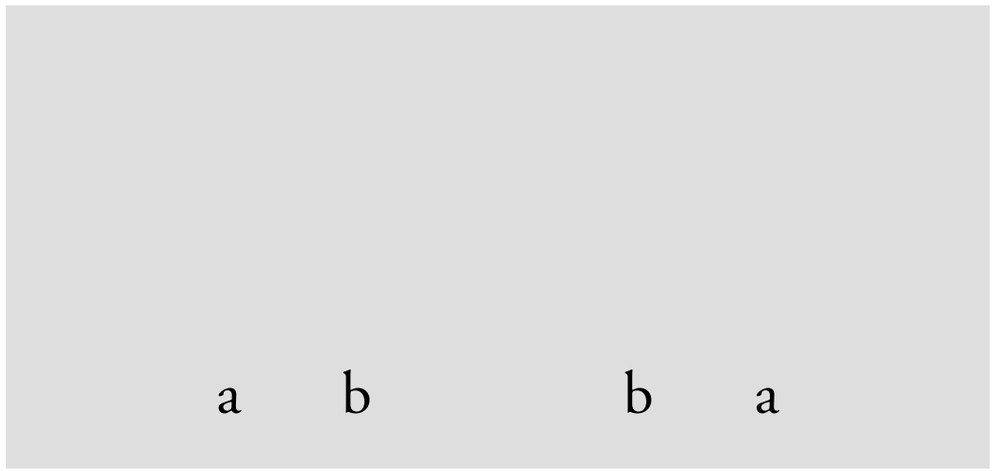

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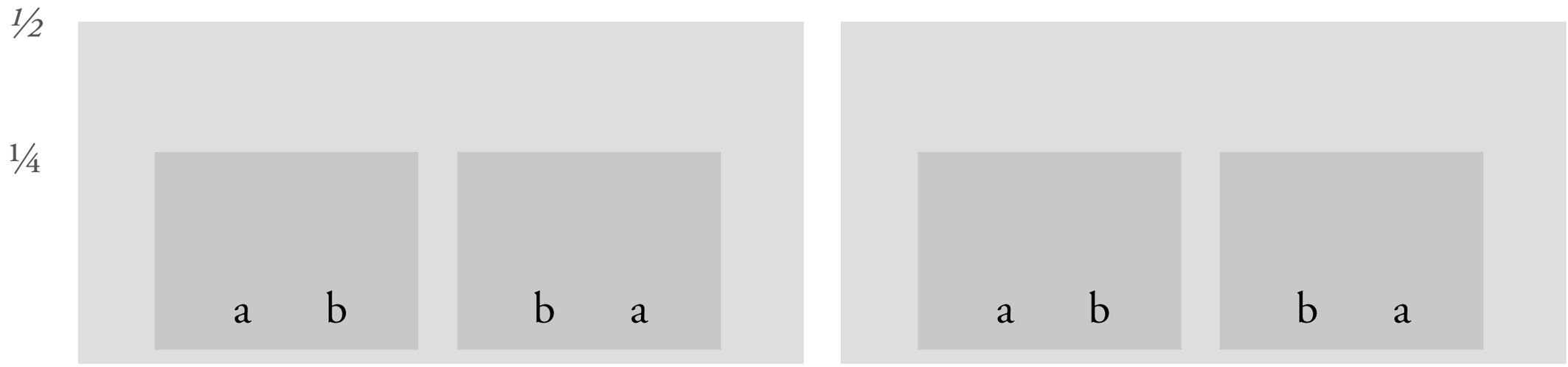

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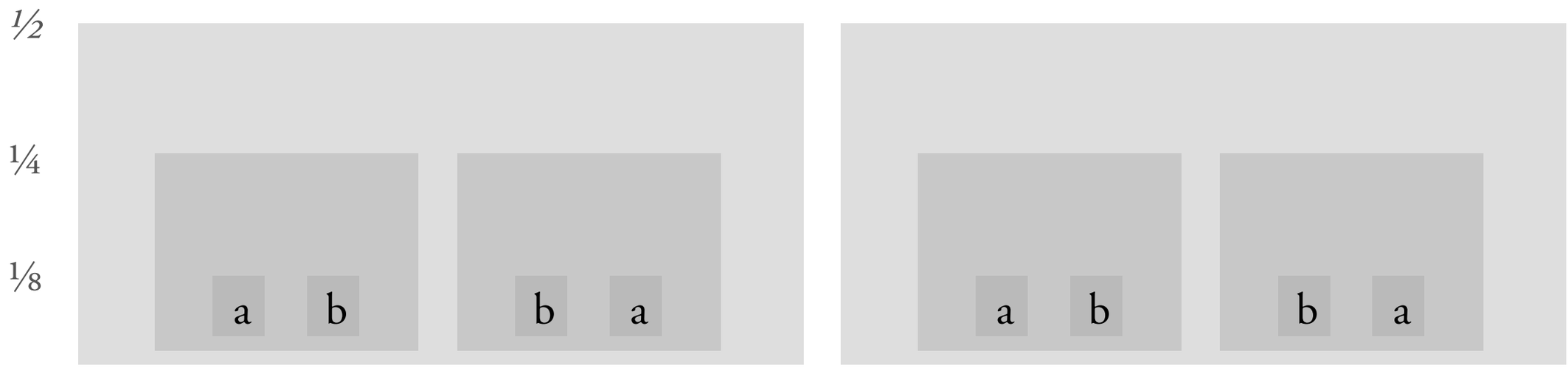

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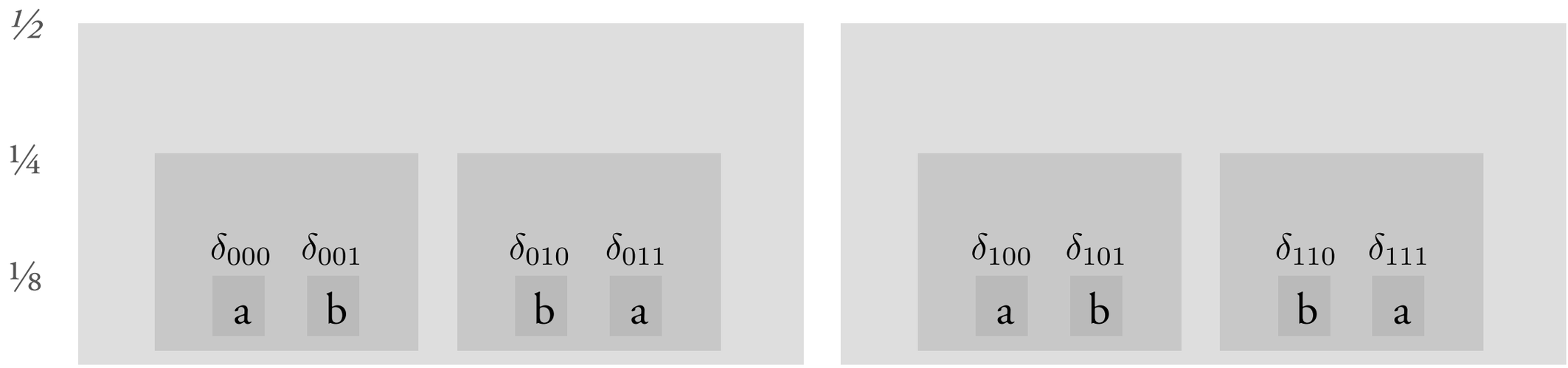

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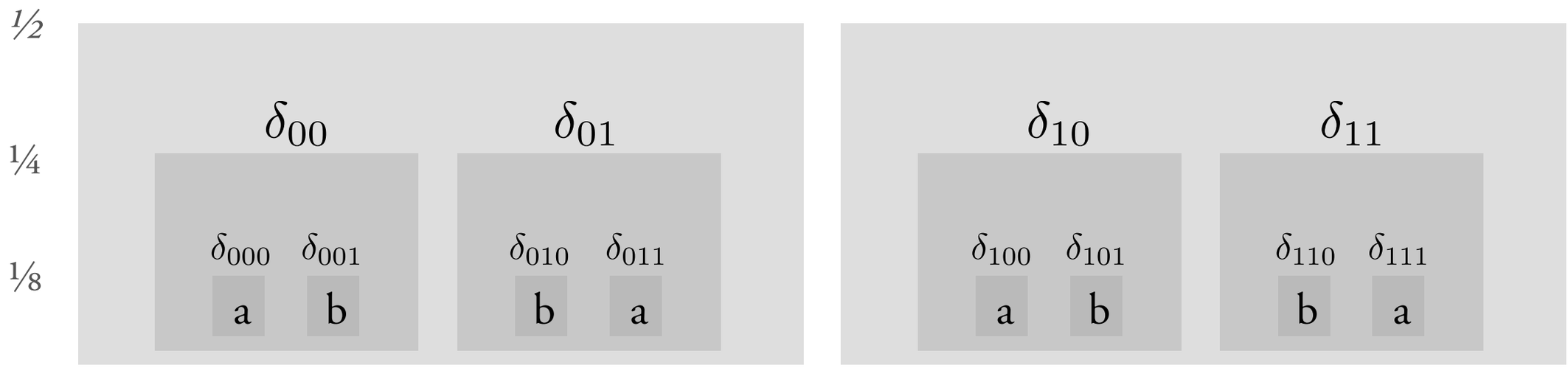

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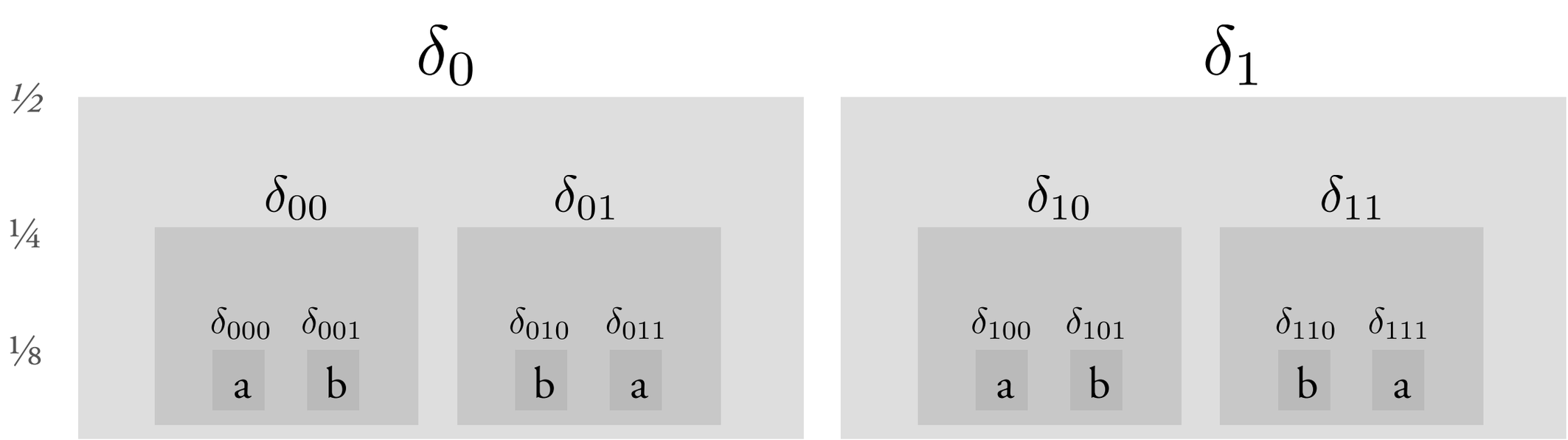

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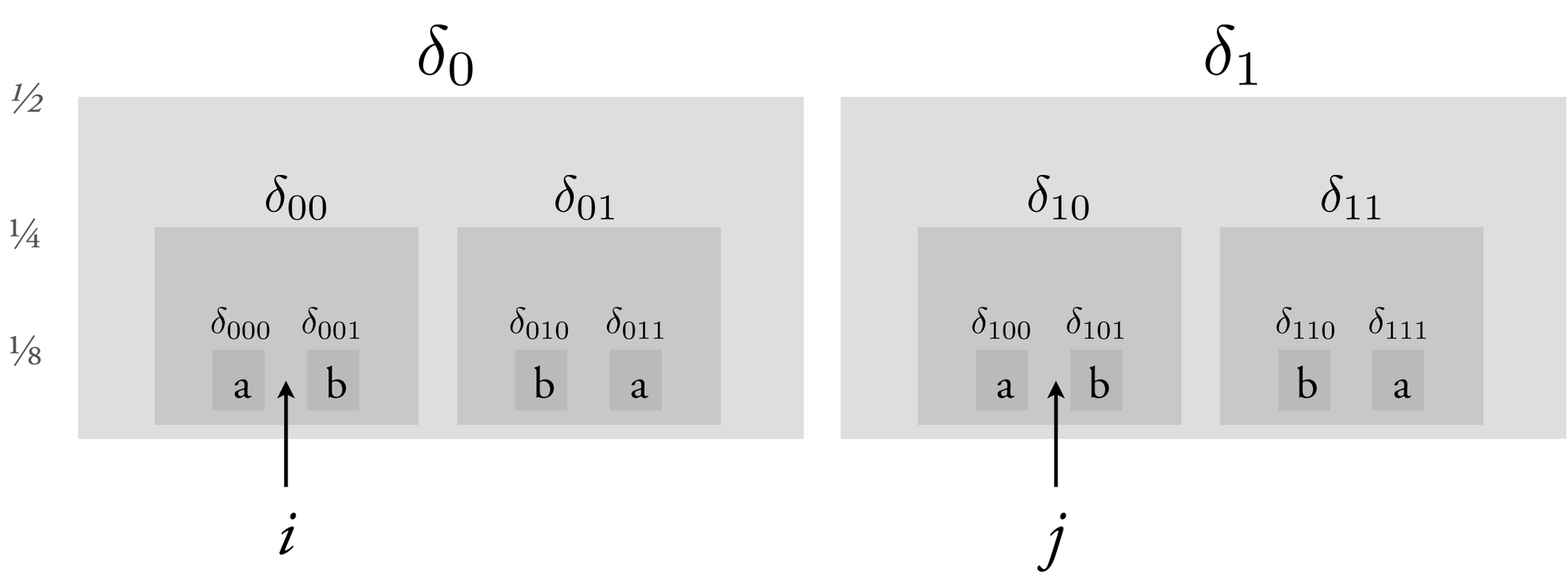

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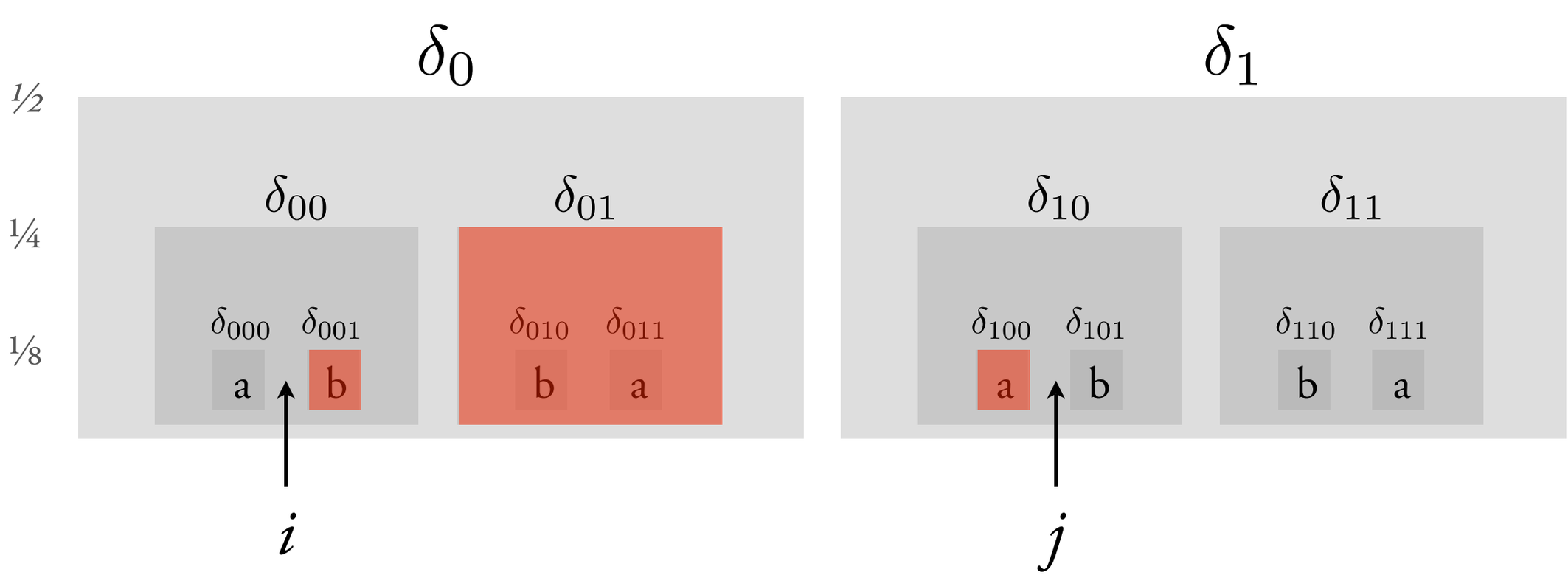

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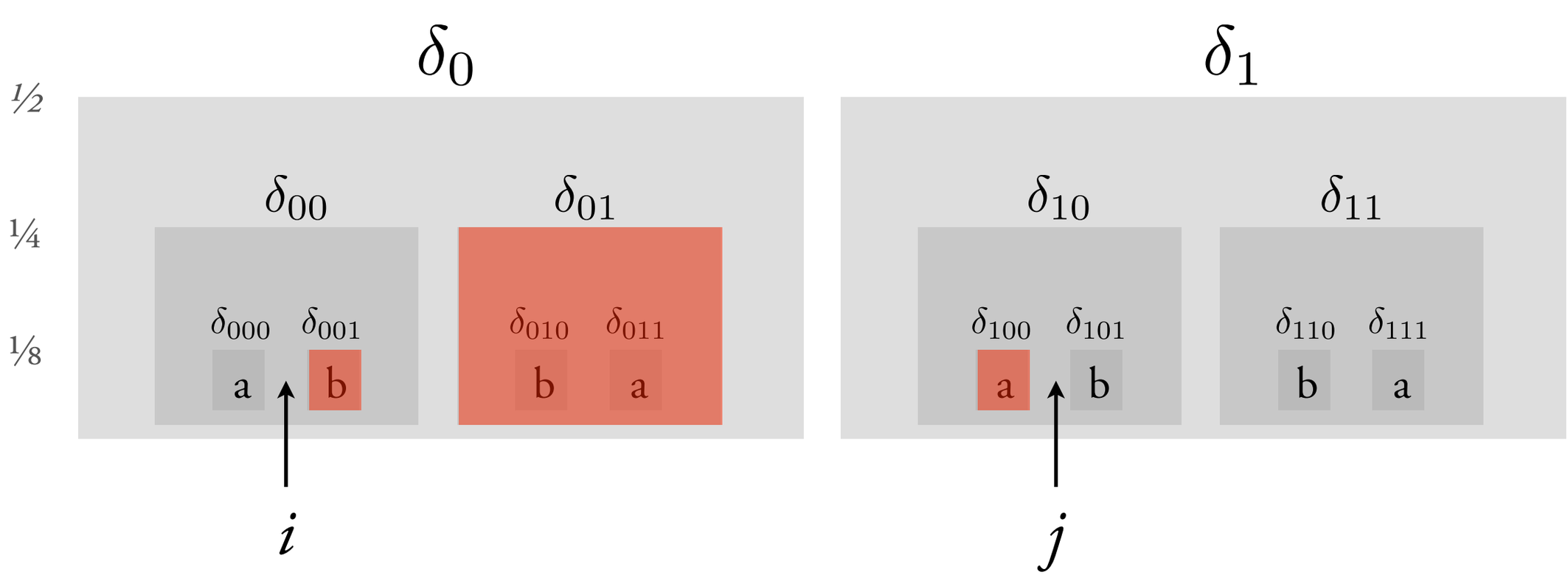

Big news: Simon decomposition does this with constant depth!

Back to XPath evaluation...

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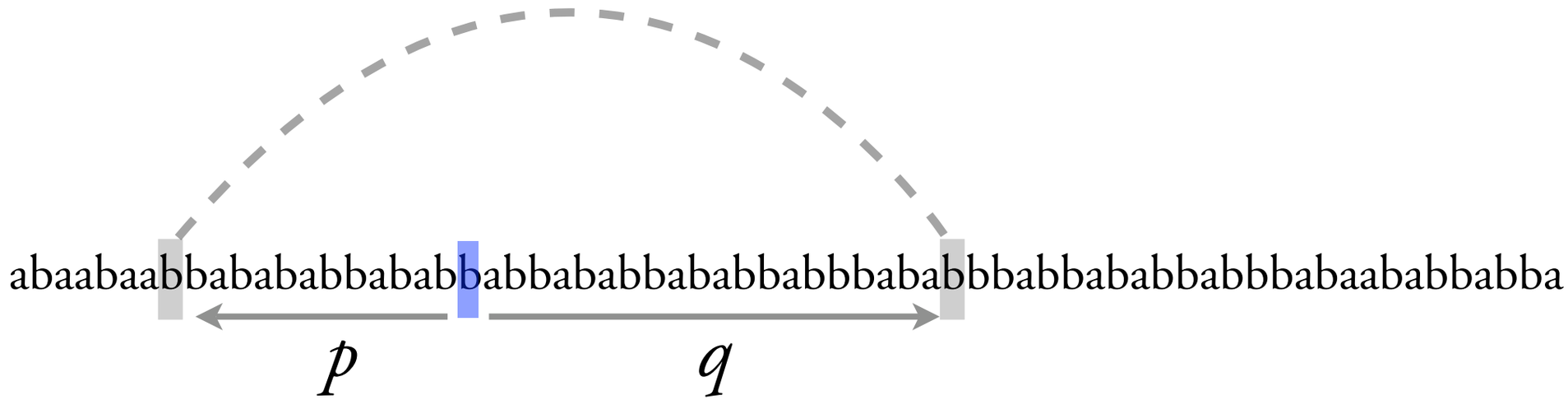

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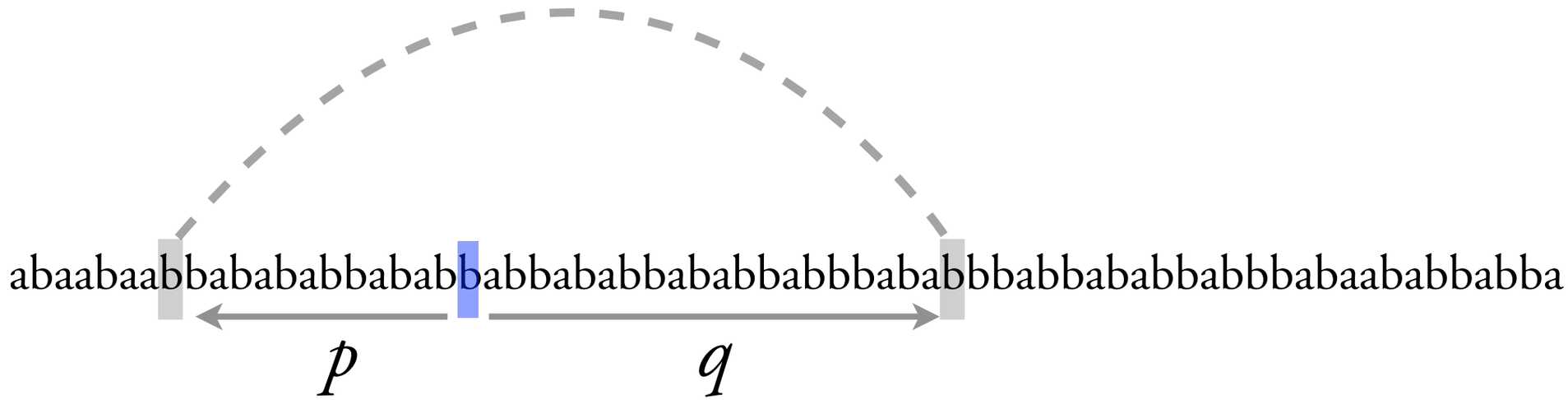

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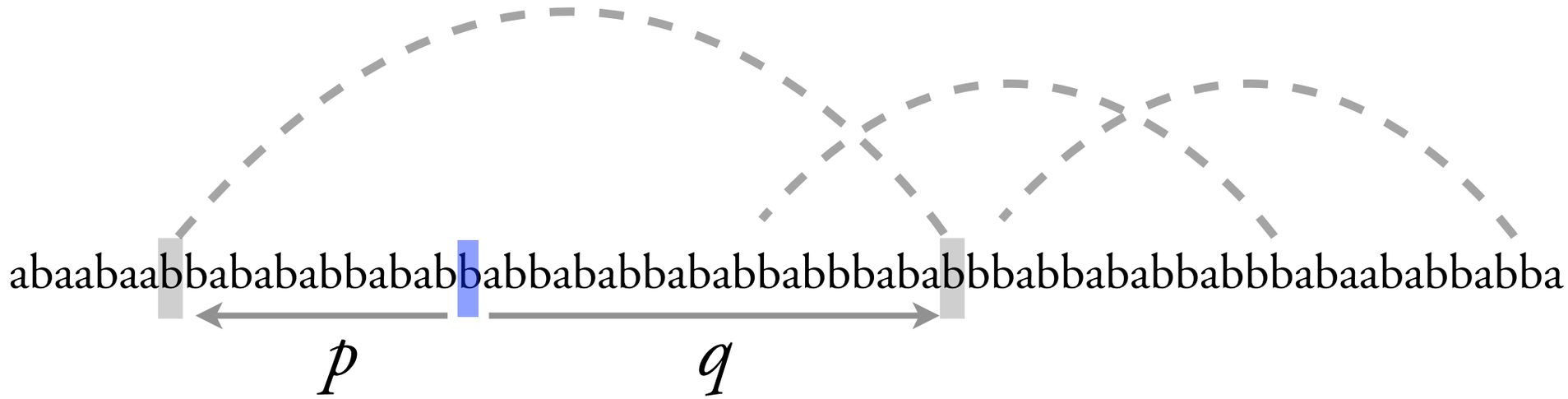

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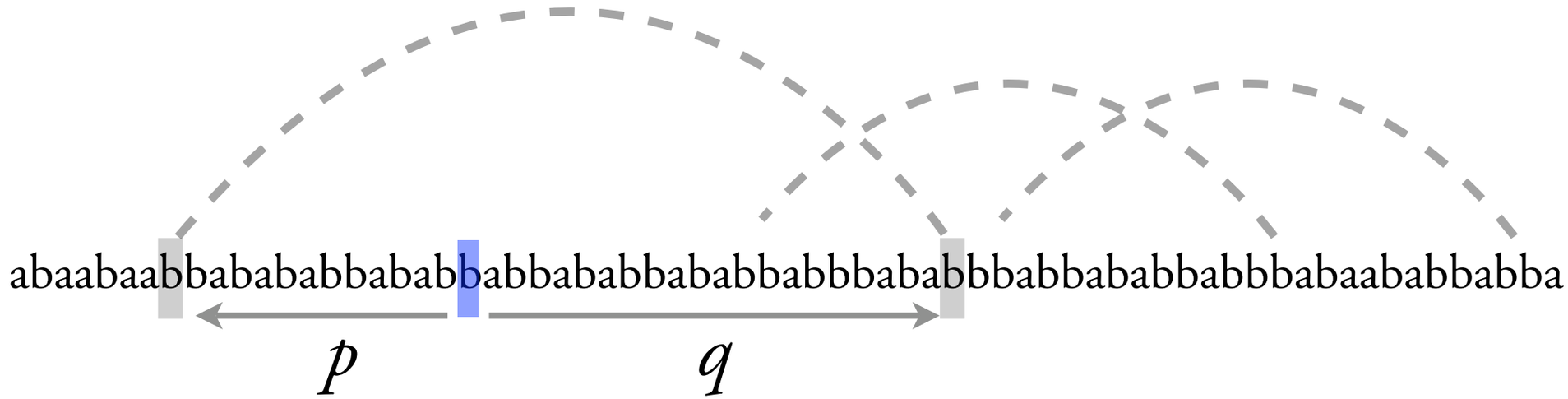

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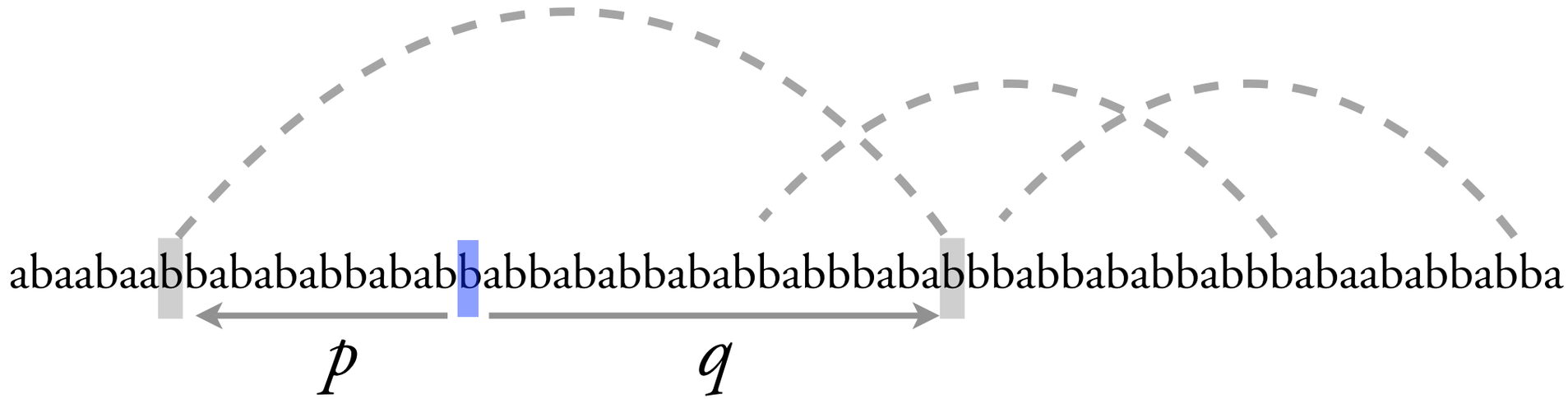

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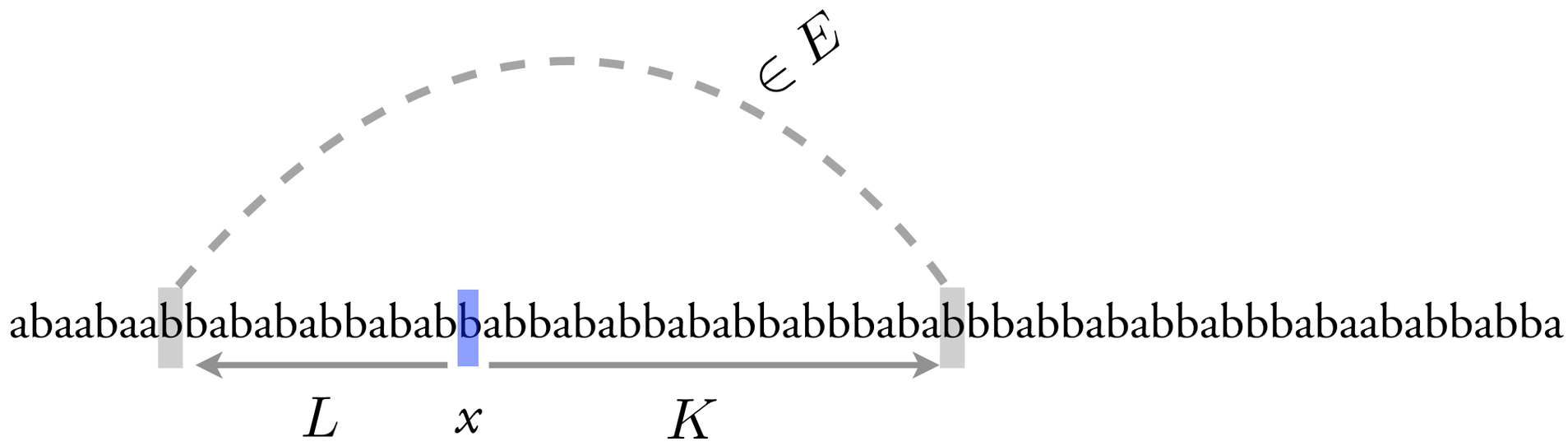
Input. $a_1 \cdots a_n \in \Sigma^*$ $E \subseteq \{1, \ldots, n\}^2$ matching

Output. Set of nodes x with



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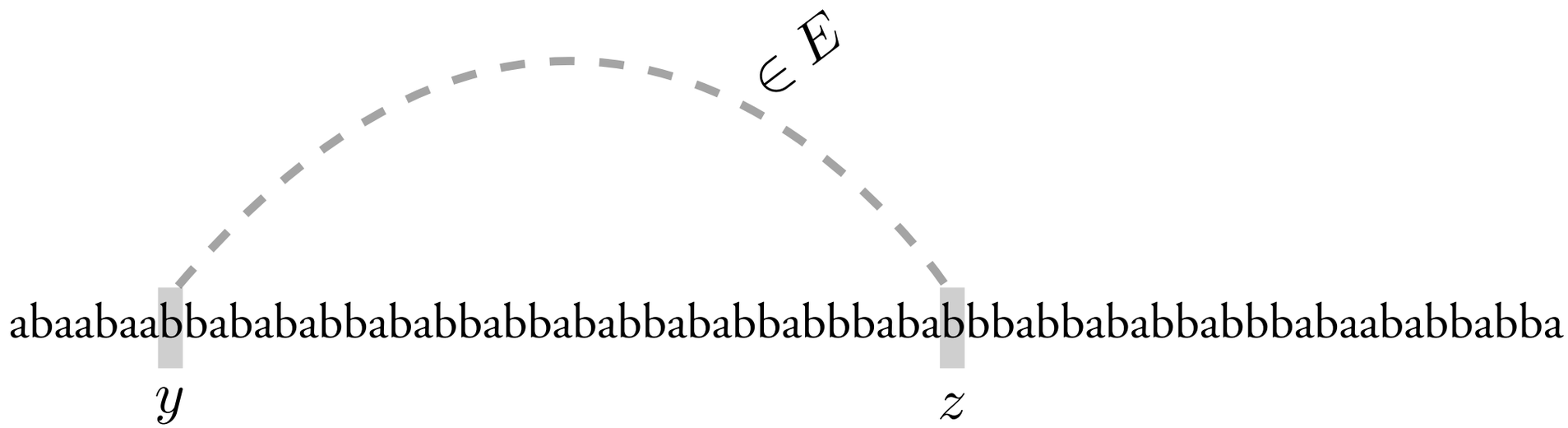

Naive algorithm.

For every match $(y, z) \in E$

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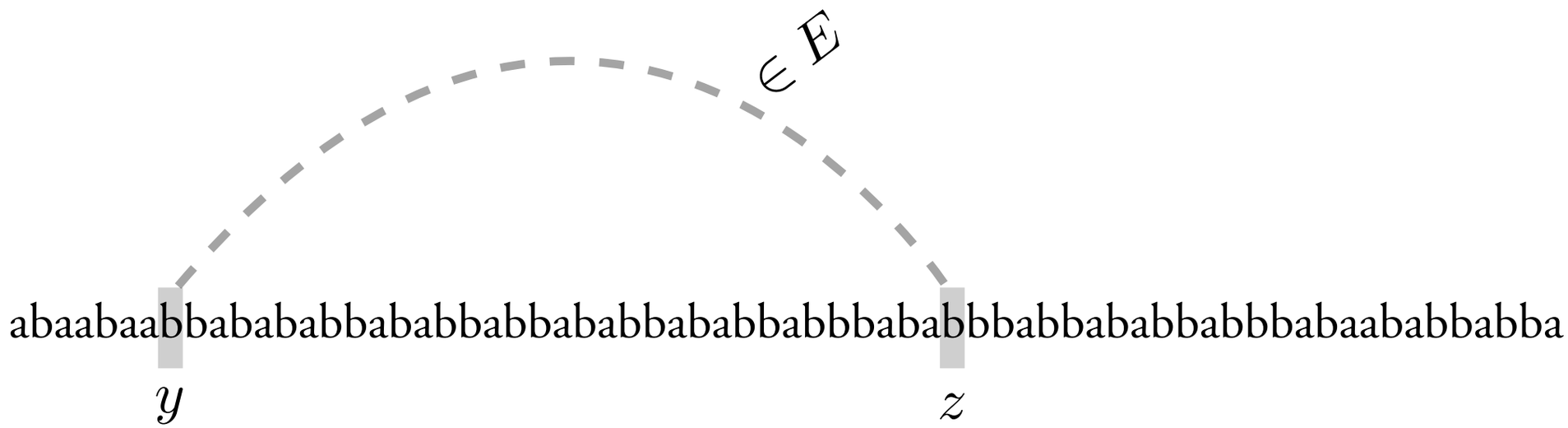

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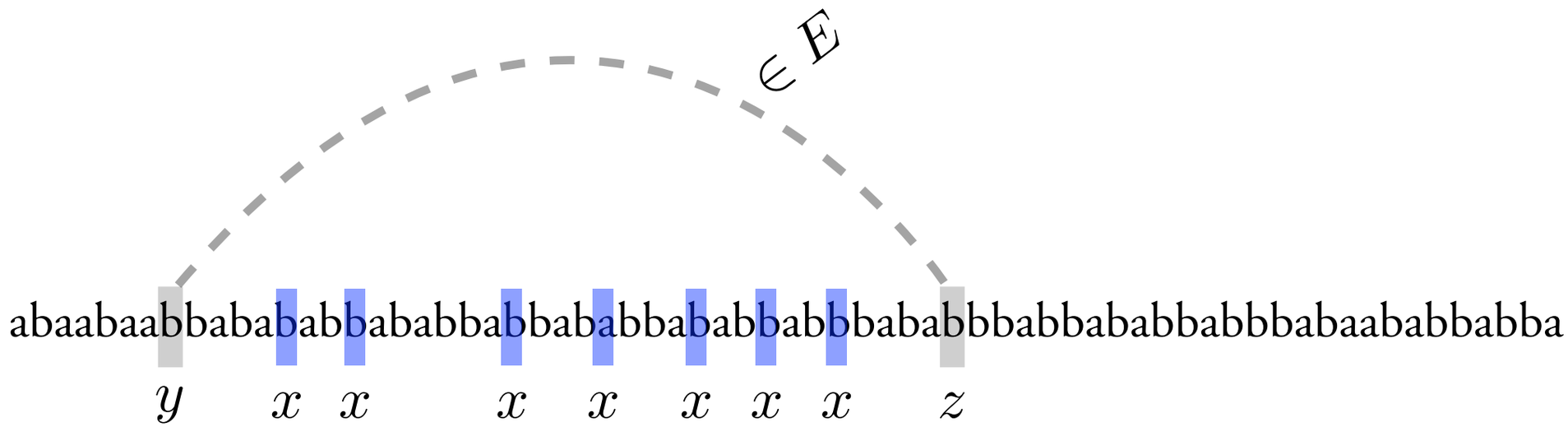

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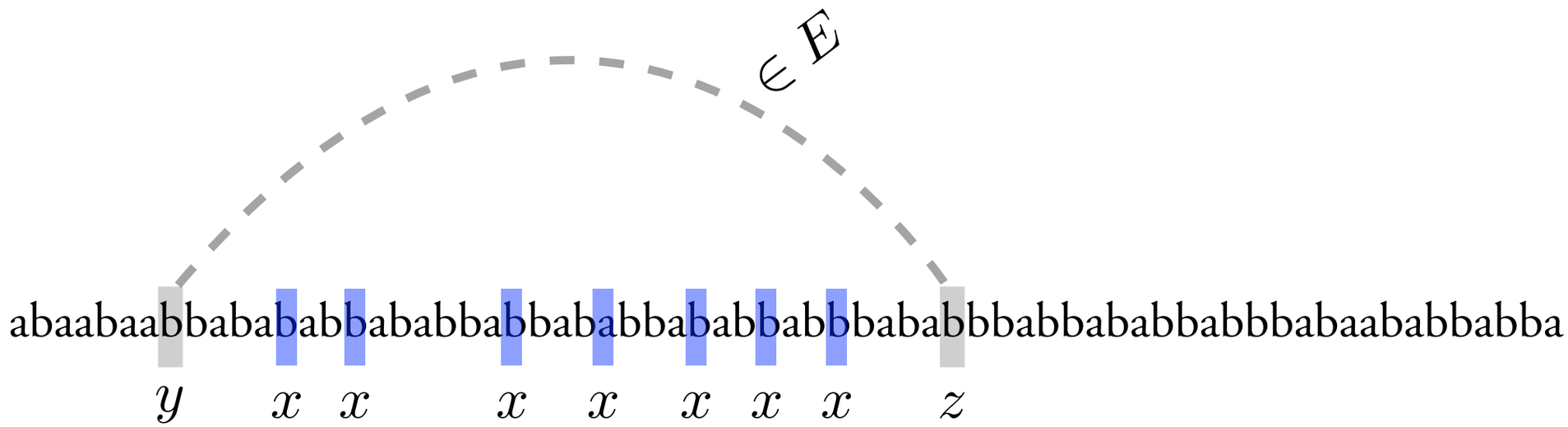

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Find nodes x such that $w[y..x] \in L$ $w[x..z] \in K$ time $O(n^2)$

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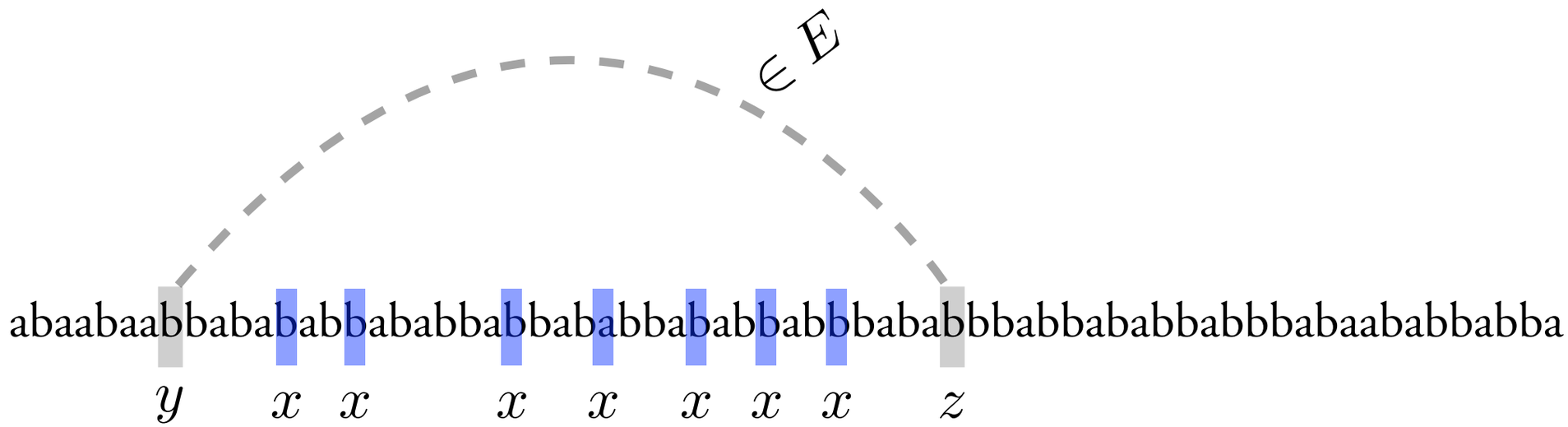

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Divide and conquer dynamic algorithm.

Solves the problem in time $O(n \log(n))$

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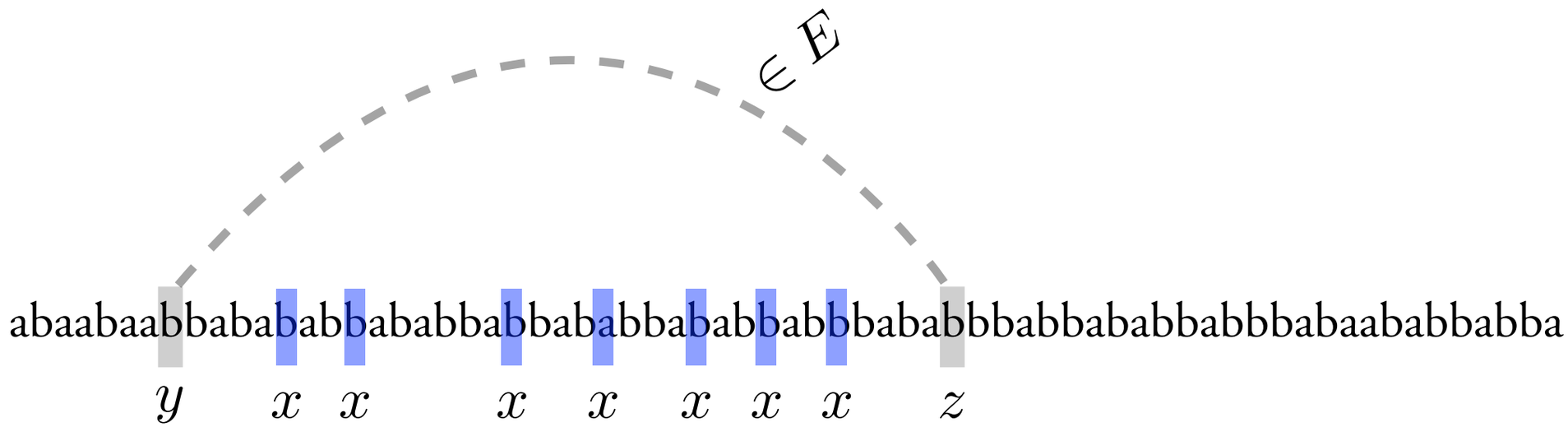

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An algorithm that uses the Simon decomposition

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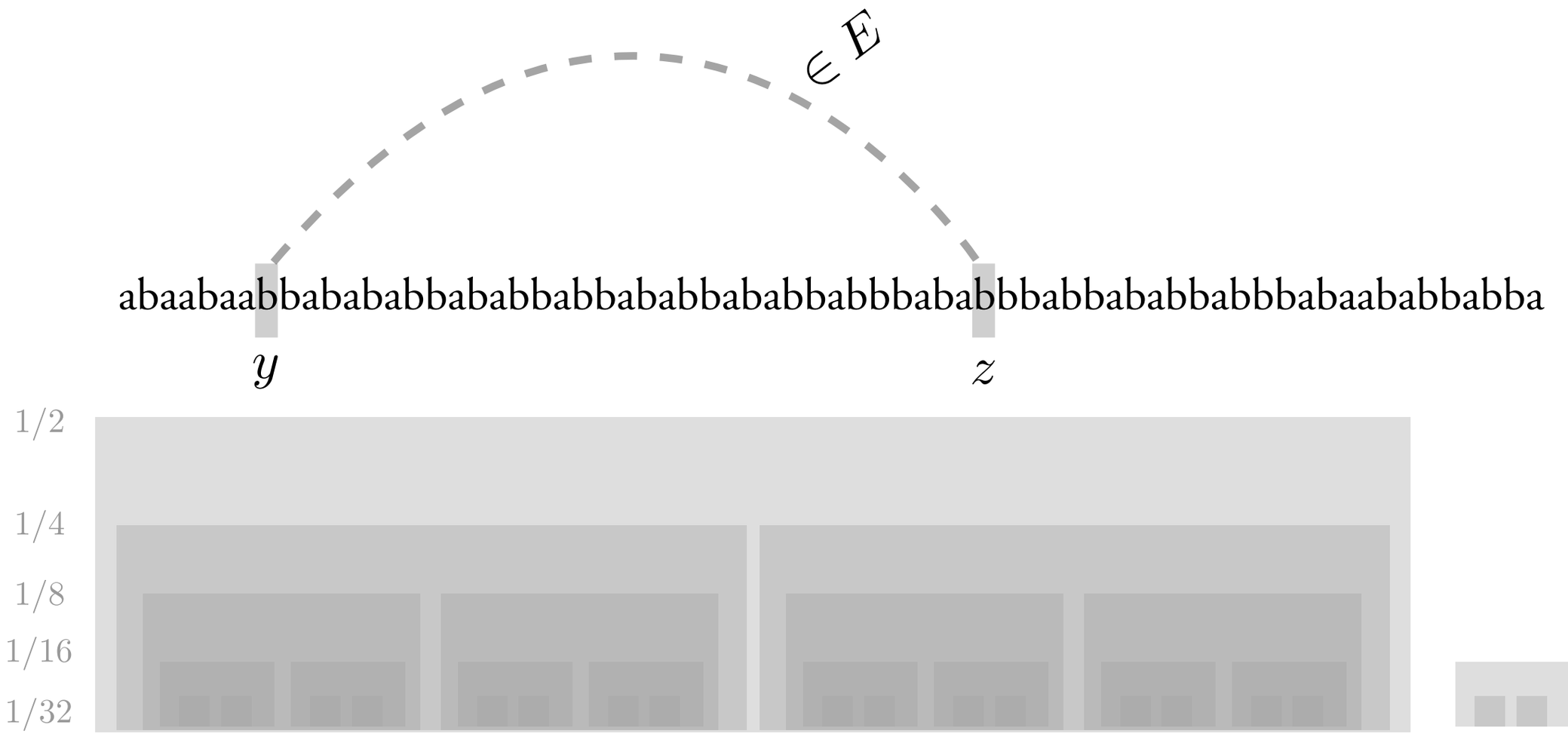

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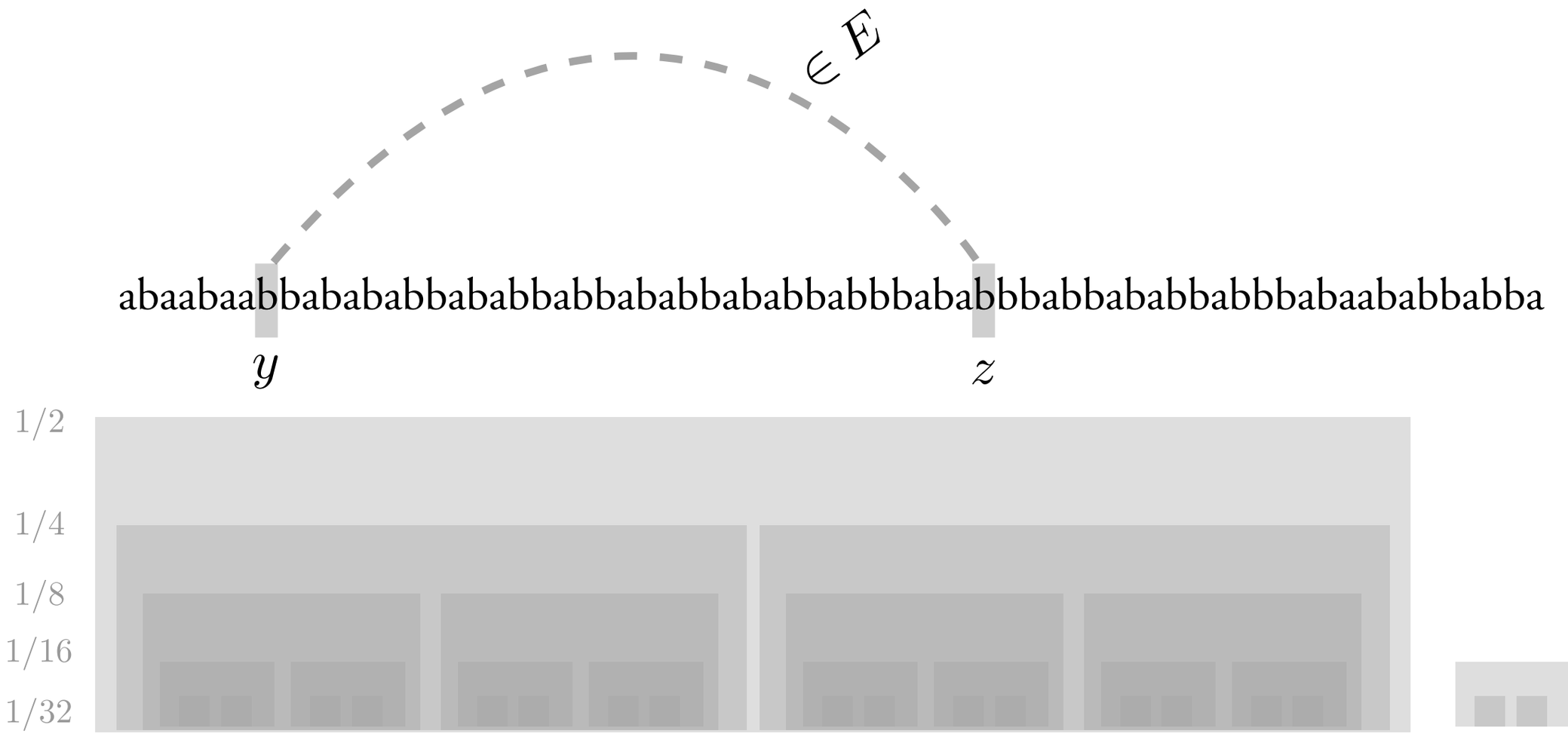

Divide and conquer dynamic algorithm.

For every match $(y, z) \in E$

Find nodes x such that $w[y..x] \in L$ $w[x..z] \in K$

but only do logarithmically many operations each time

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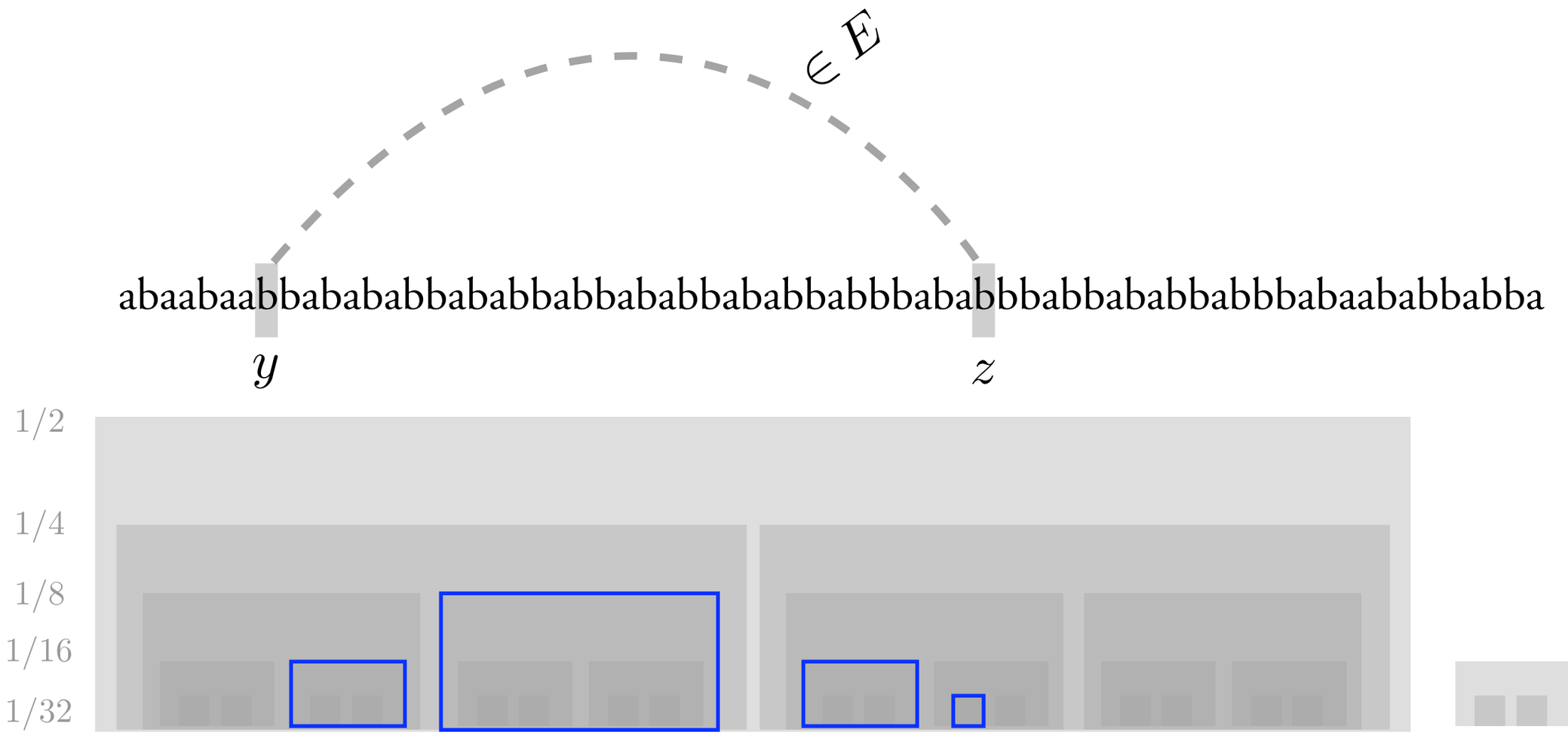

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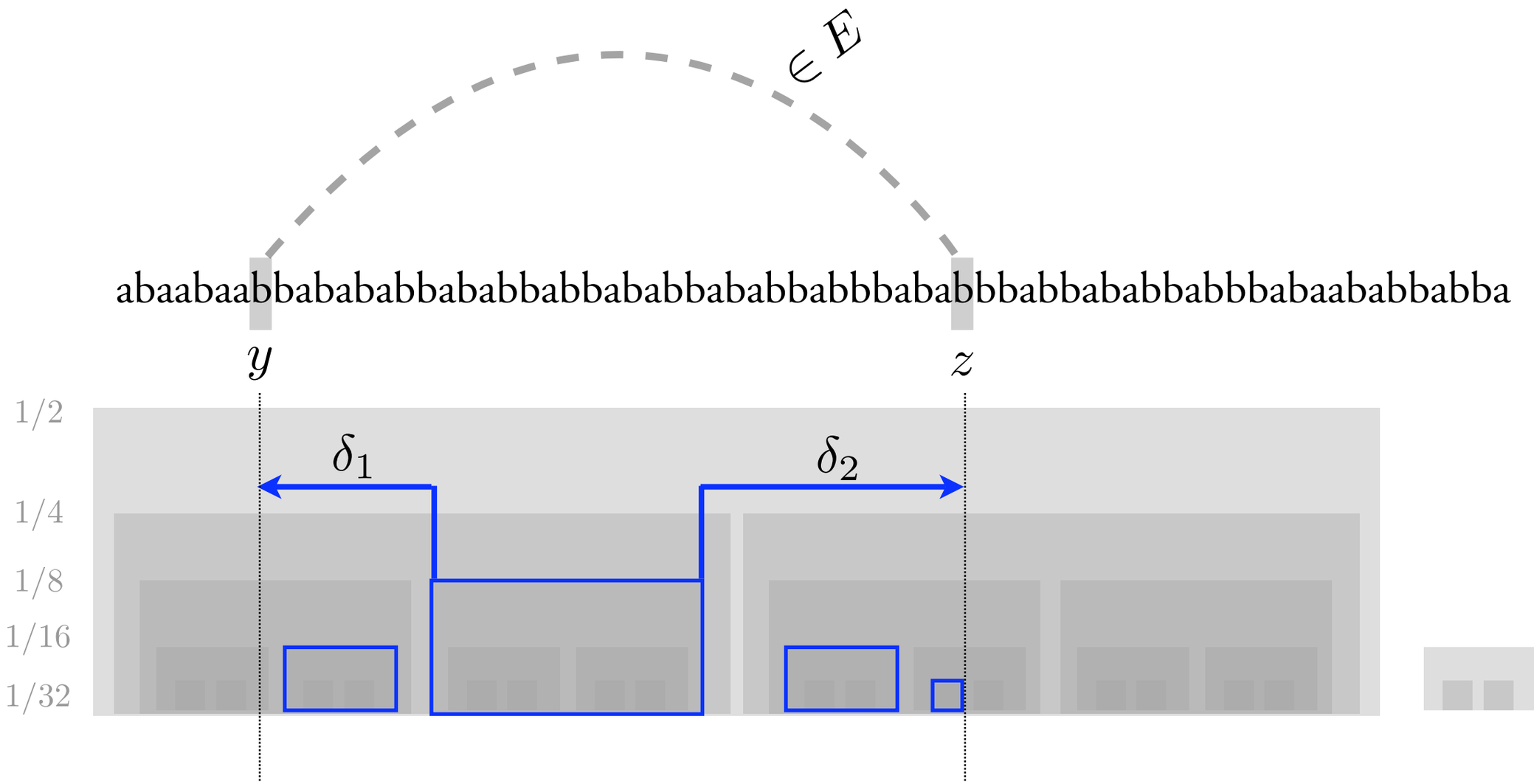

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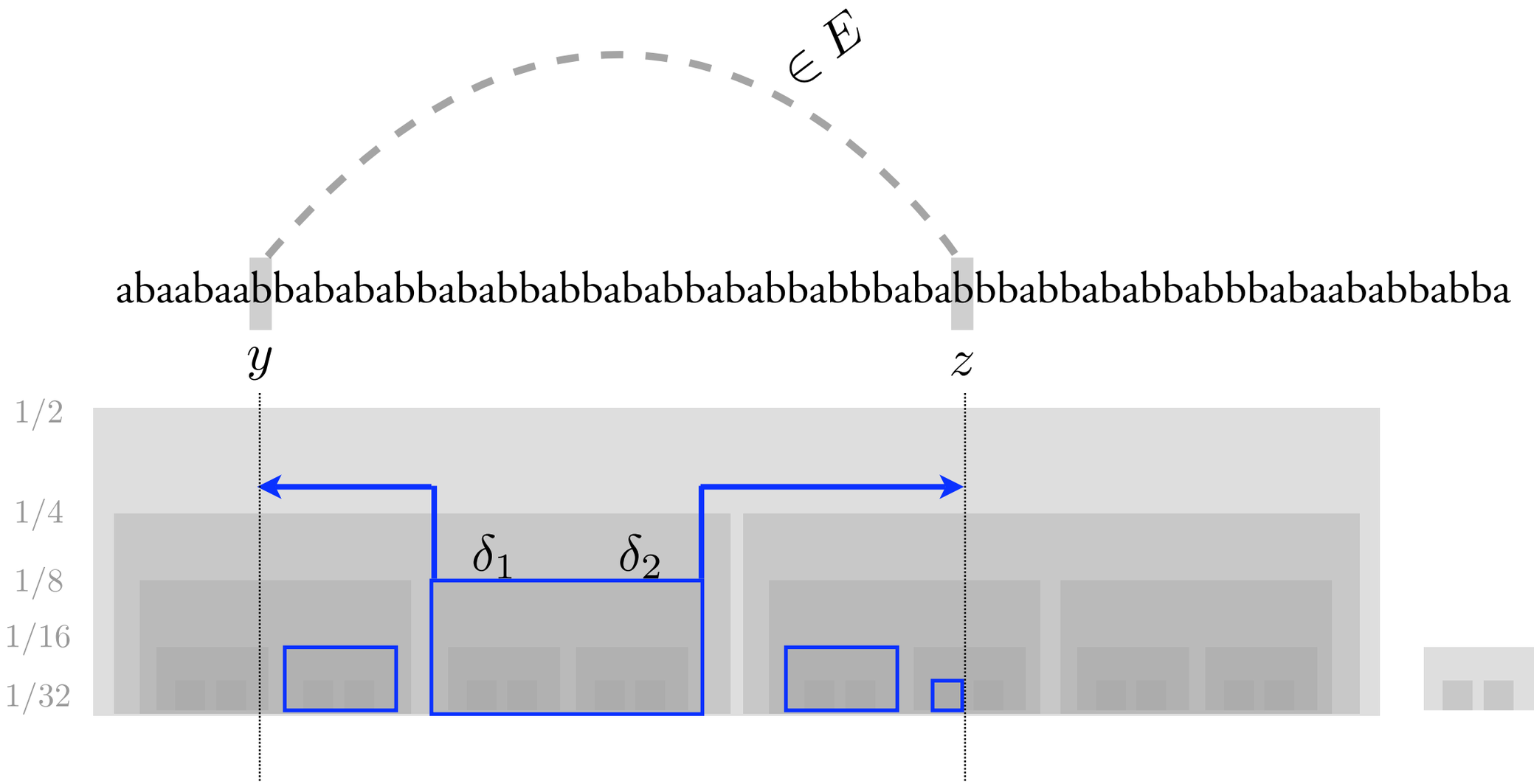

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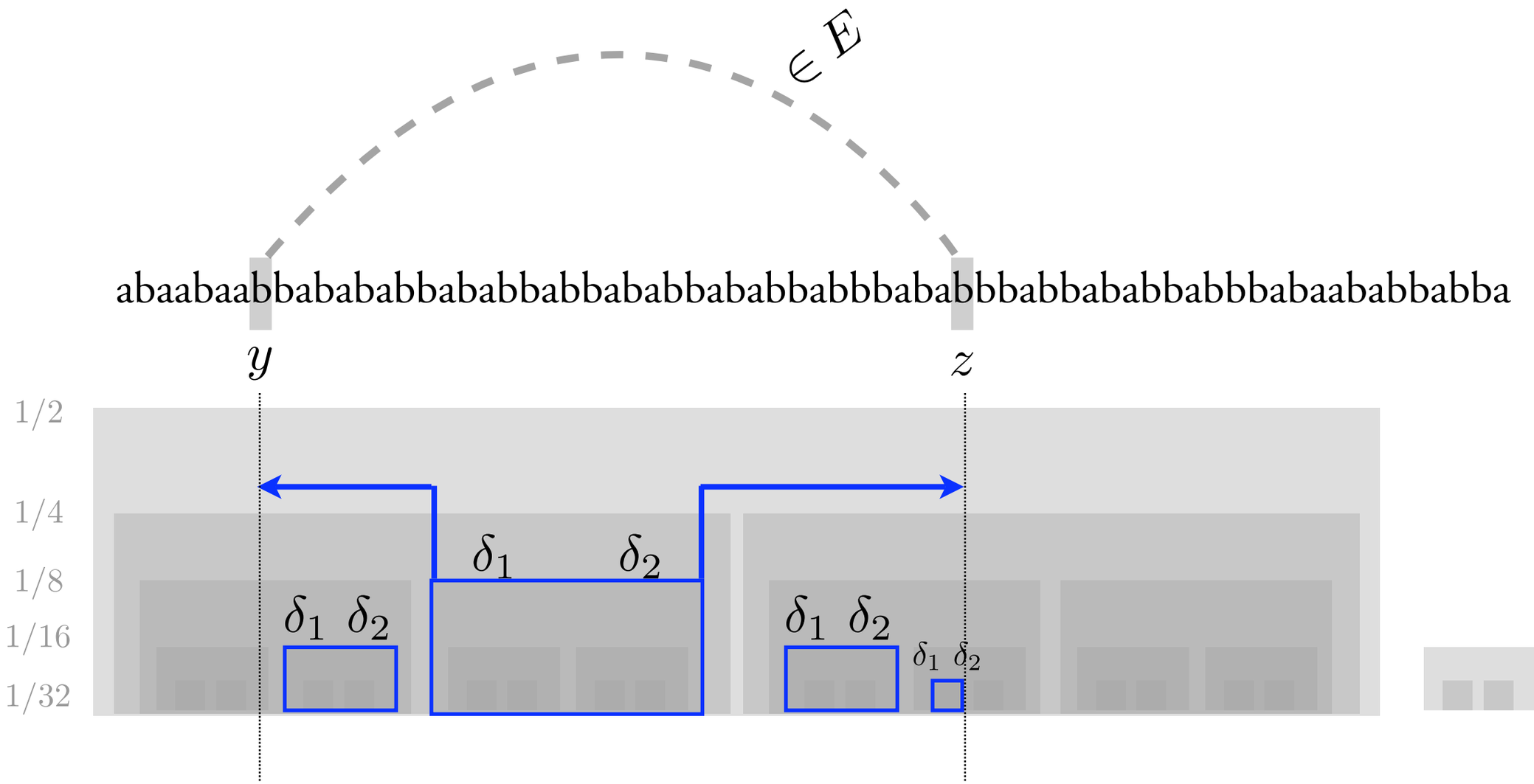

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– Preliminary results indicate that semigroups can be avoided, and the constant becomes polynomial in the query.

– We want to investigate more of XPath, and other languages

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Two rules for splitting words.

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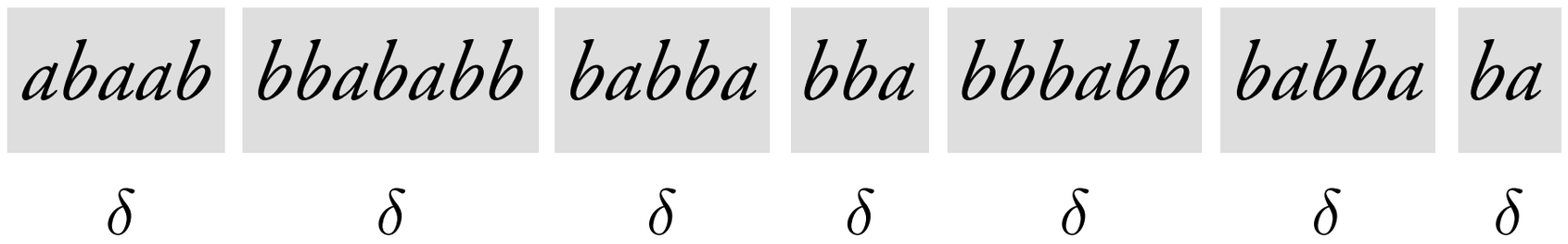

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