A robust extension of ω-regular word languages.

Mikołaj Bojańczyk
Warsaw University

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– regular expressions
– automata
– monadic second-order logic
– closure properties
– semigroups
– Myhill-Nerode equivalence

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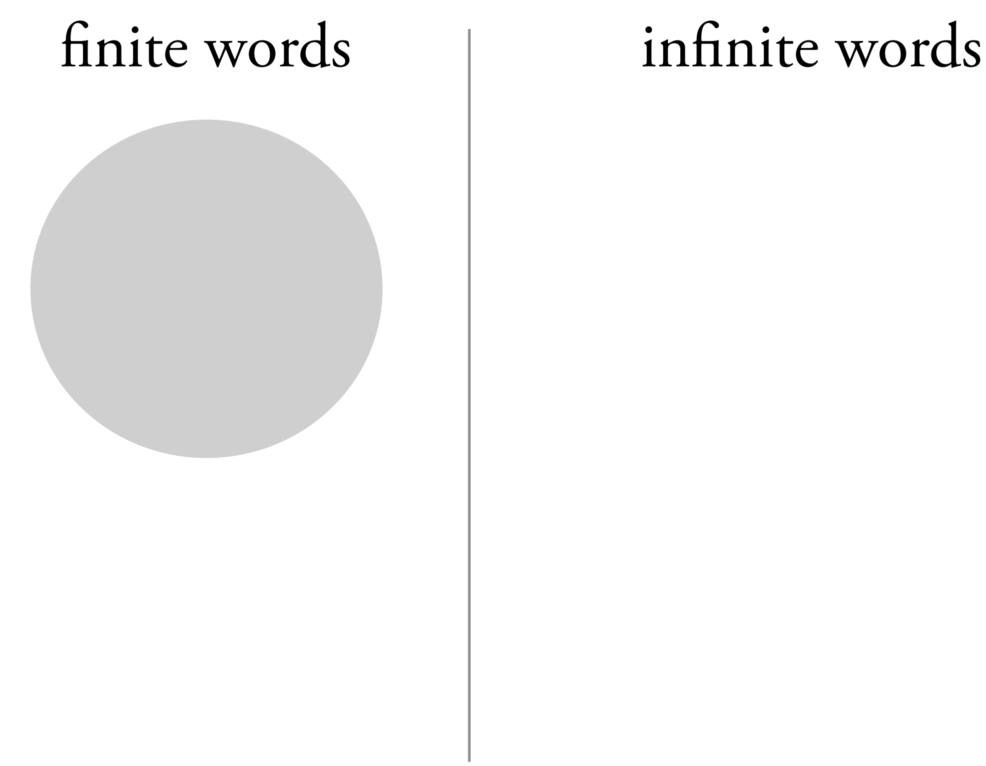

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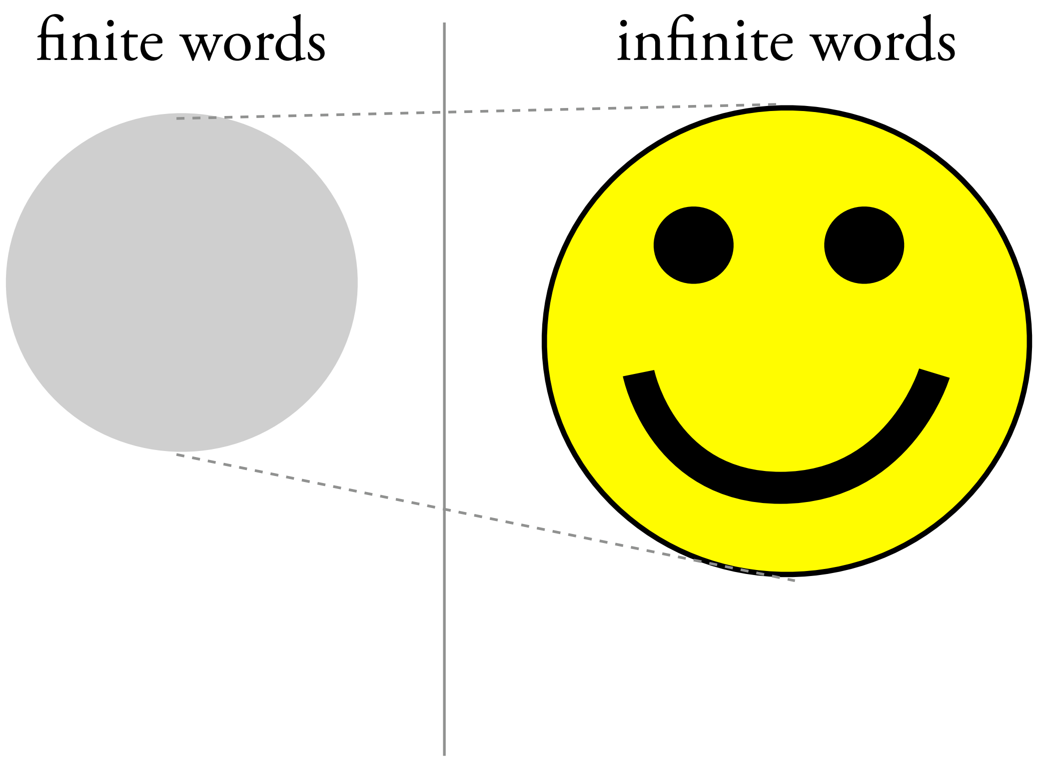

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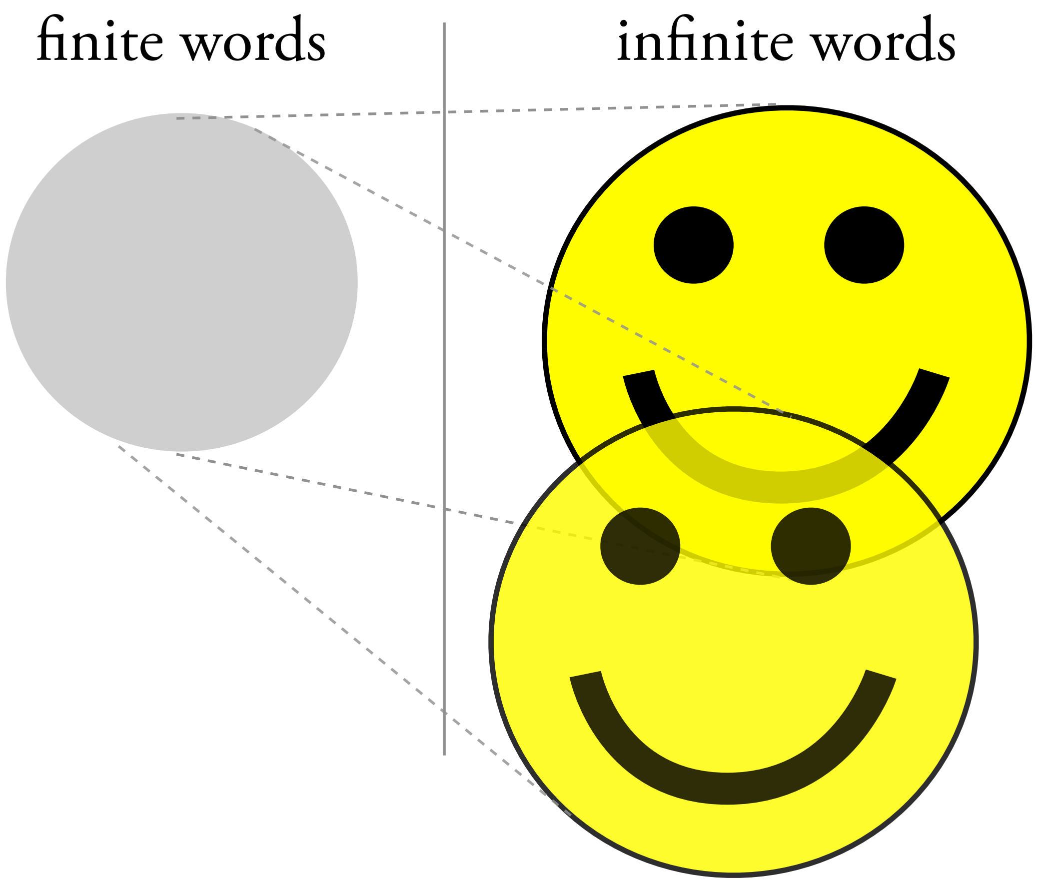

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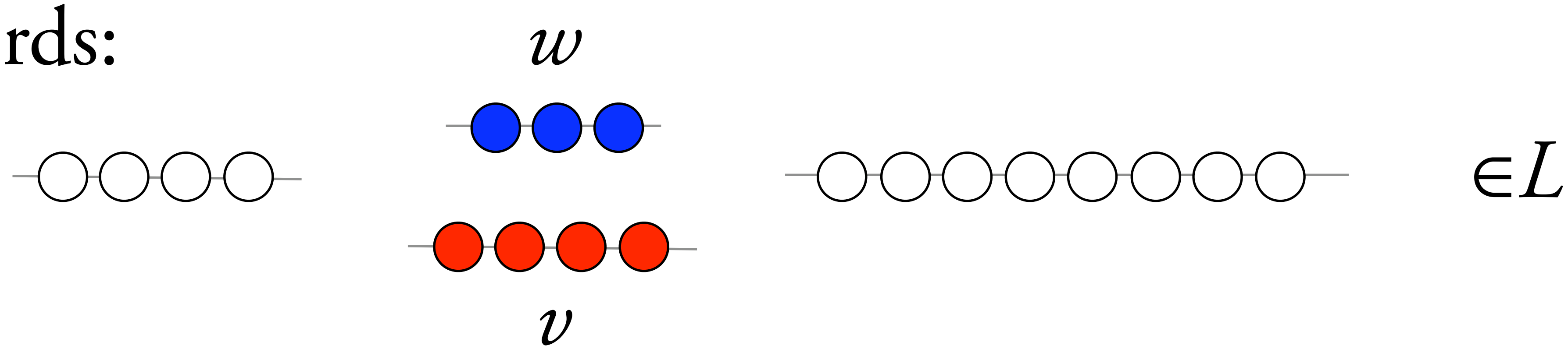

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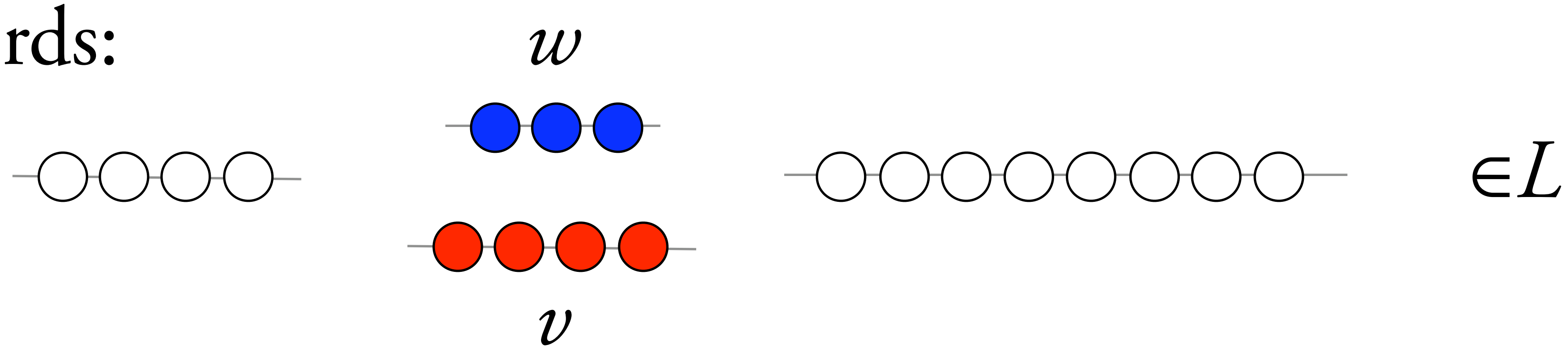

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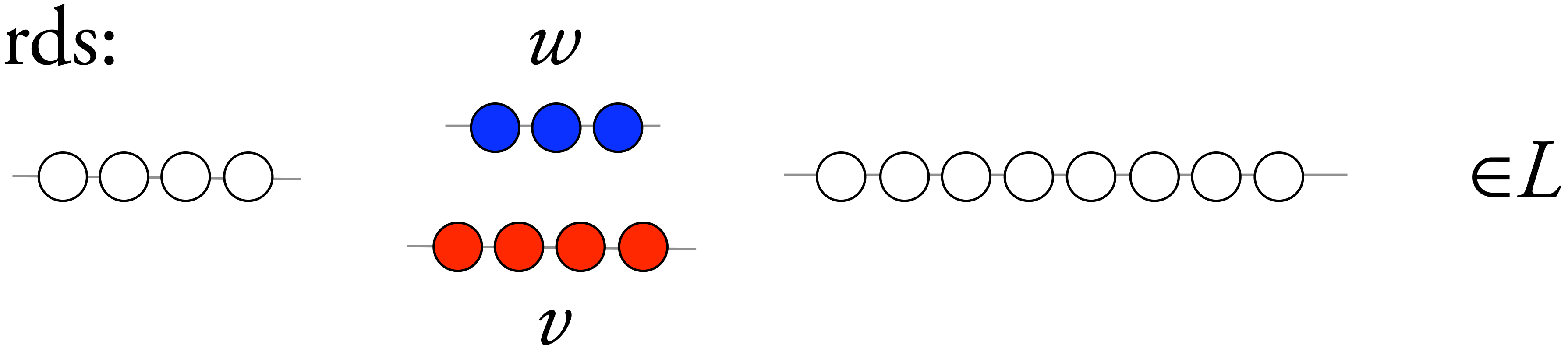

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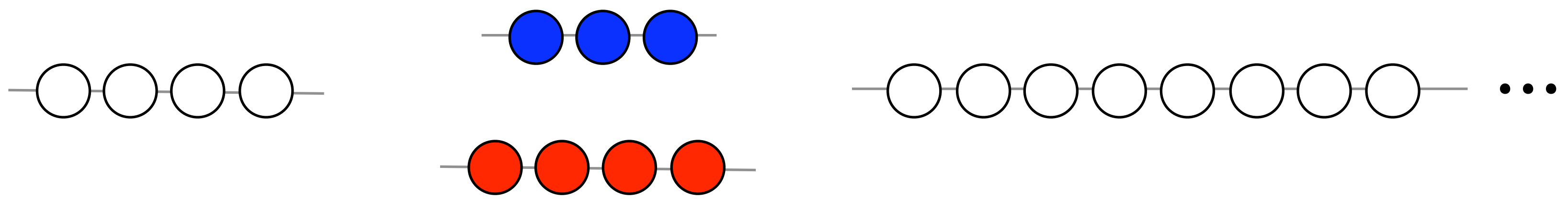

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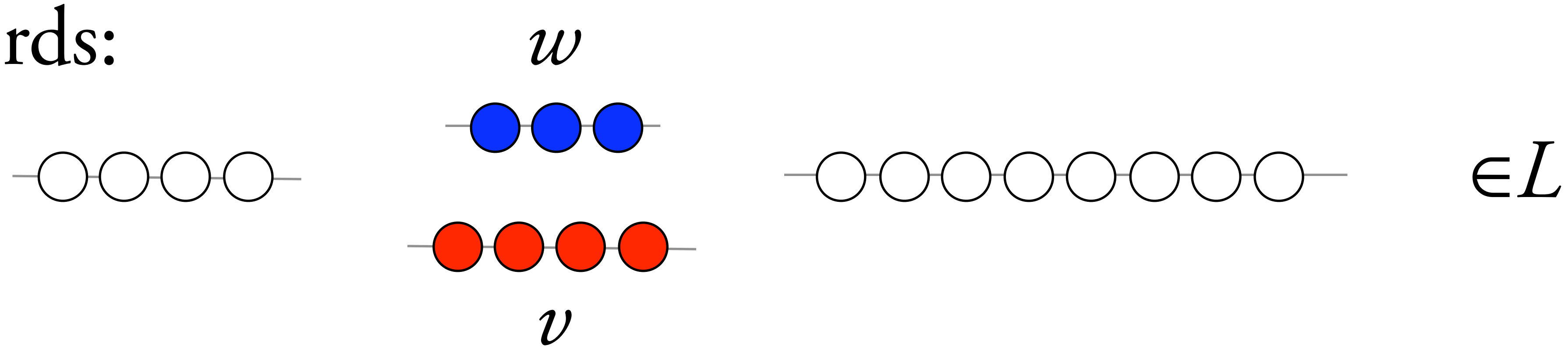

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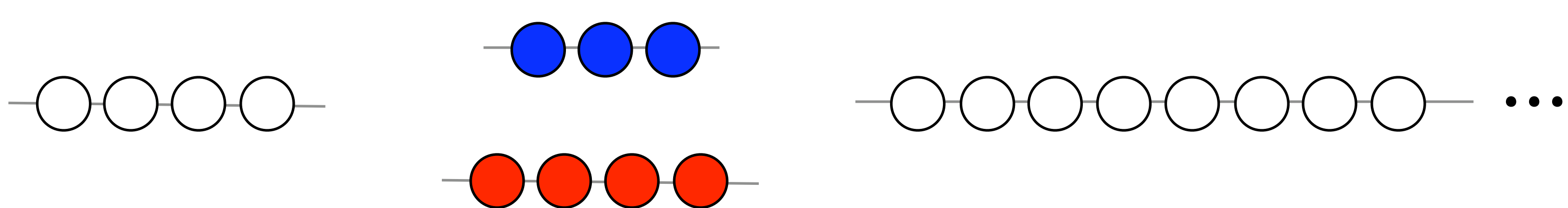

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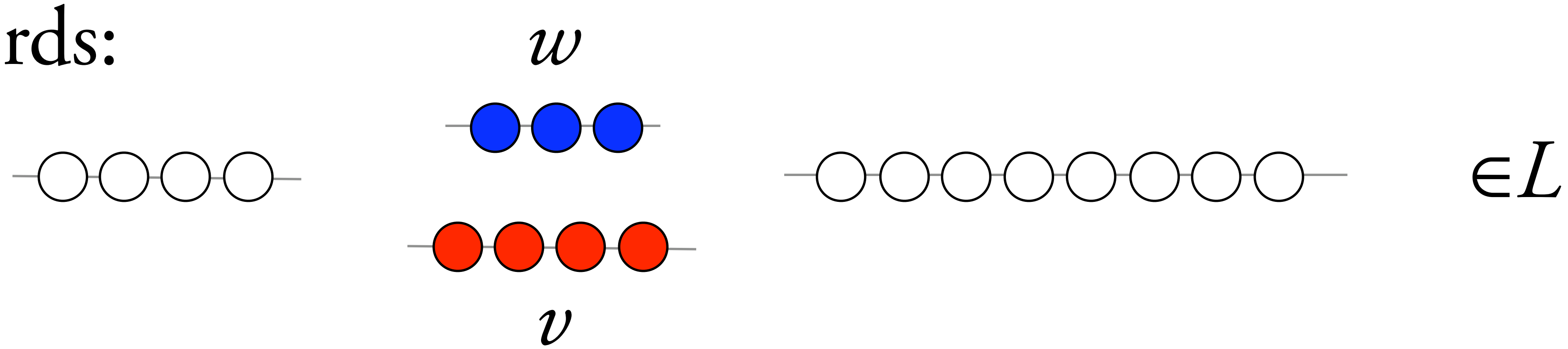

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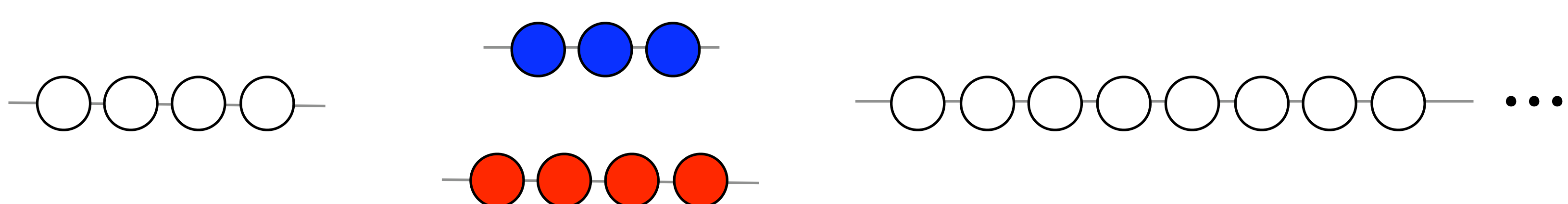

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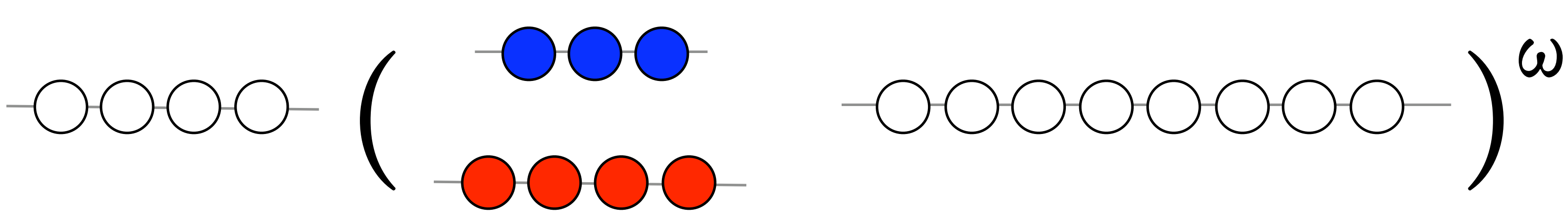

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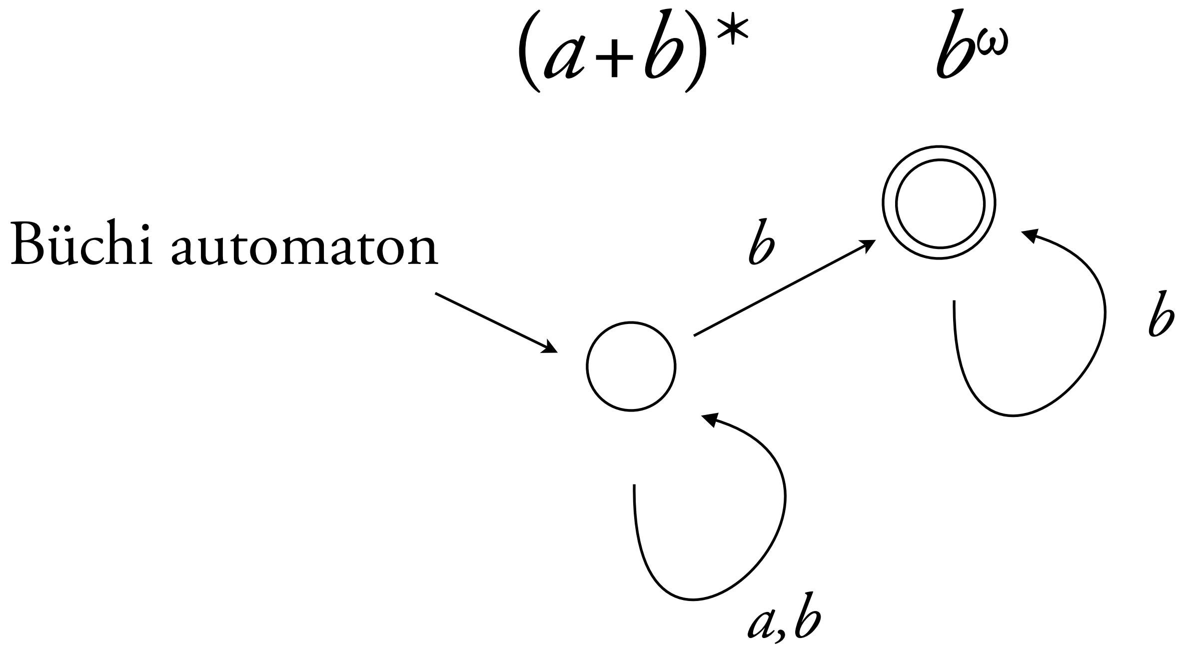

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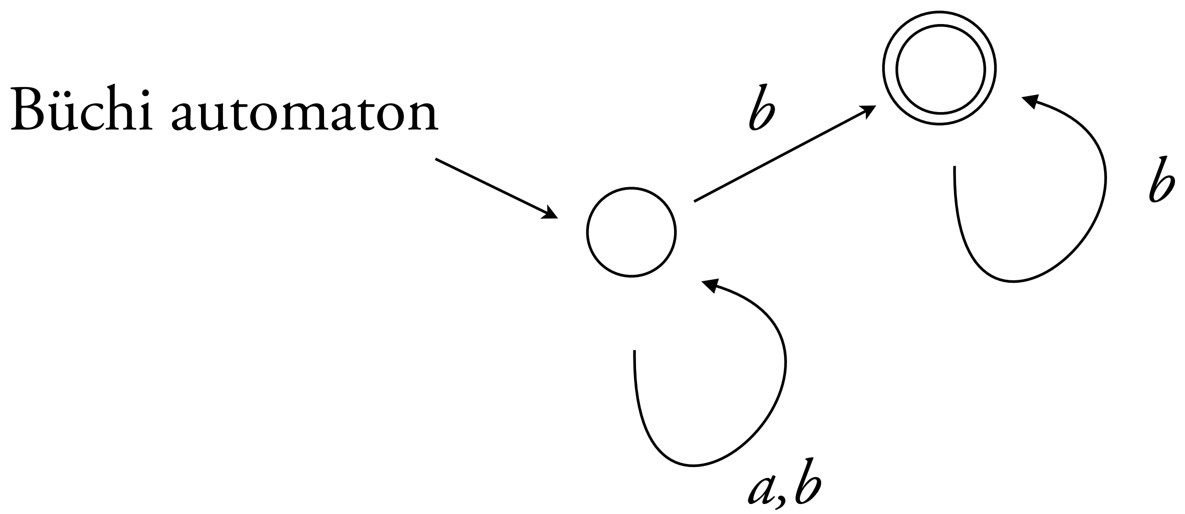

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For every nondeterministic Büchi automaton, there is an equivalent deterministic Muller automaton.

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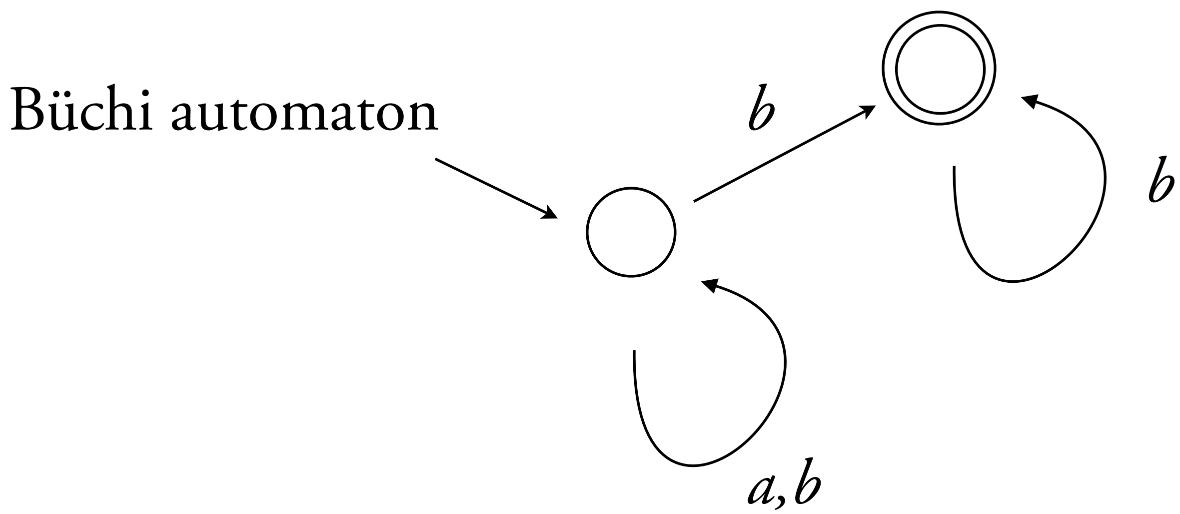

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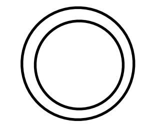 appears infinitely often,

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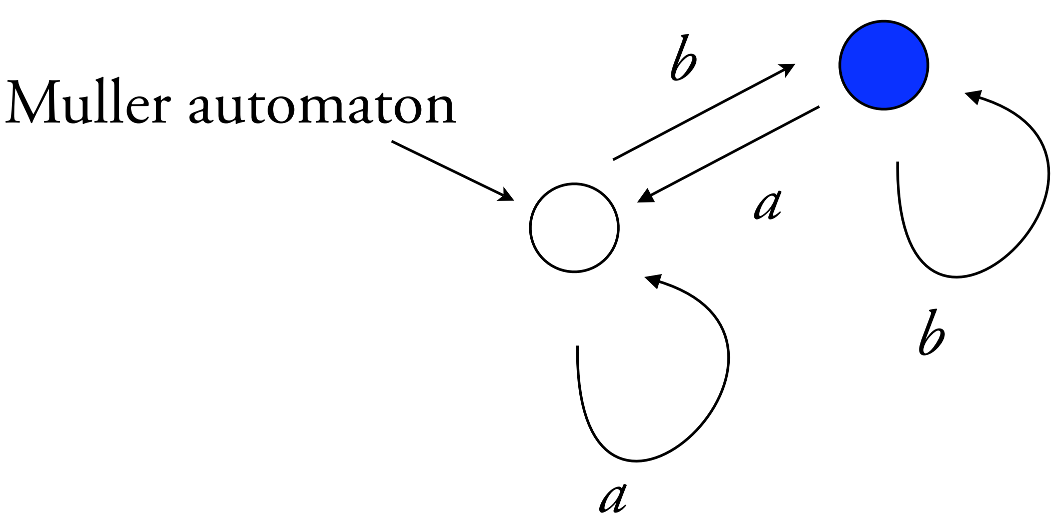

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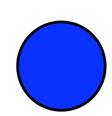
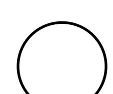
 appears infinitely often,

and  appears finitely often

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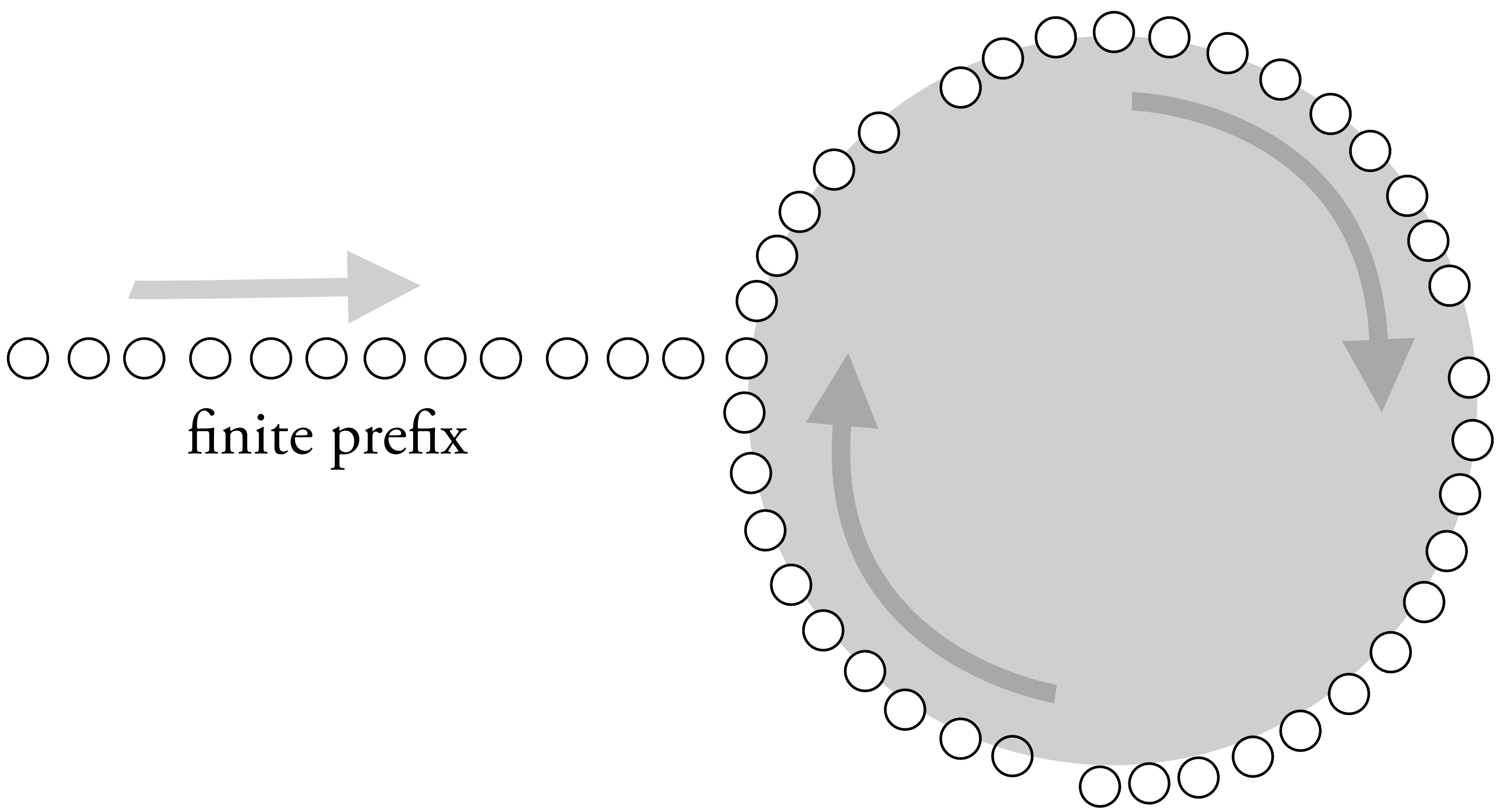

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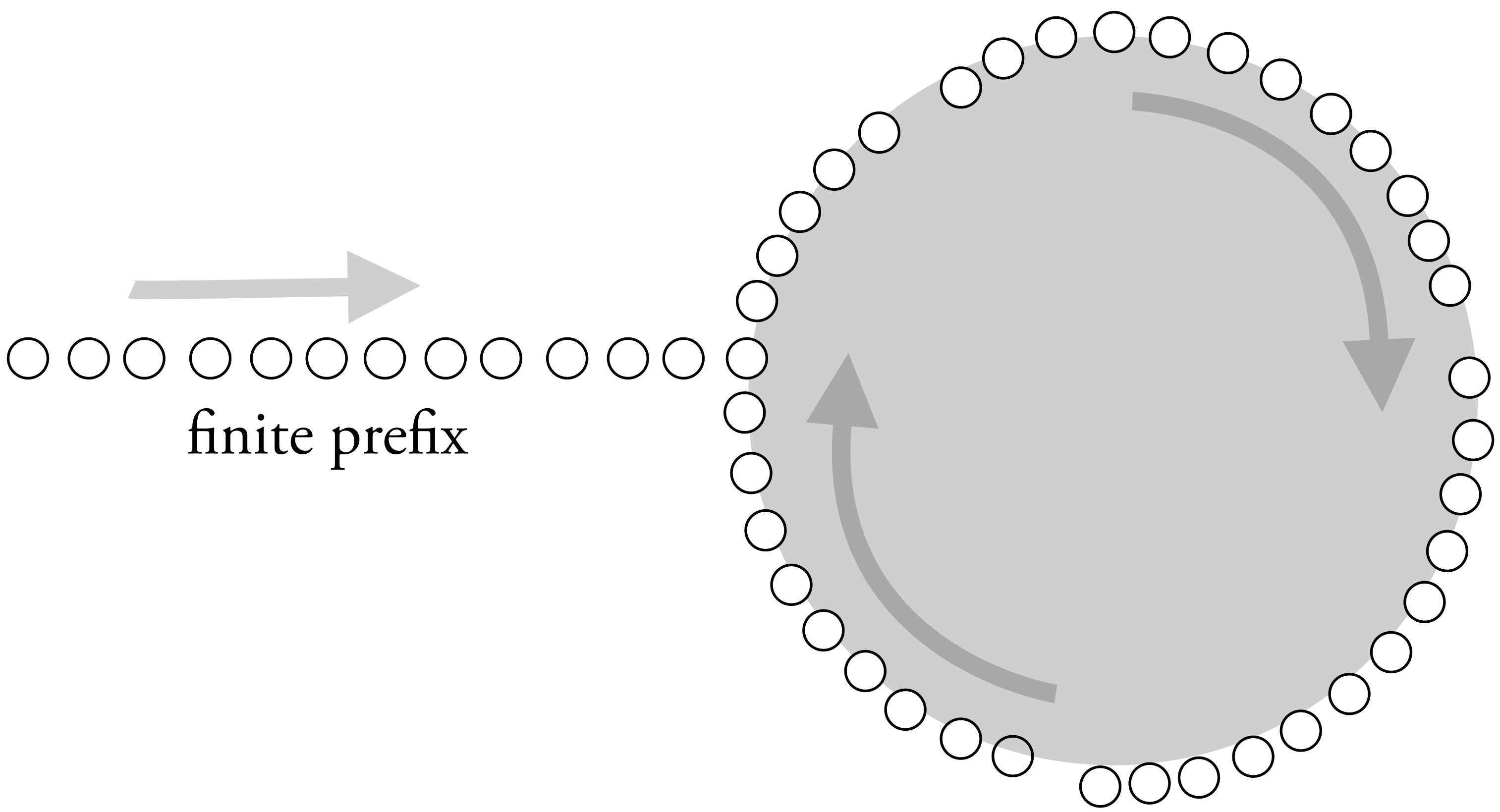

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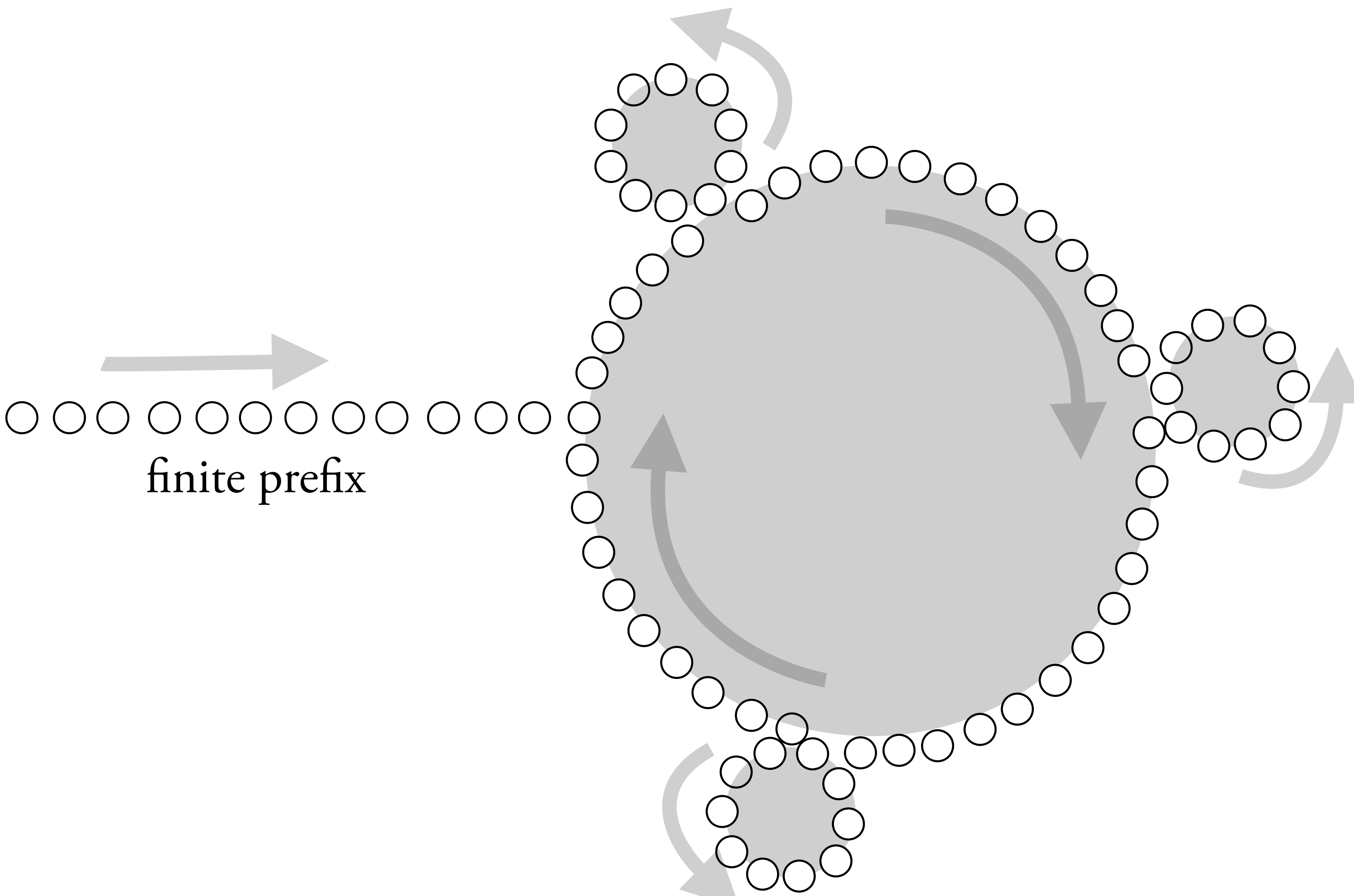

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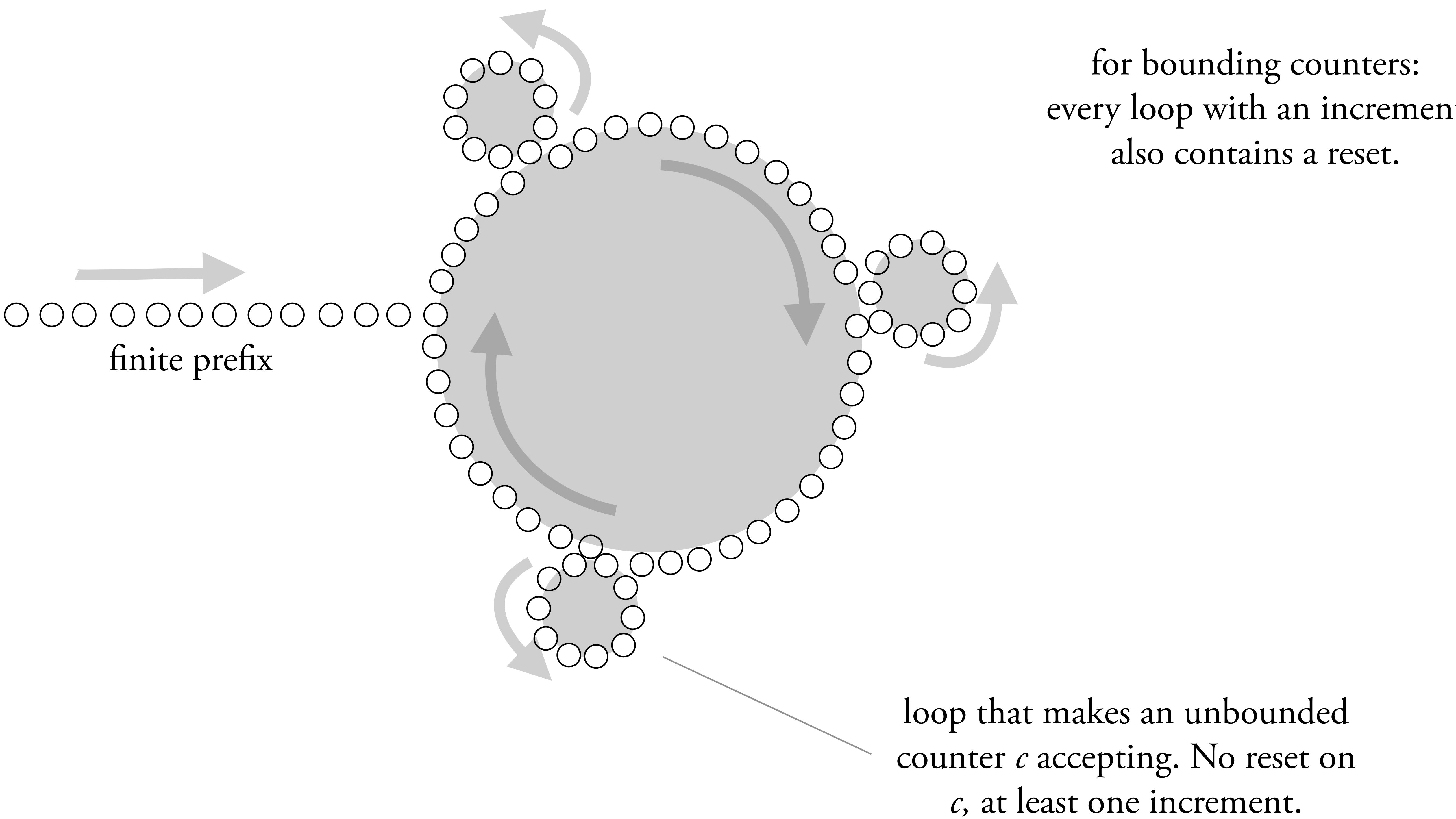

What is the logic for max-automata?

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$\mathrm{U}X$ "X is a set of consecutive a's"

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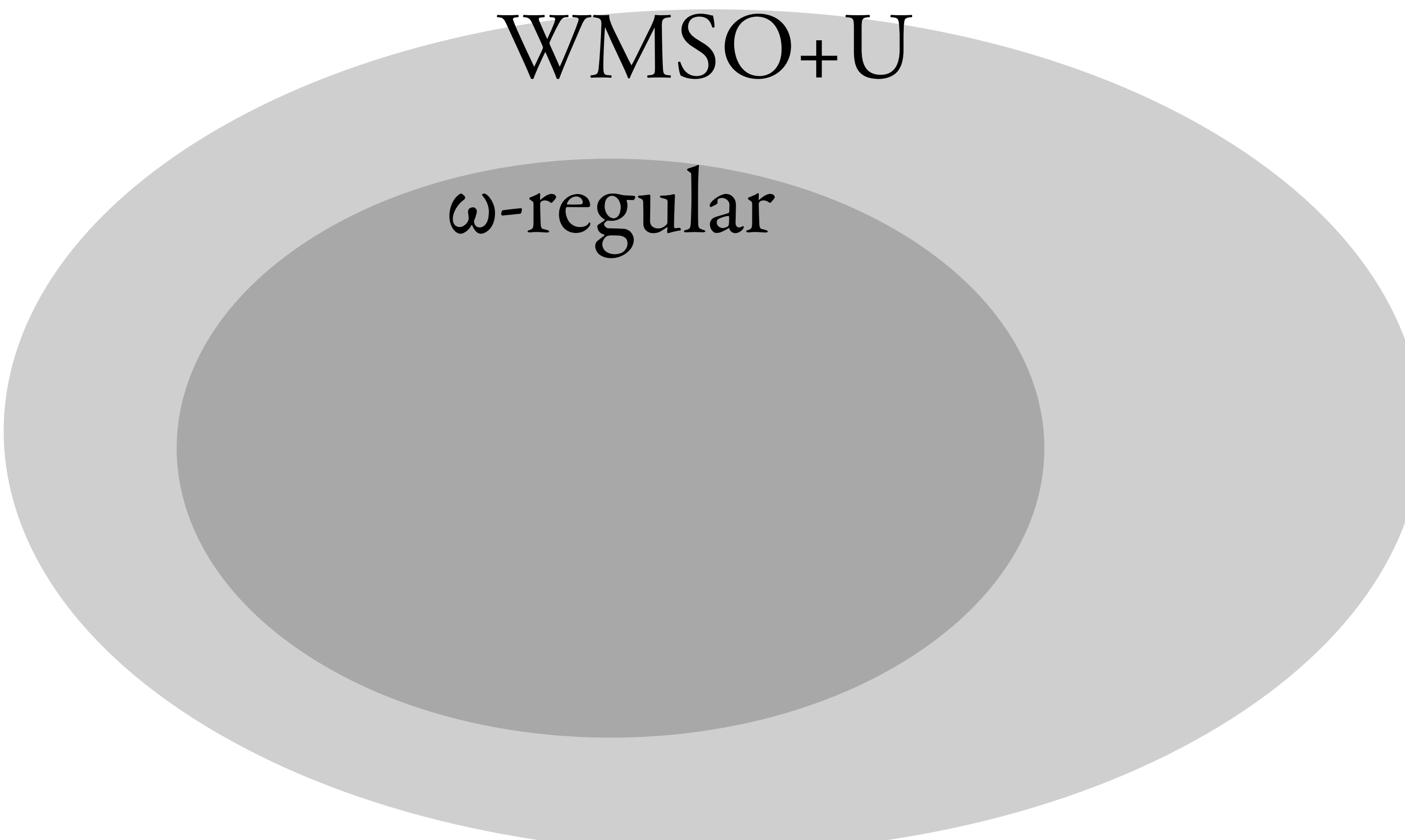

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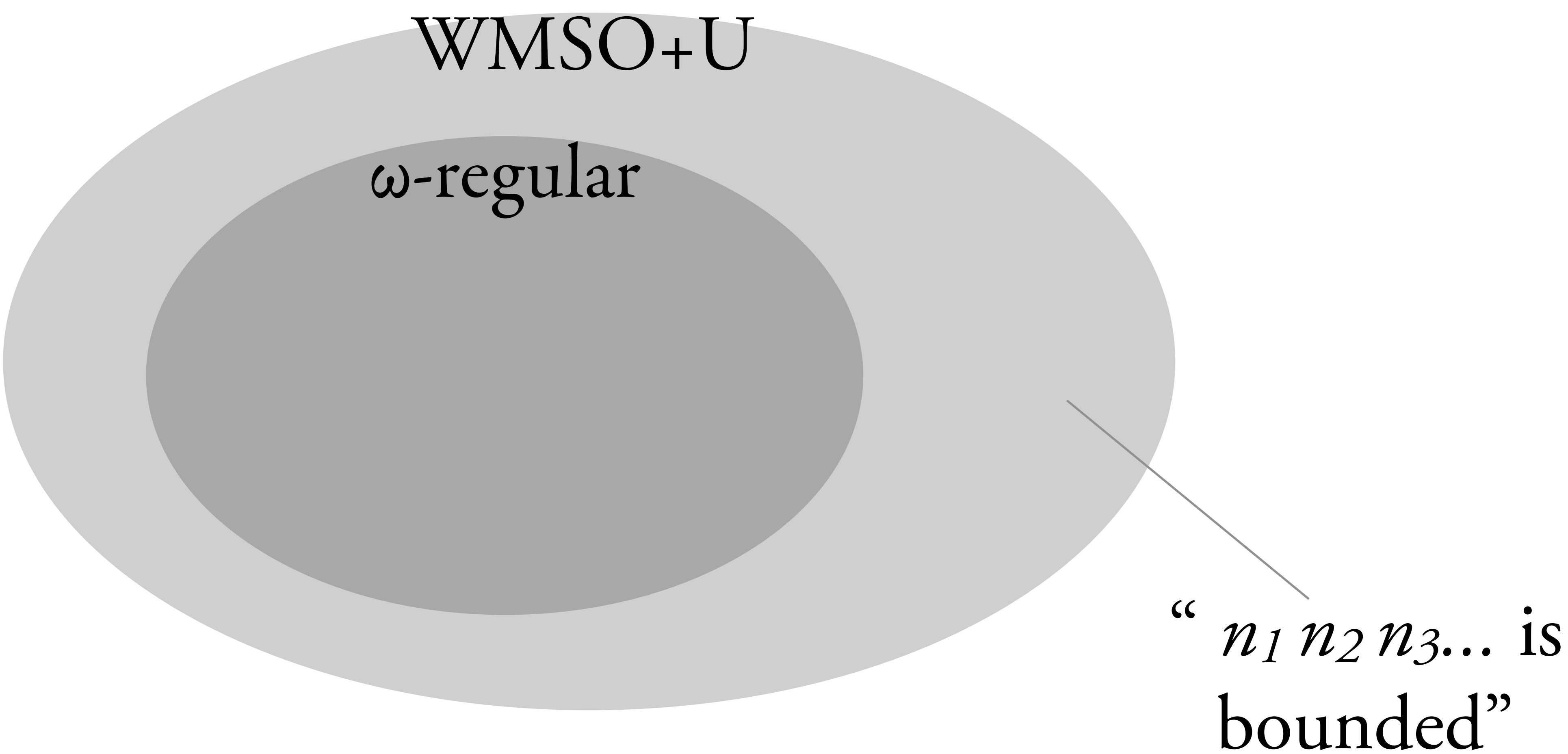

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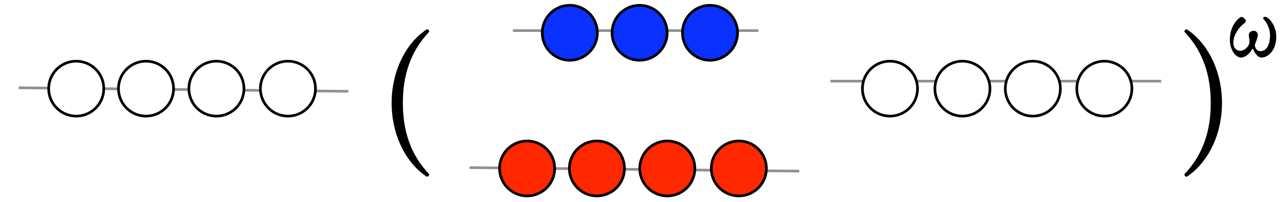

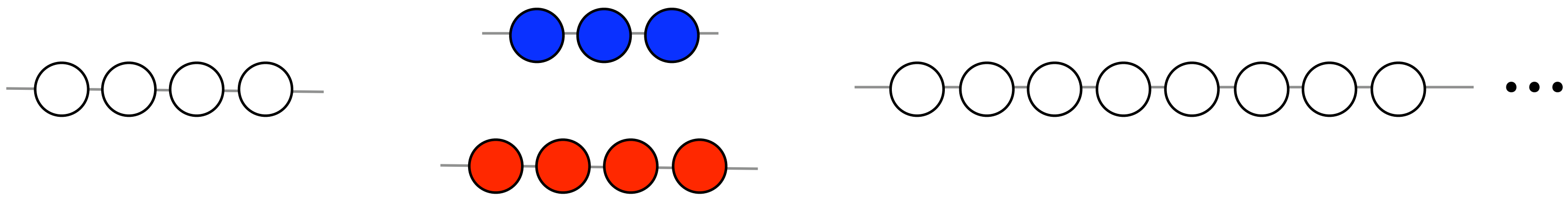

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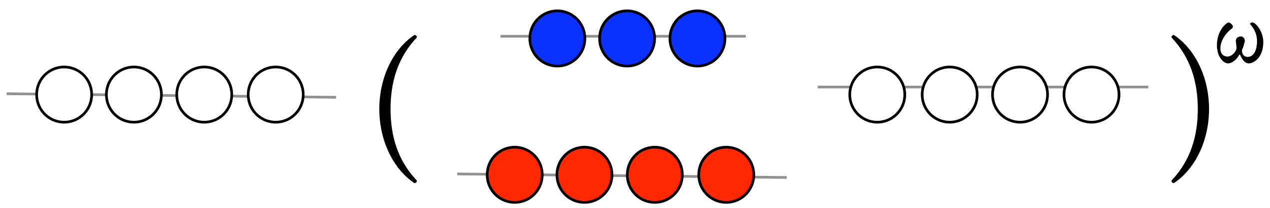

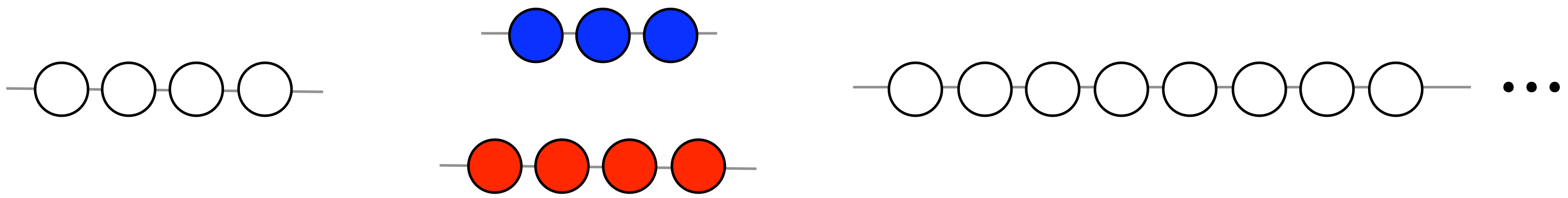

Prop. Languages recognized by max-automata have finitely many equivalence classes. Each class is a regular language of finite words.

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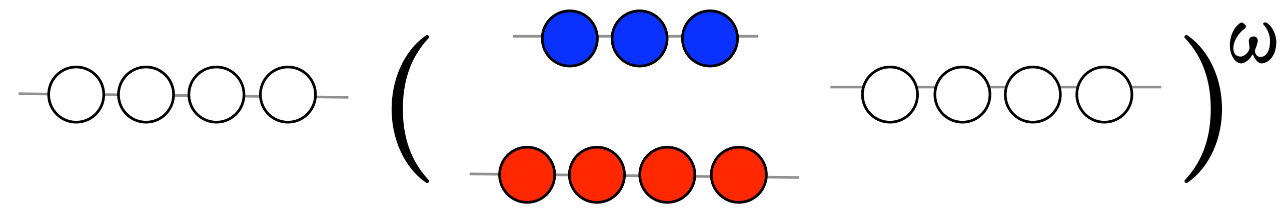

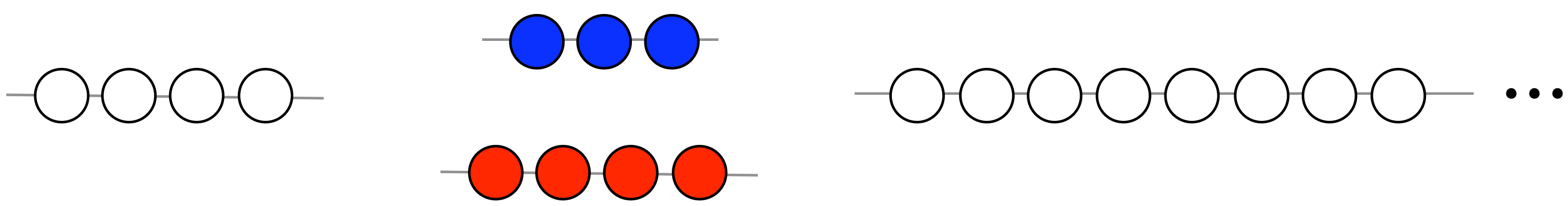

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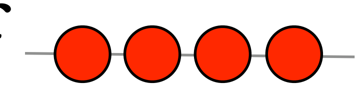
Proof sketch. Equivalence class of  depends on state transformations, which counters are incremented (but not how much), and which counters are reset.

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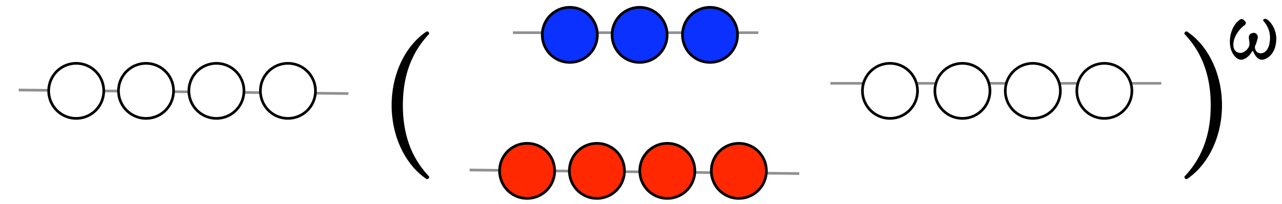

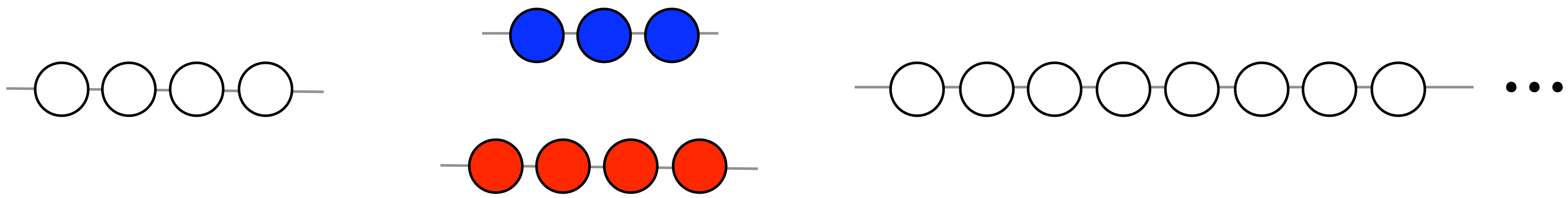

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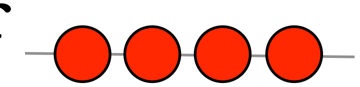
Proof sketch. Equivalence class of  depends on state transformations, which counters are incremented (but not how much), and which counters are reset.

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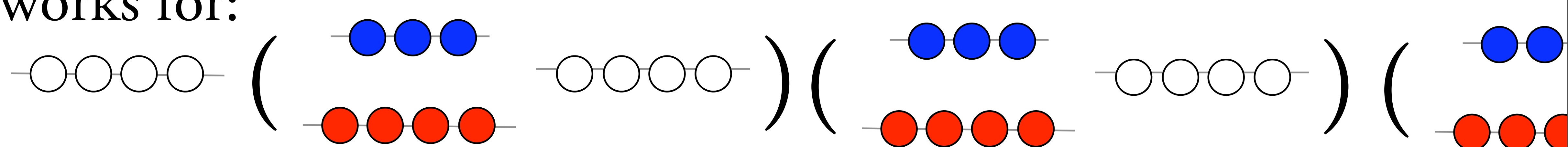

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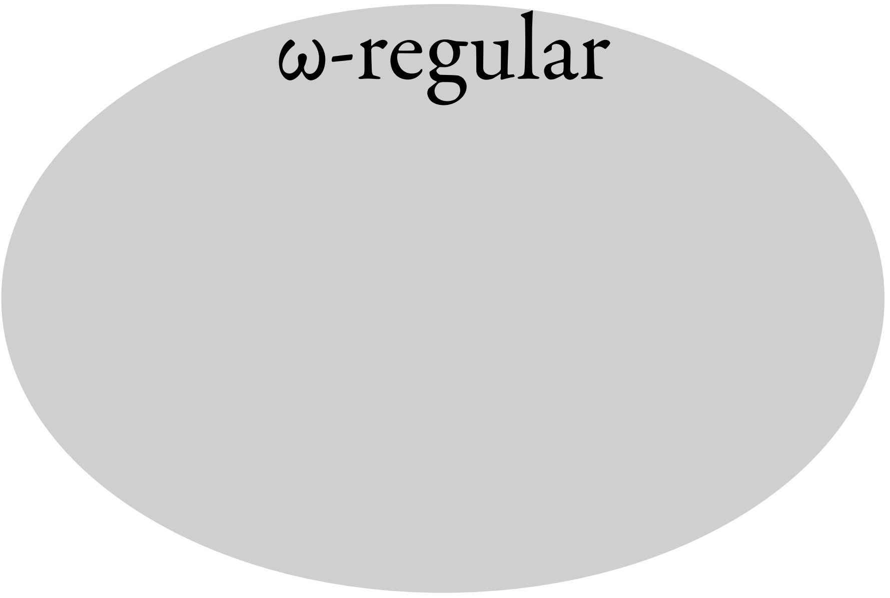
Prop. A language recognized by a max automaton is a boolean combination of Σ_2 sets, while L is not.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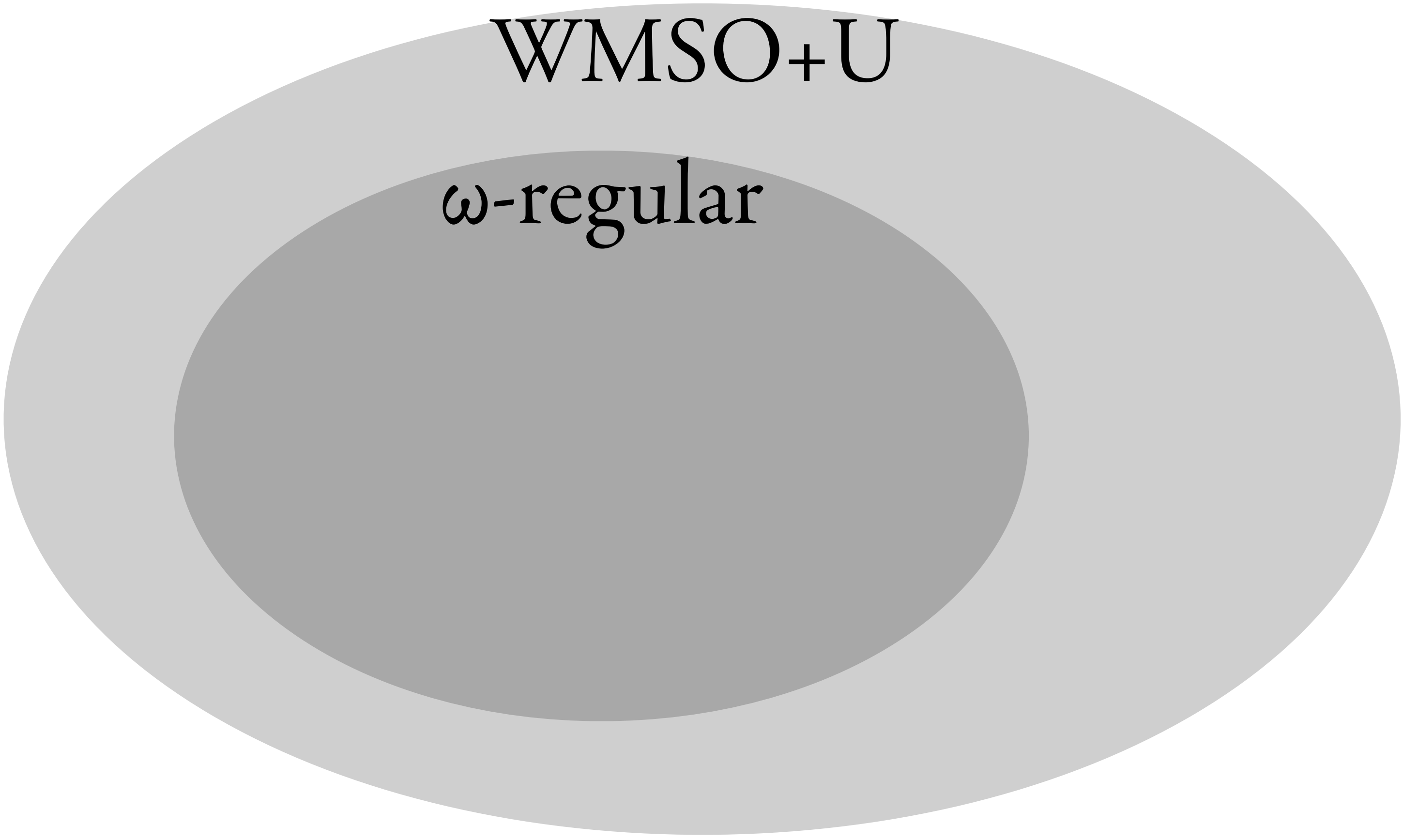
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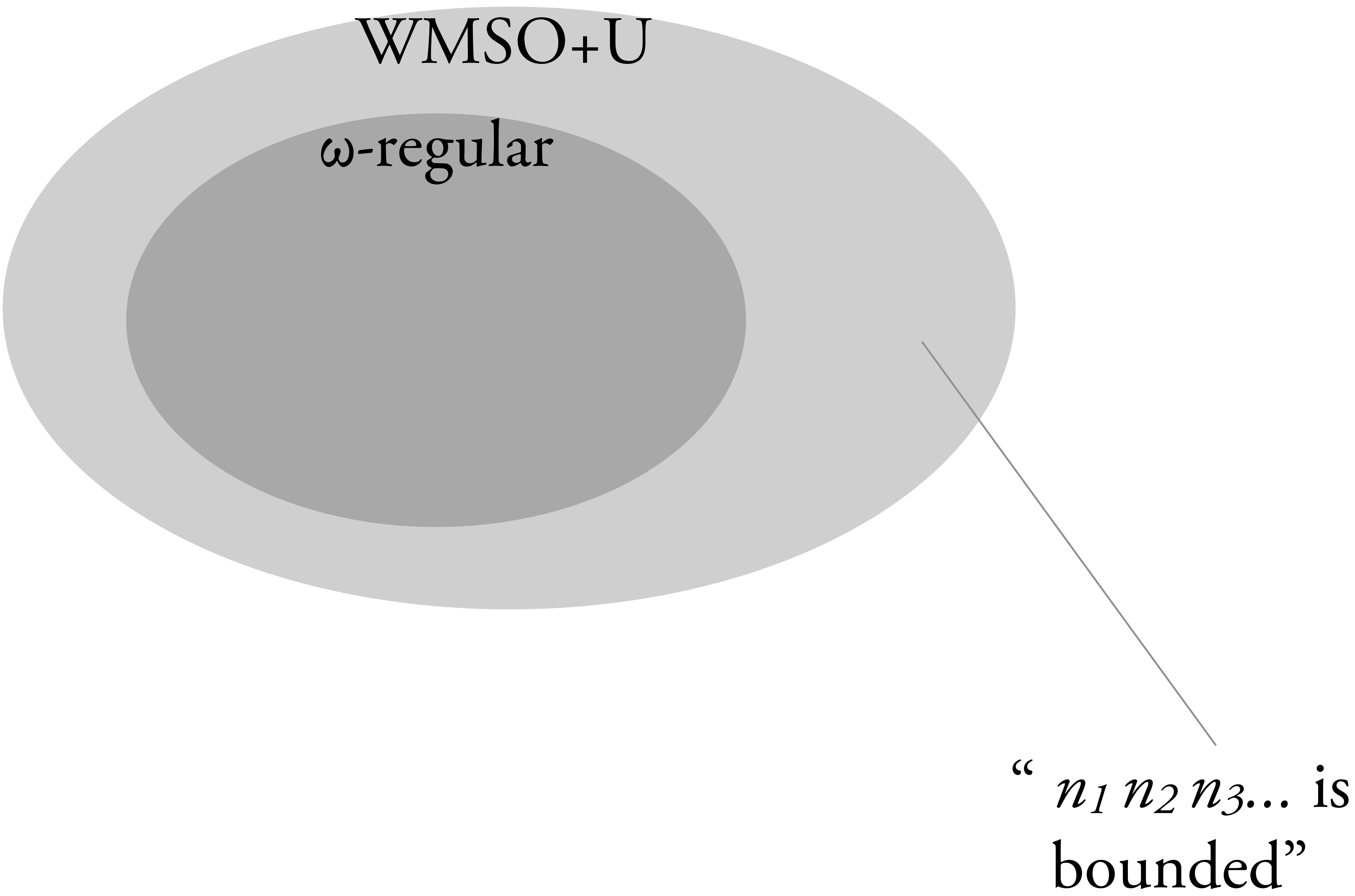
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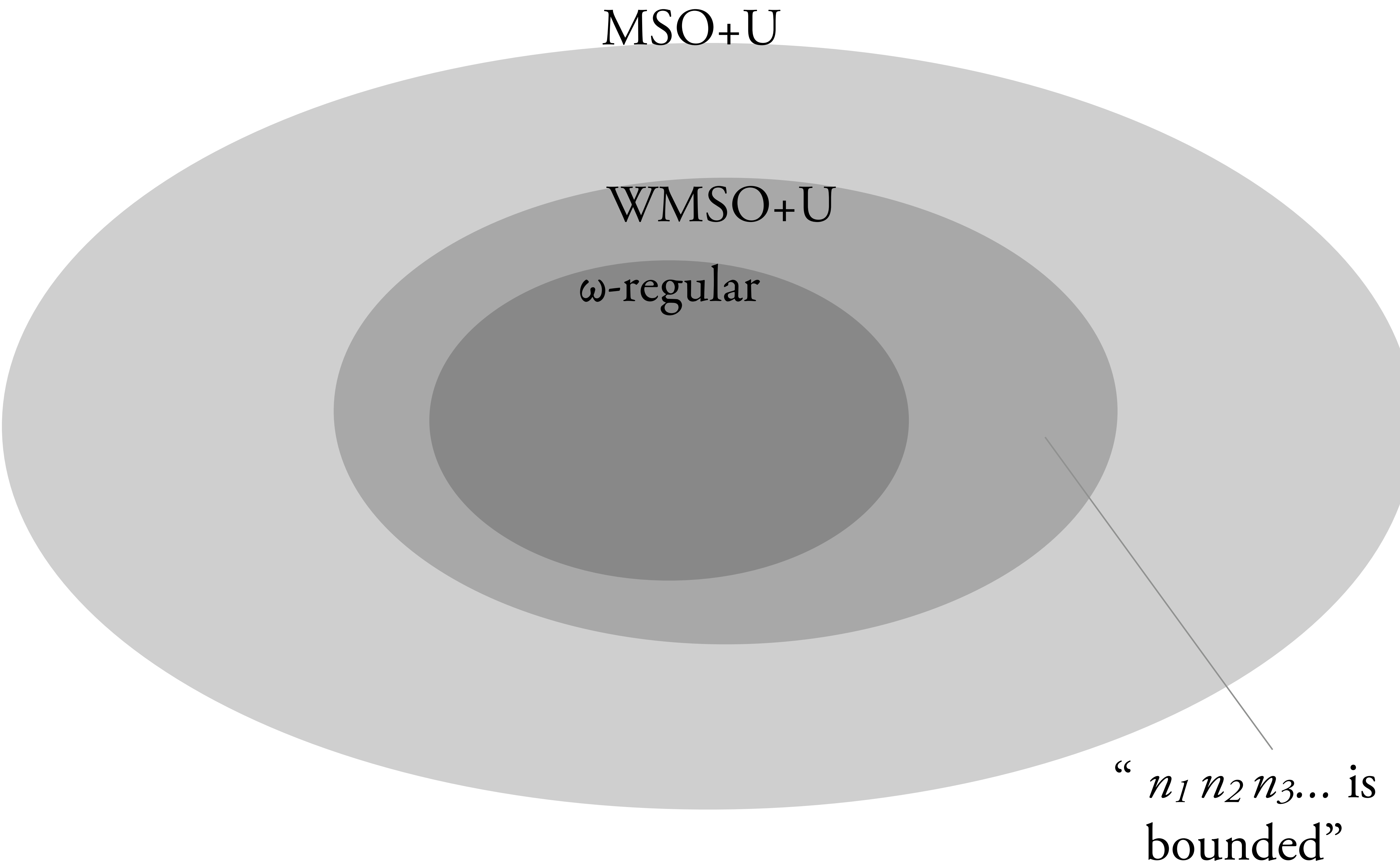
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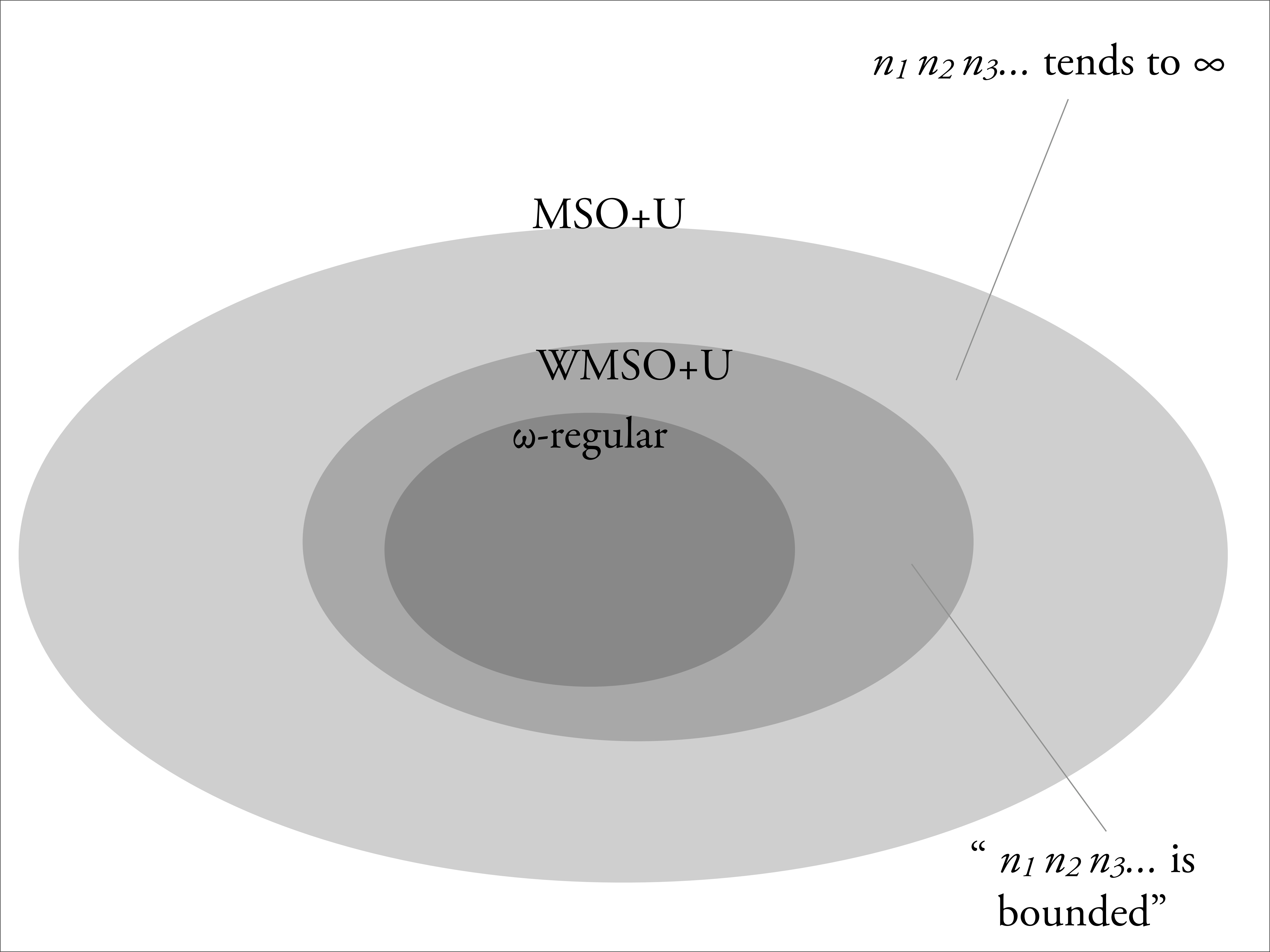

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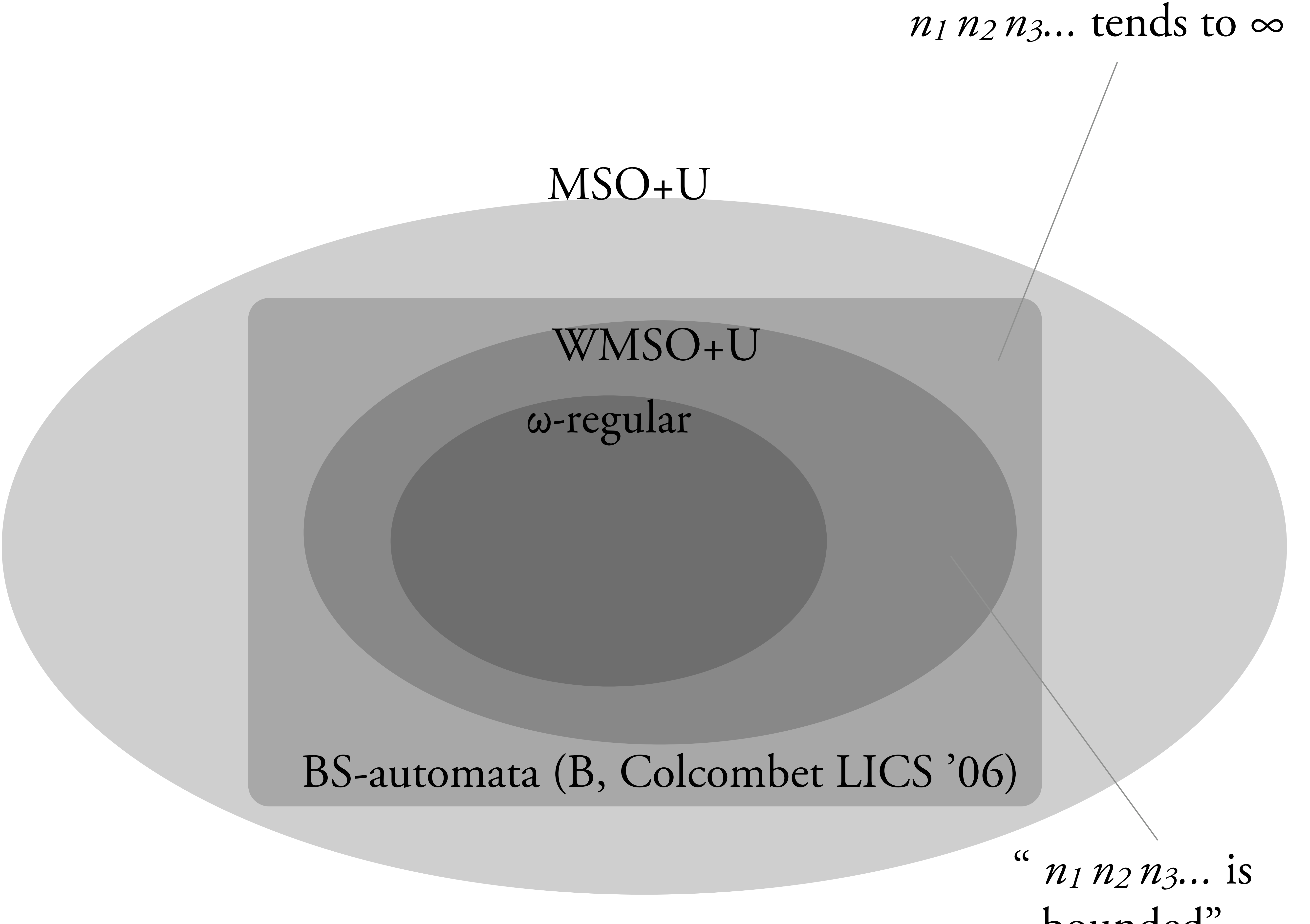

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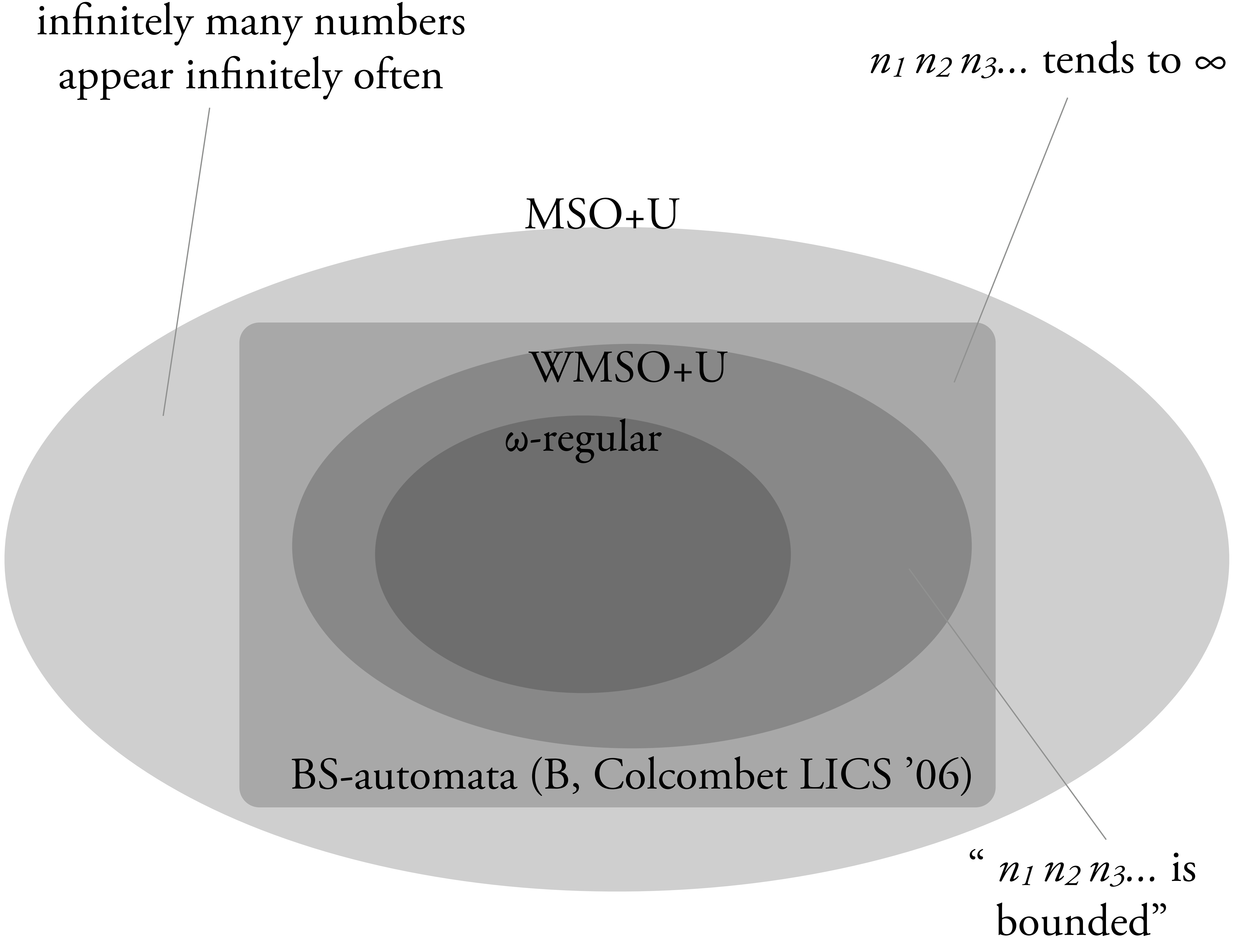
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a bit about the proofs

WMSO+U → deterministic max-automata

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The proof uses a combinatoric theorem of I. Simon.

Let A be an automaton with state space Q

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δ	δ	δ	δ	δ	δ	δ

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$$\delta \circ \delta = \delta$$

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“even number of *a*’s” has a decomposition of depth 5

two transition functions: even (0) and odd (1)

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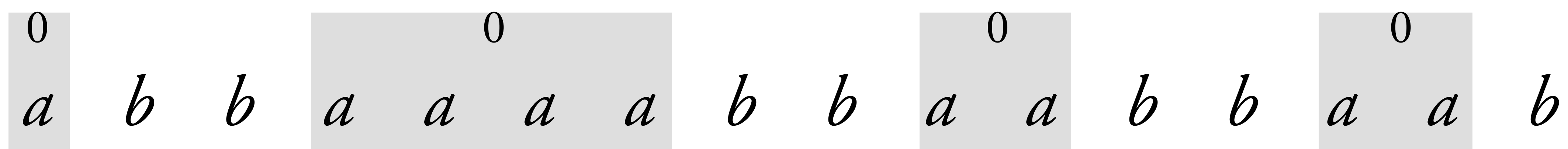

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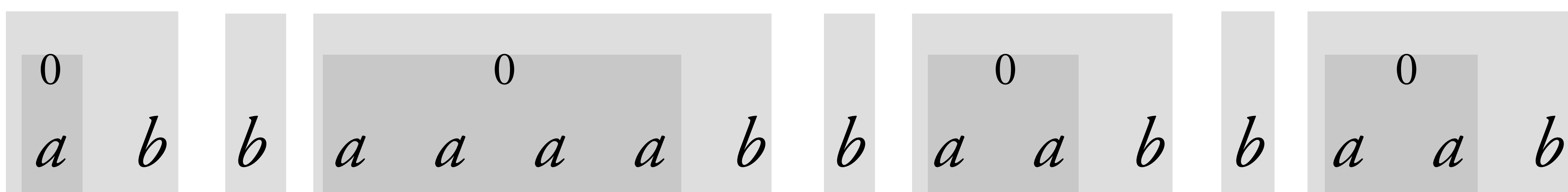

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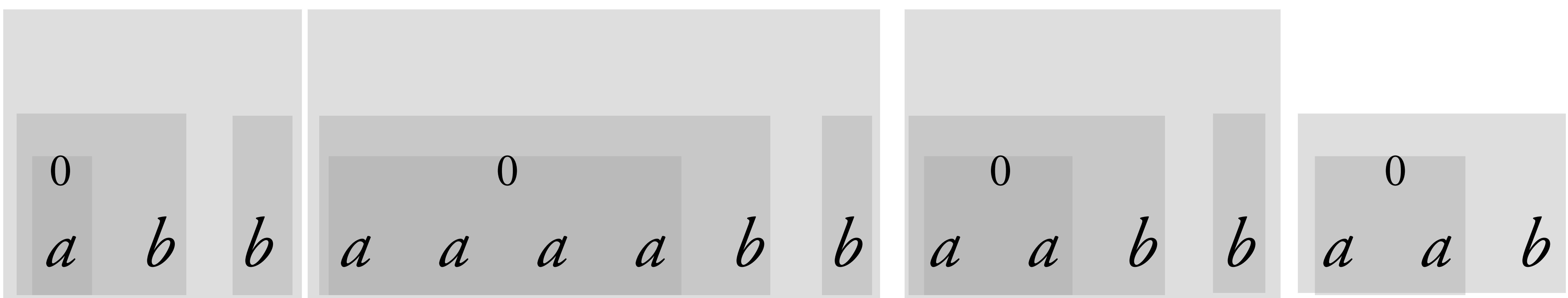

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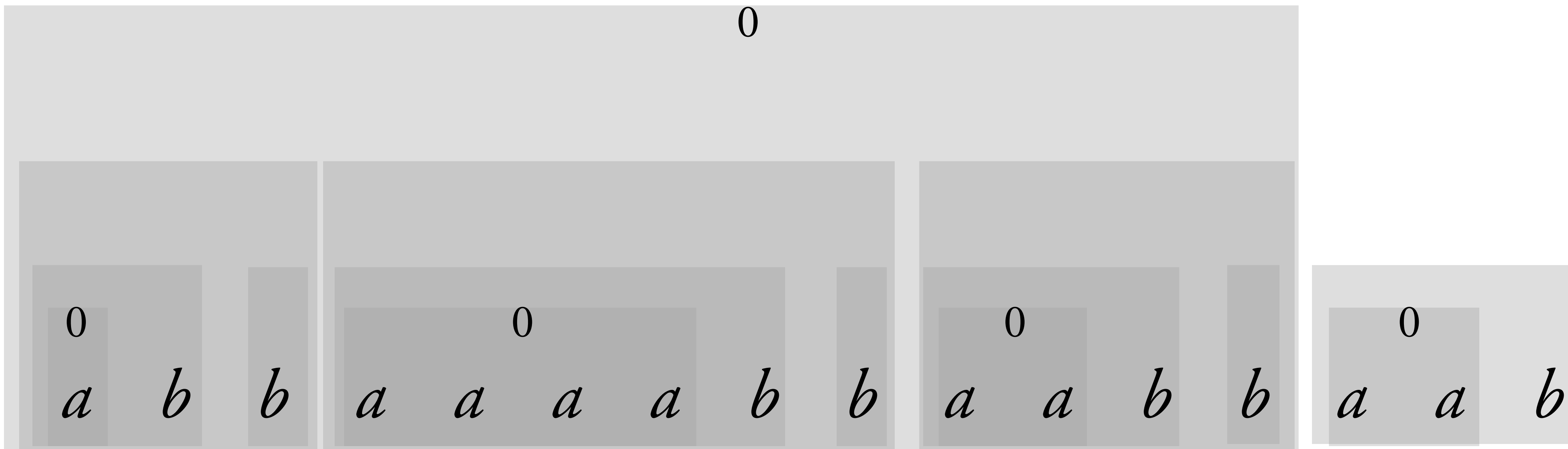

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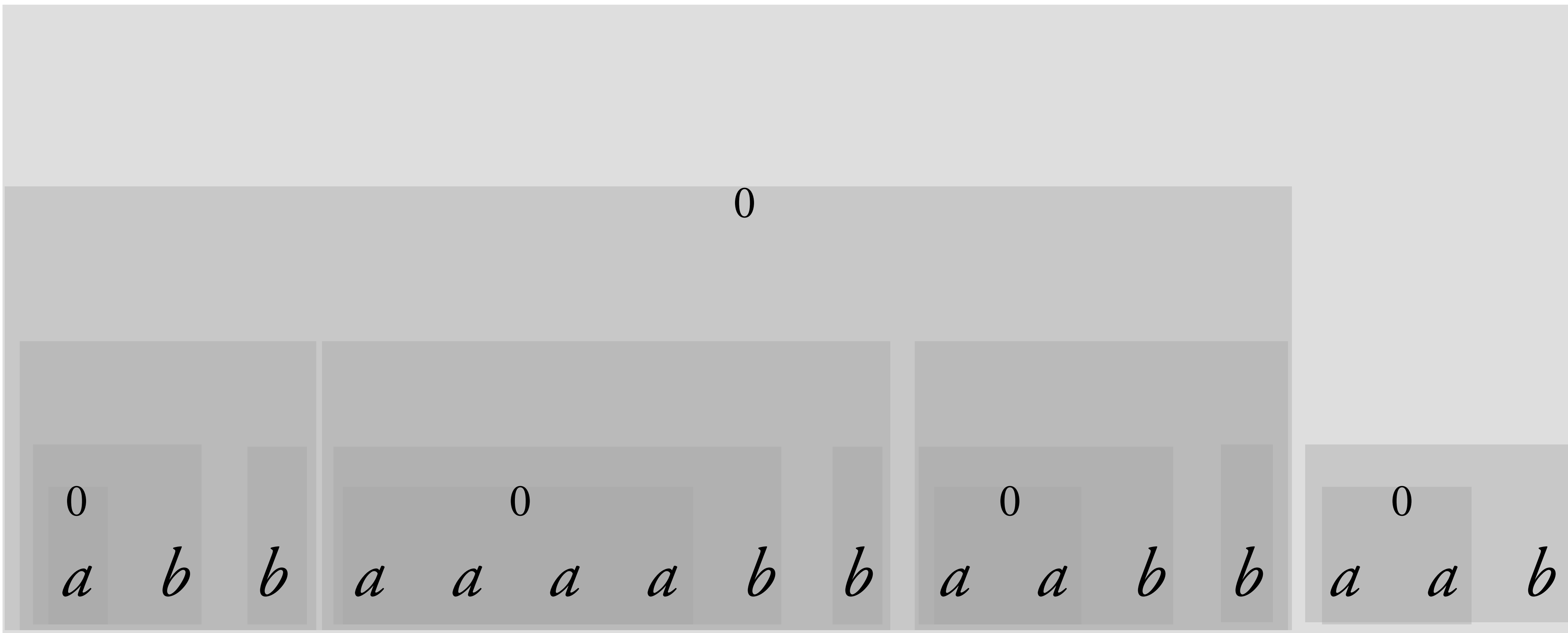

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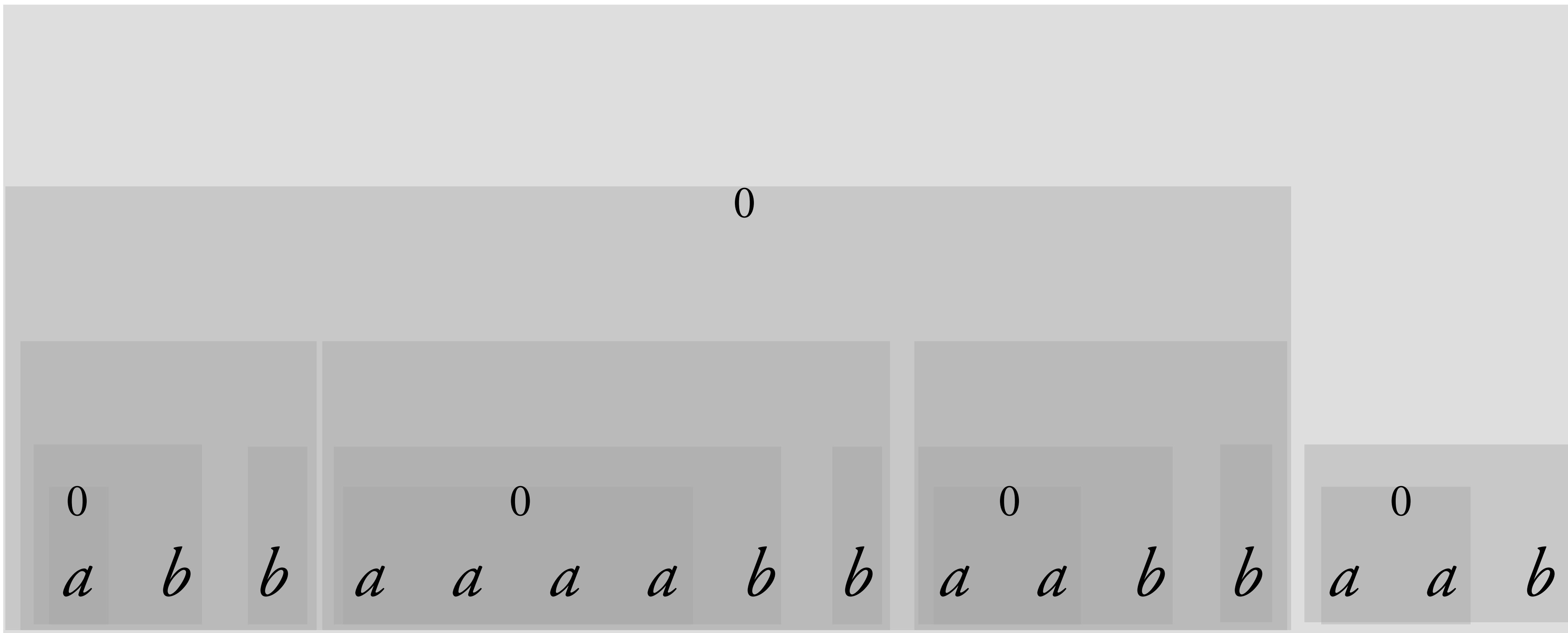

Thm. (Colcombet '07)
The decomposition can be output by a deterministic finite state transducer.

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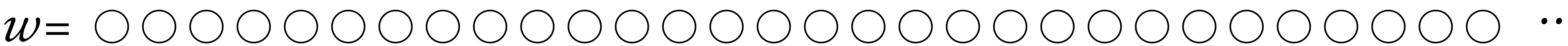

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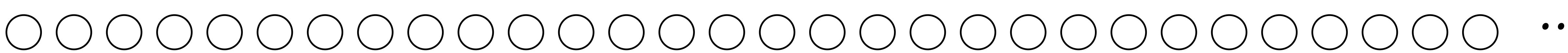
$w =$ 

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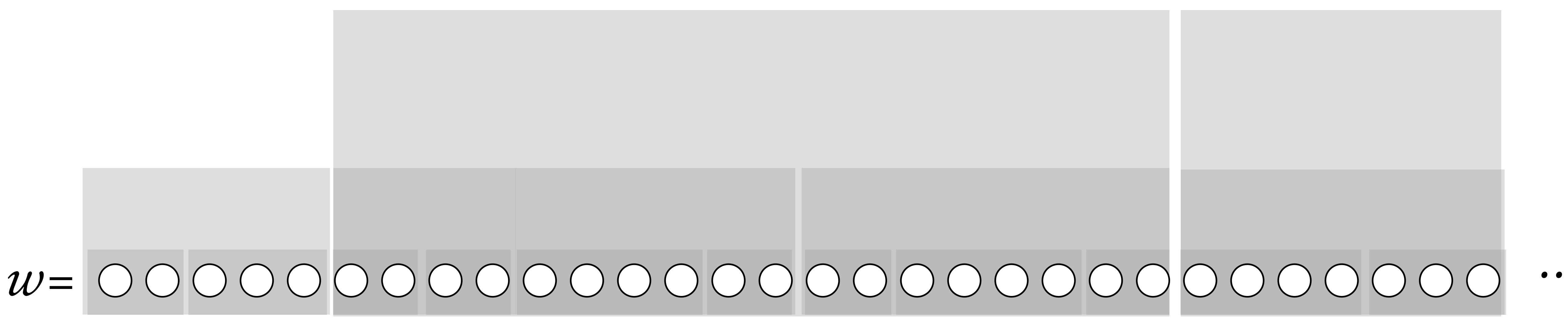

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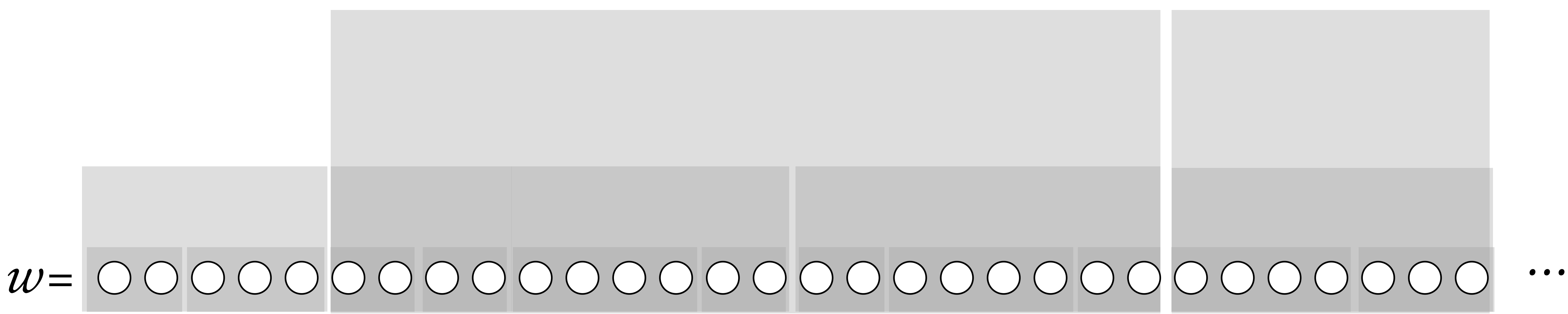

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