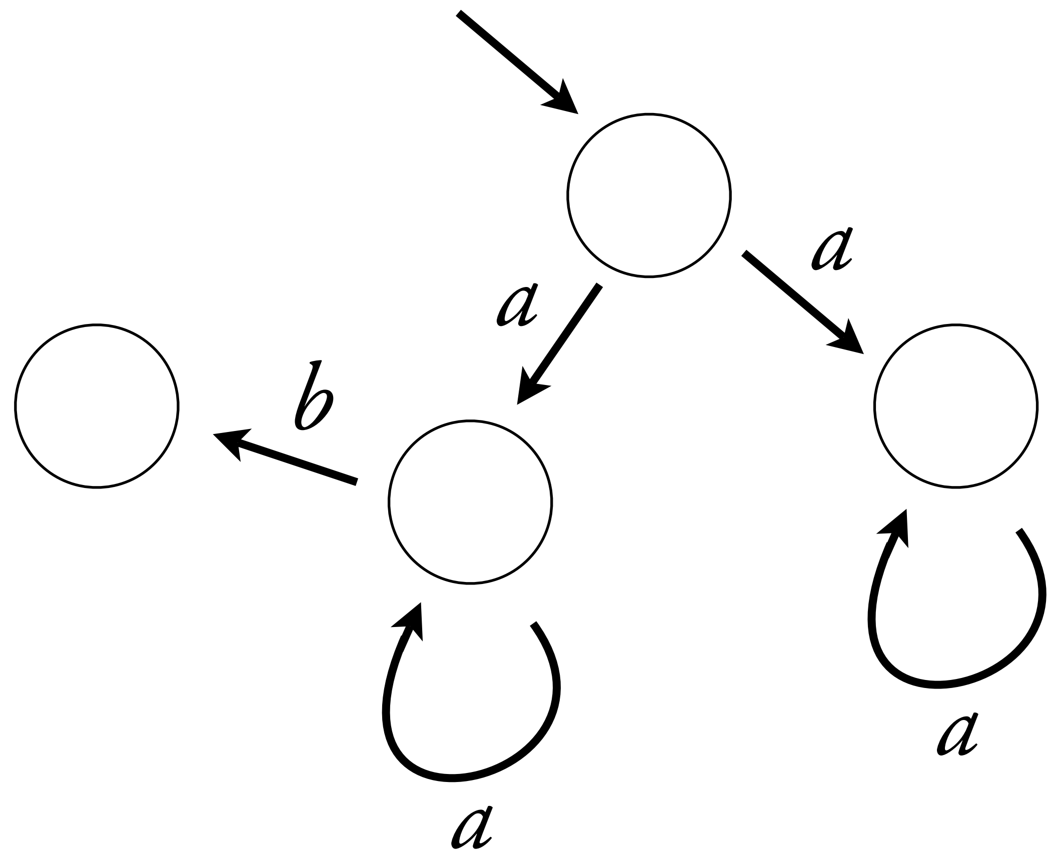
The common fragment of ACTL and CTL

Mikołaj Bojańczyk
Warsaw University

Logics that talk about transition systems



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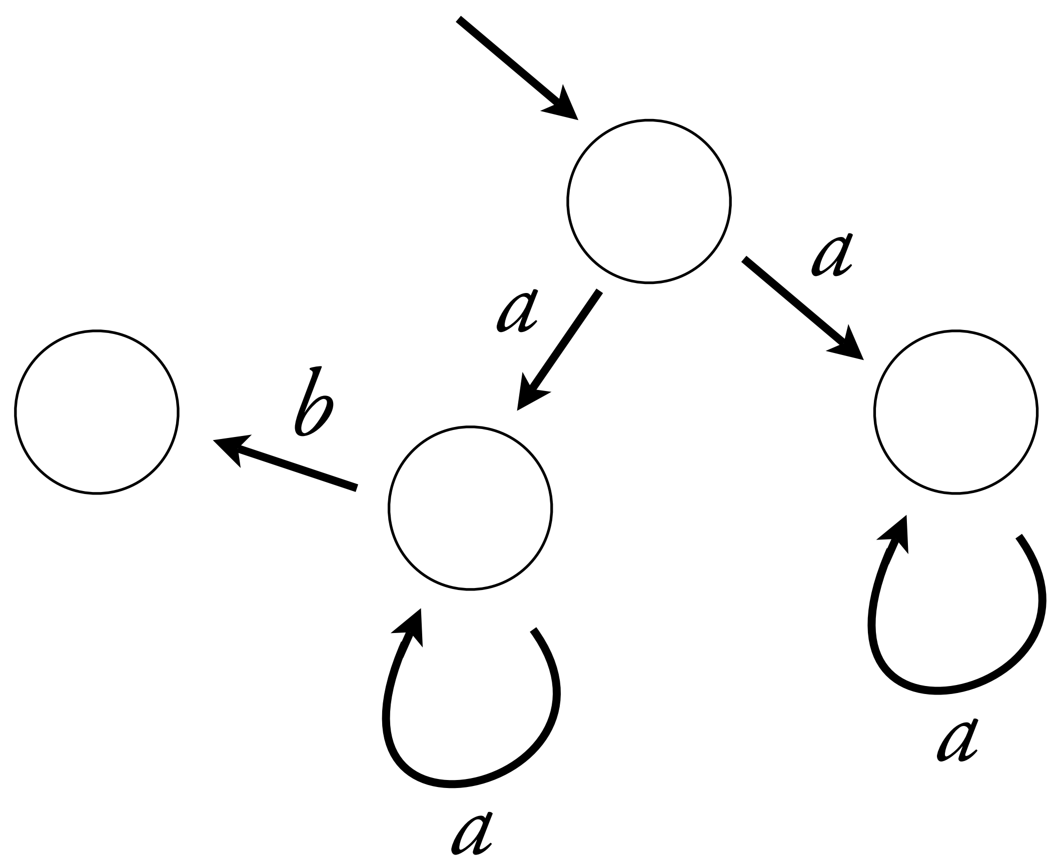

Linear time logic.

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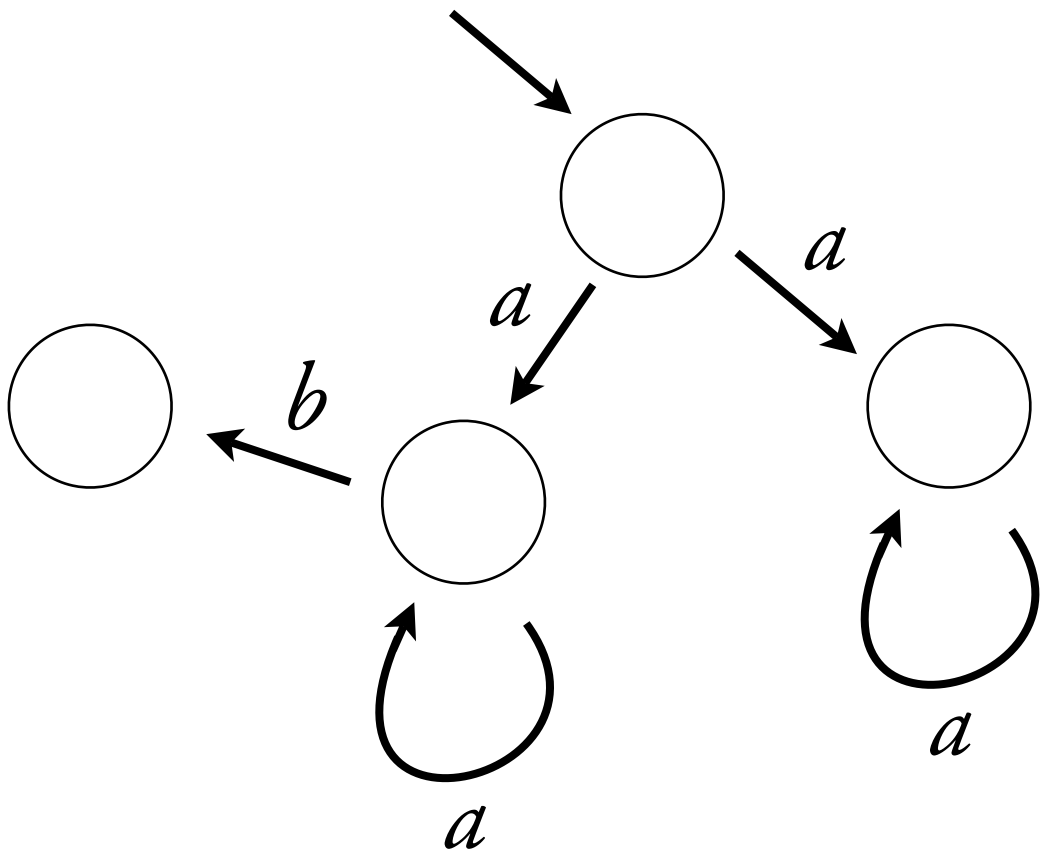

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after every a^* prefix, b is possible

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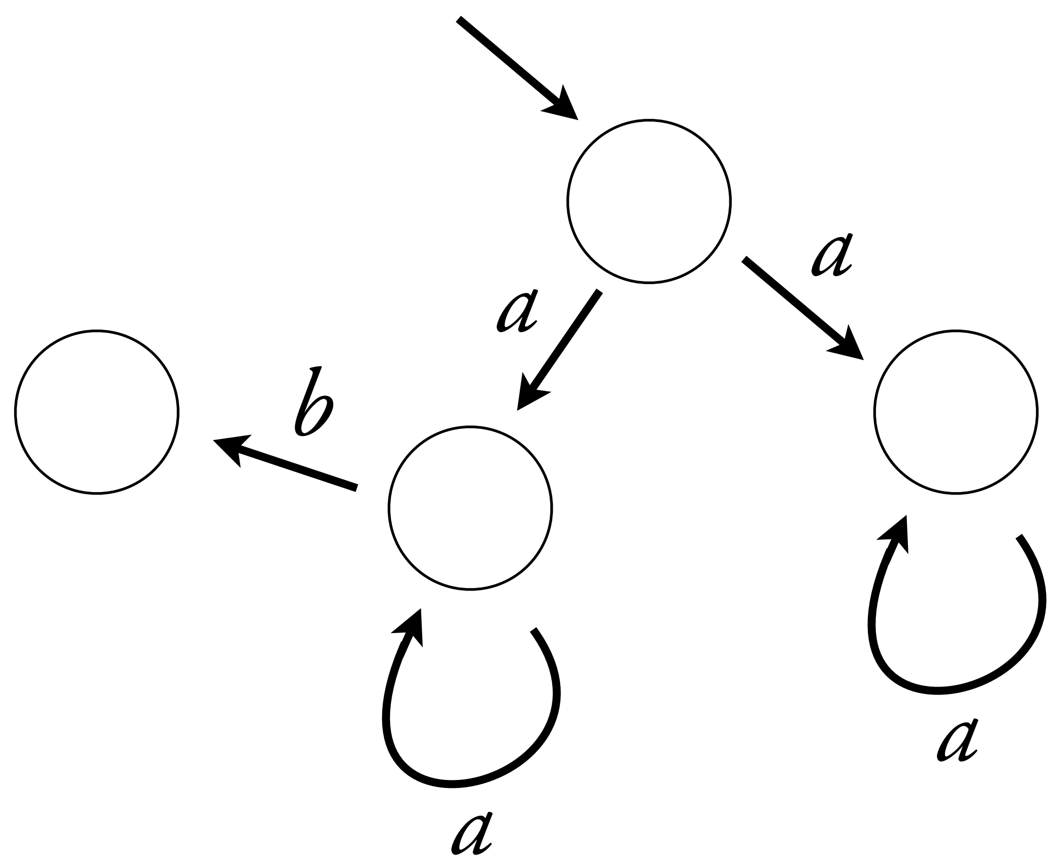

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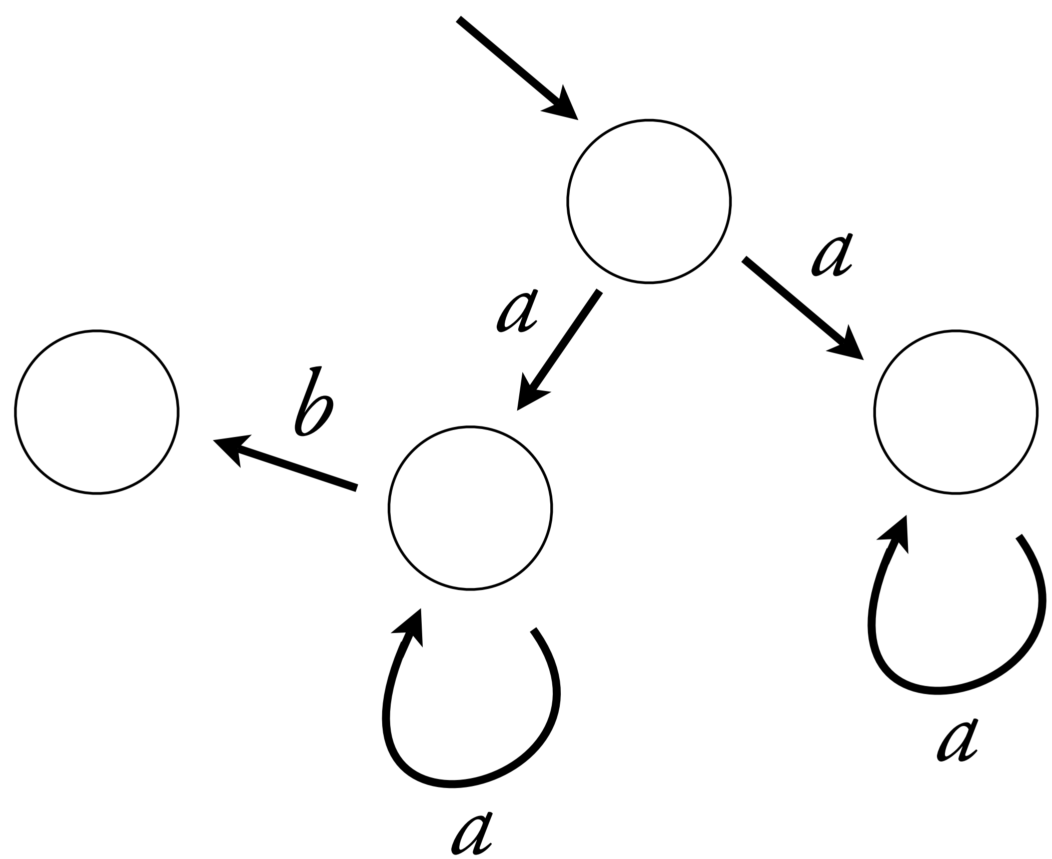

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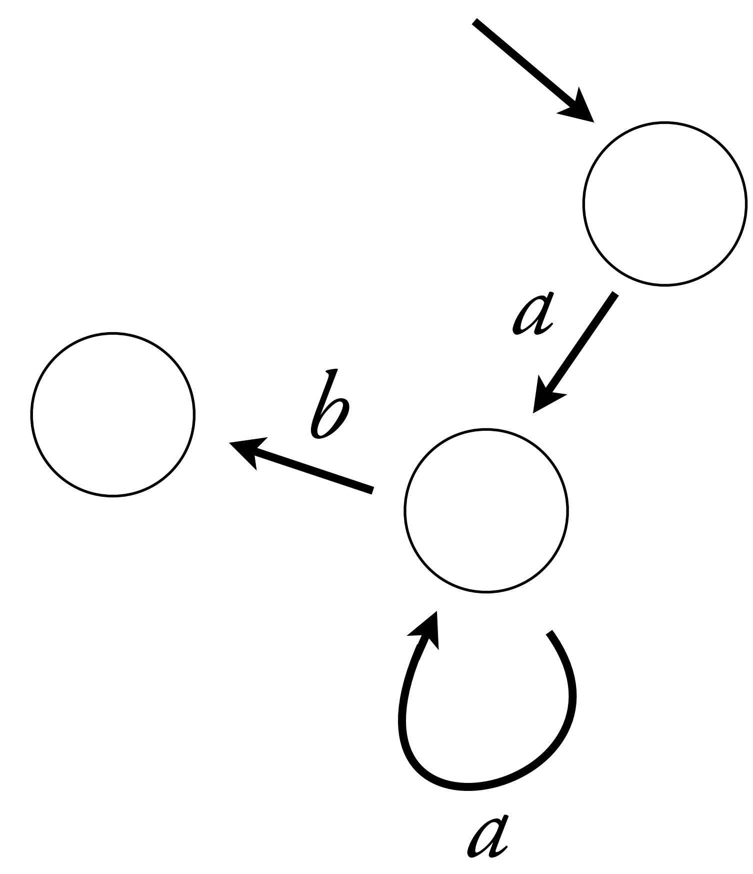

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The results generalize to transition systems.

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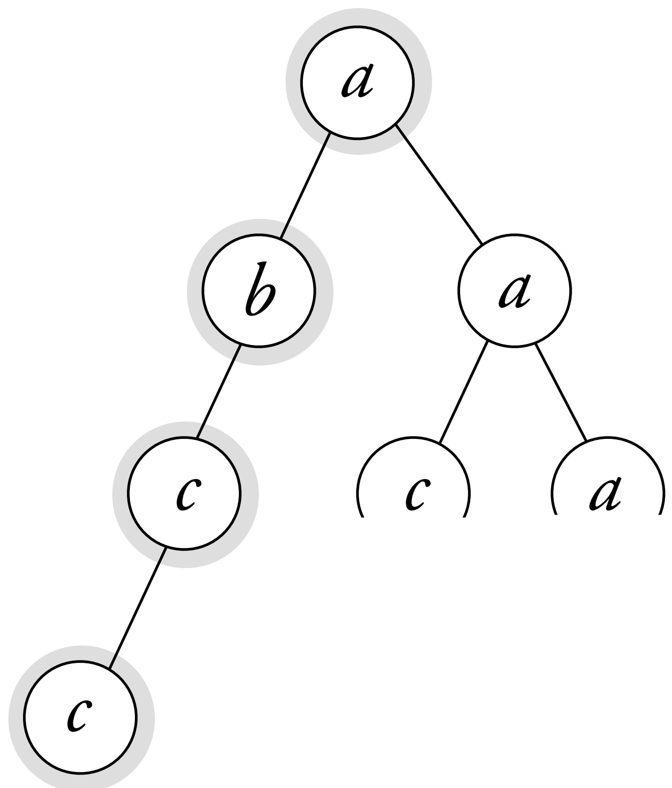

LTL

Languages of the form:
every path satisfies φ

φ a word property defined in LTL, e.g. $\mathsf{GF}\ a$

Here, instead of LTL, we use regular expressions for the word languages, e.g. $((b+c)^*a)^\omega$

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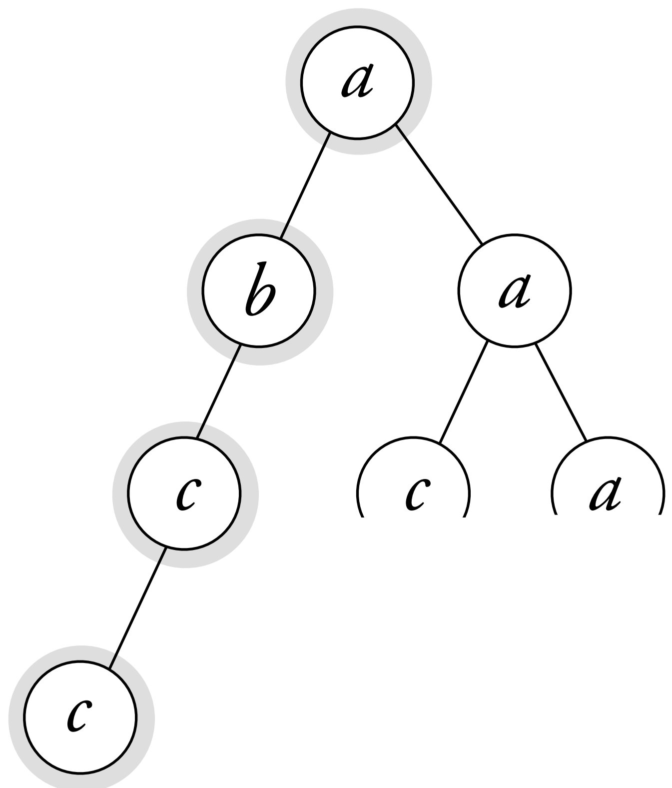

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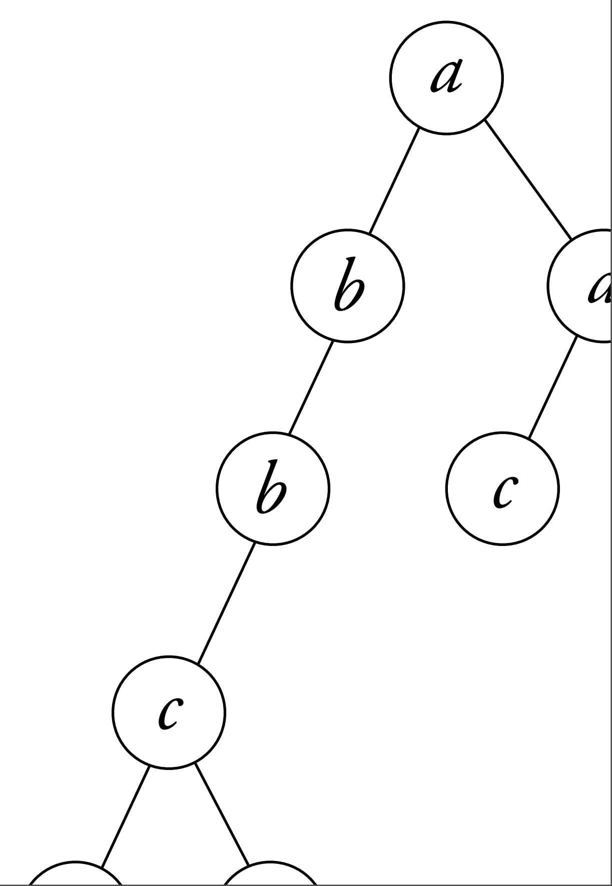

CTL

$\mathsf{A}\,\varphi\,\mathsf{U}\,\psi$: one every path φ holds until ψ holds.

$\mathsf{AG}\,\varphi$: on every path, φ holds globally.

$\mathsf{AX}\,\varphi$: on every path, φ holds in the next position.

label tests and boolean operations.

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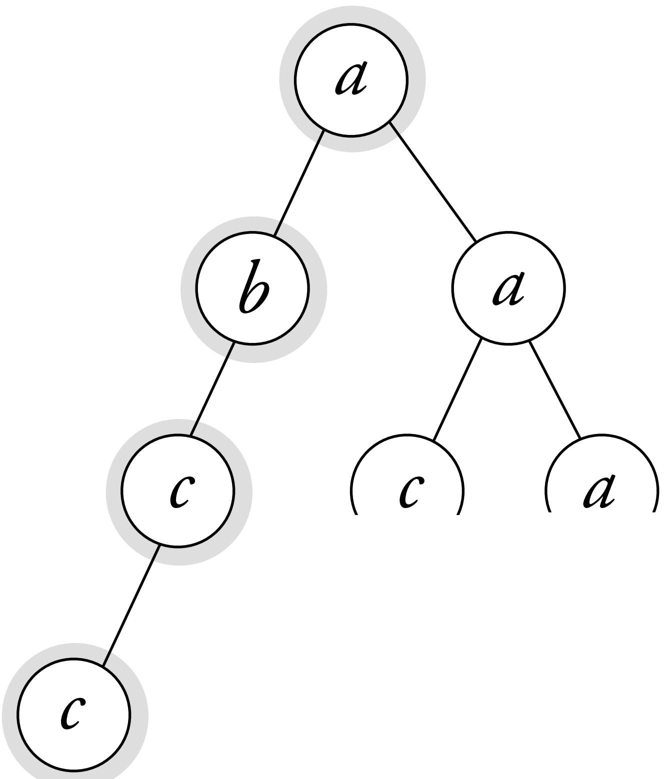

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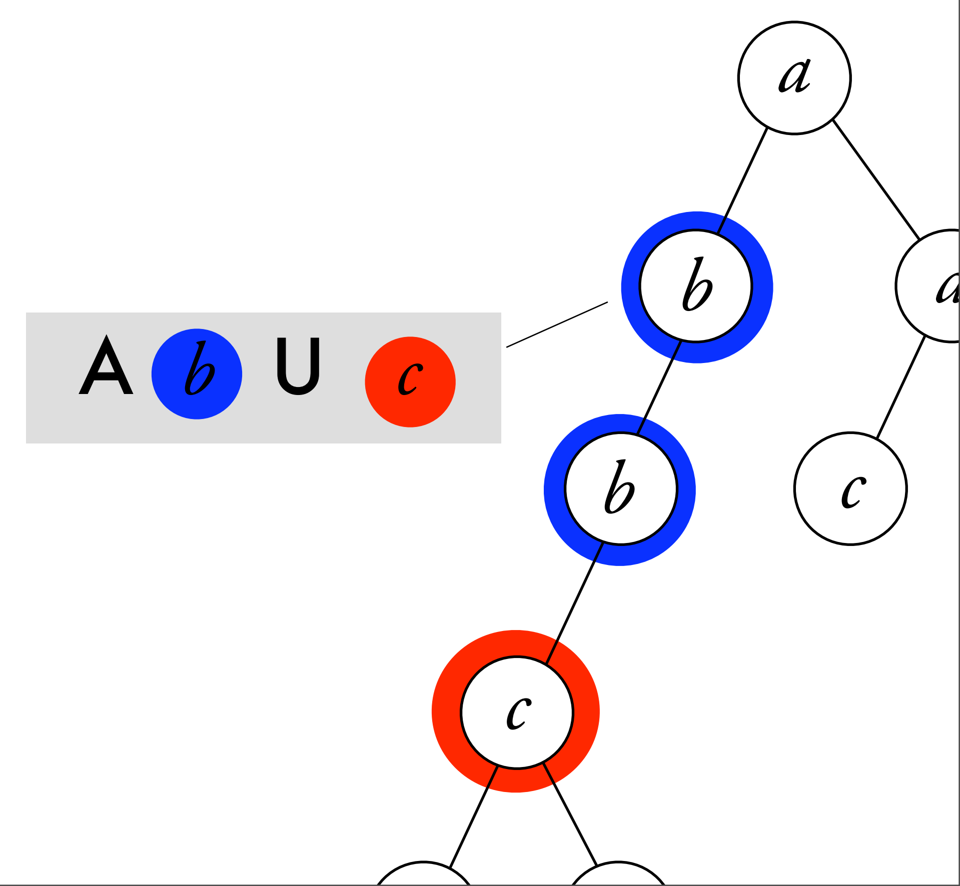

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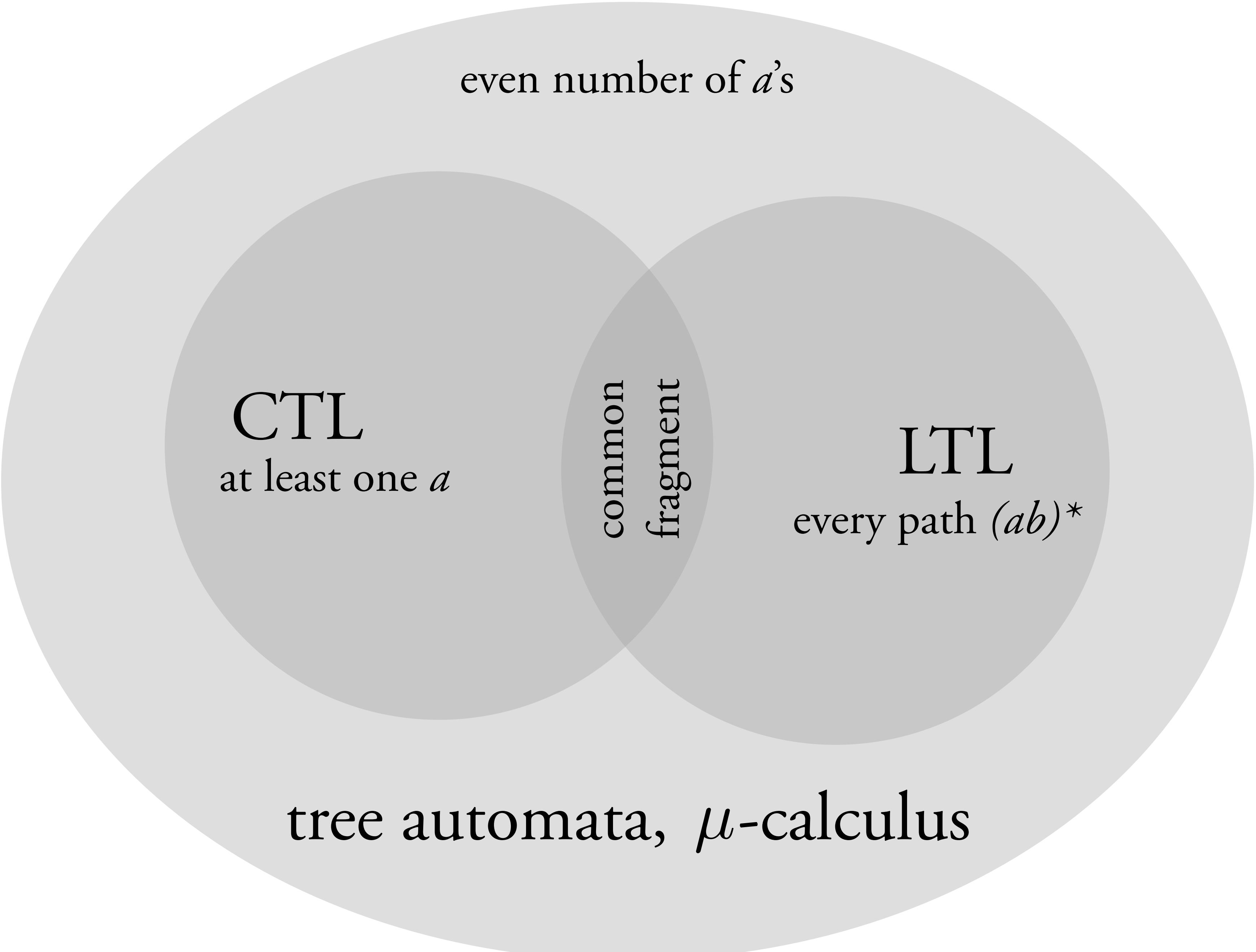
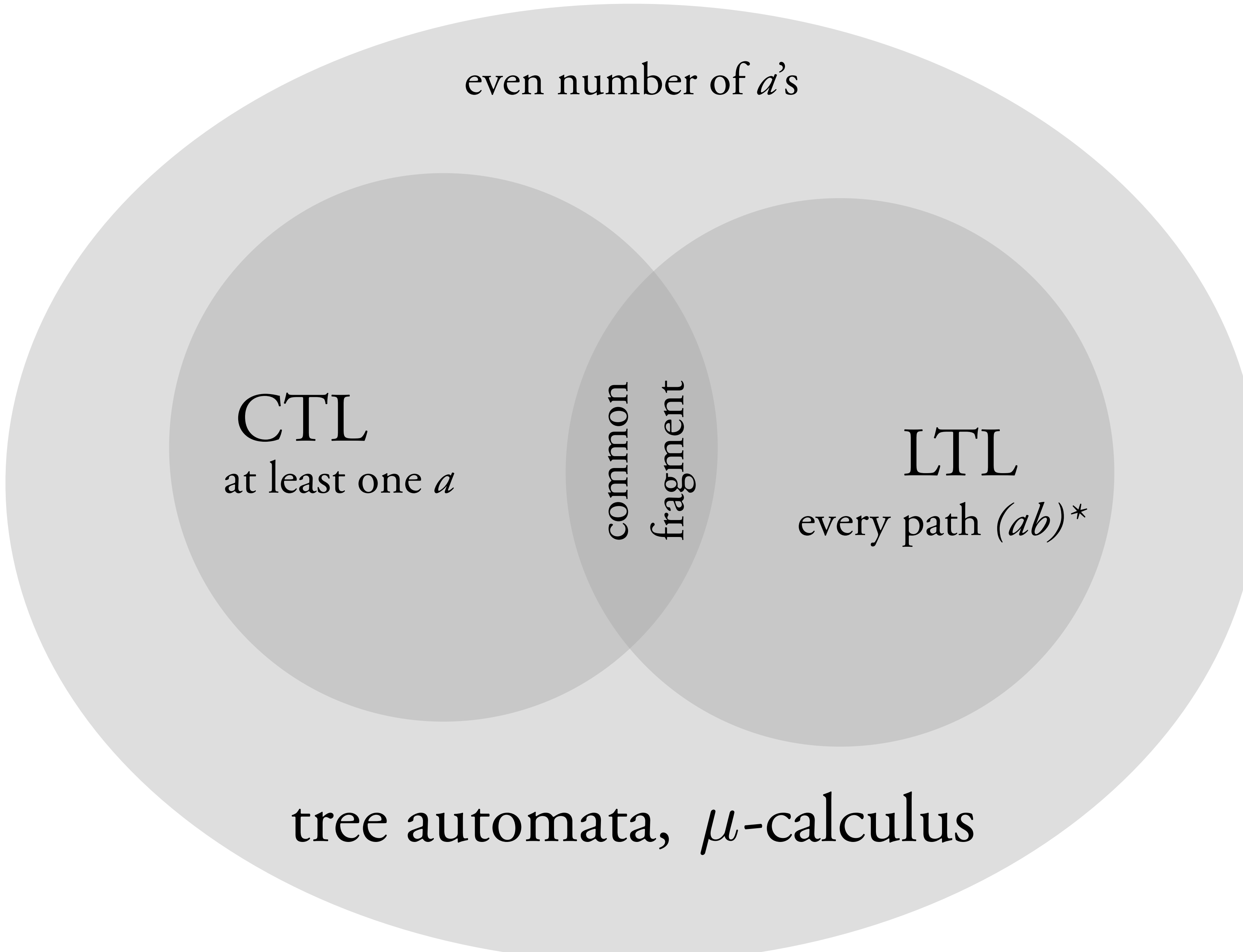
What is the common fragment?



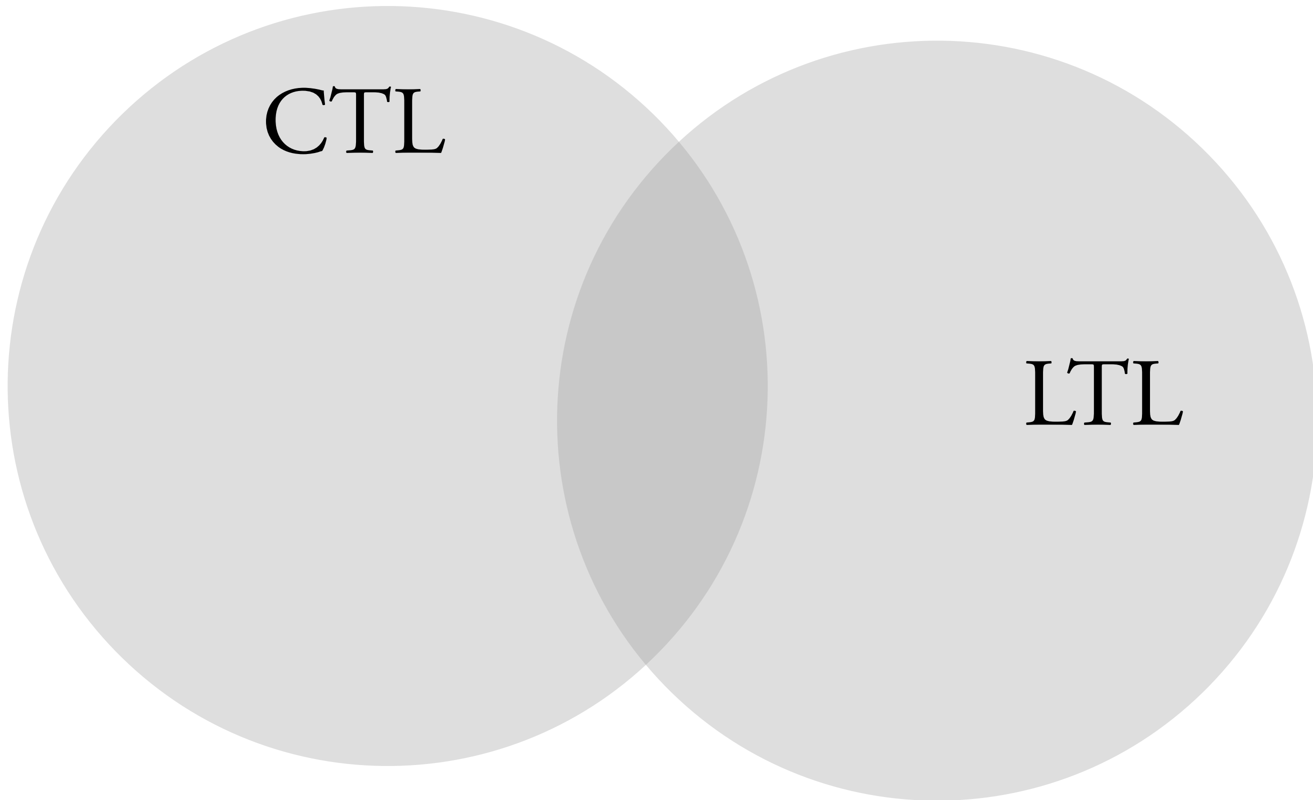
What is the common fragment?

Input:
CTL formula.
Question:
Is it in LTL?



Input:
LTL formula.
Question:
Is it in CTL?

Input:
Tree automaton.
Question:
Is it in the common fragment?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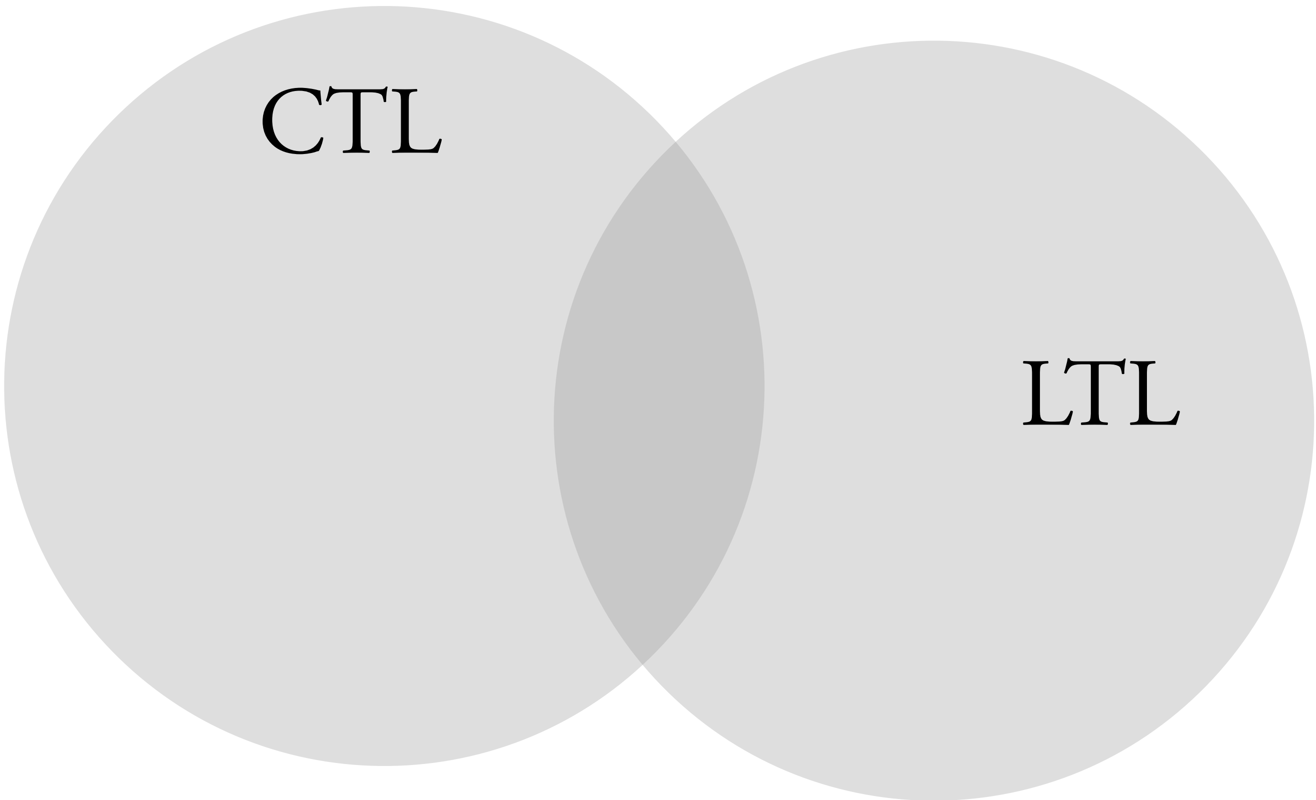
Thm.
It is decidable if a given regular language is in LTL.

Thm. [Maidl 00]
Complexity is PSPACE-complete if input is given in CTL.



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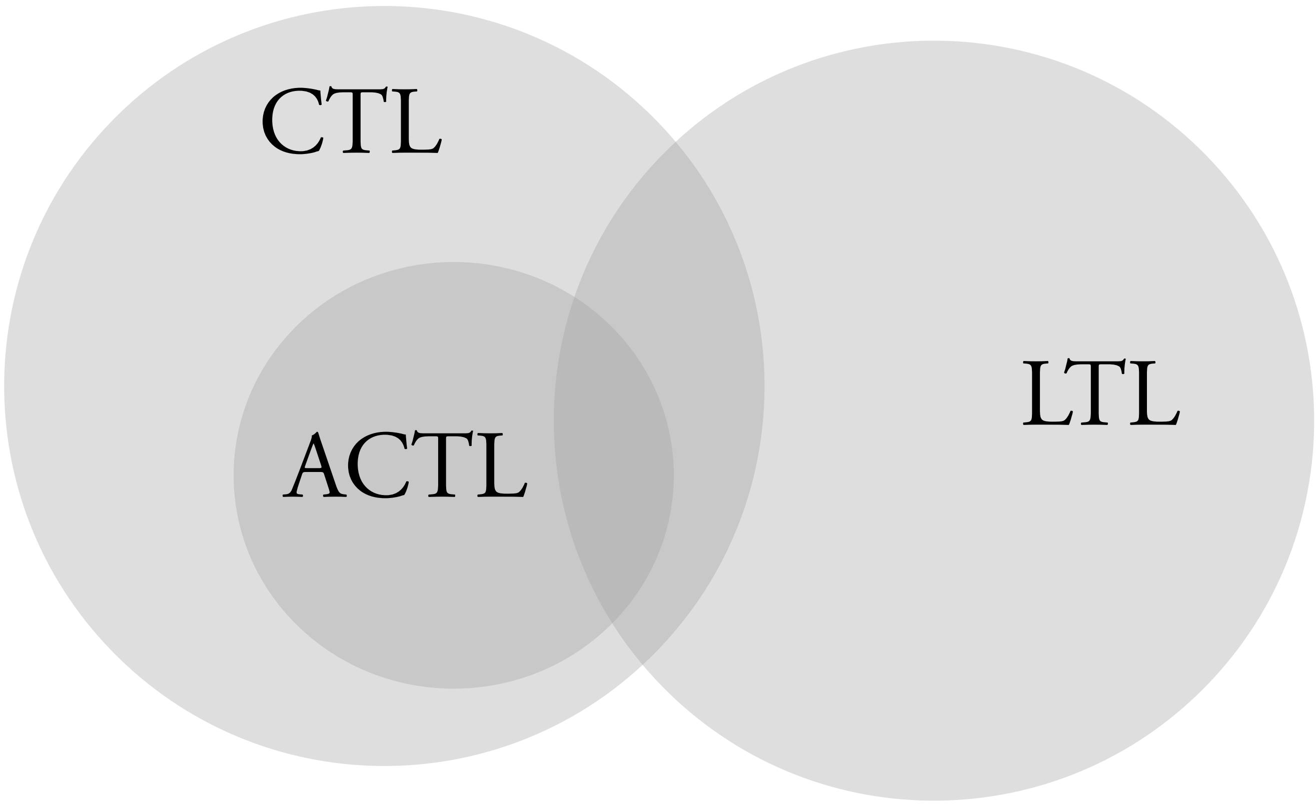

ACTL
CTL without negation,
only “on all paths...”

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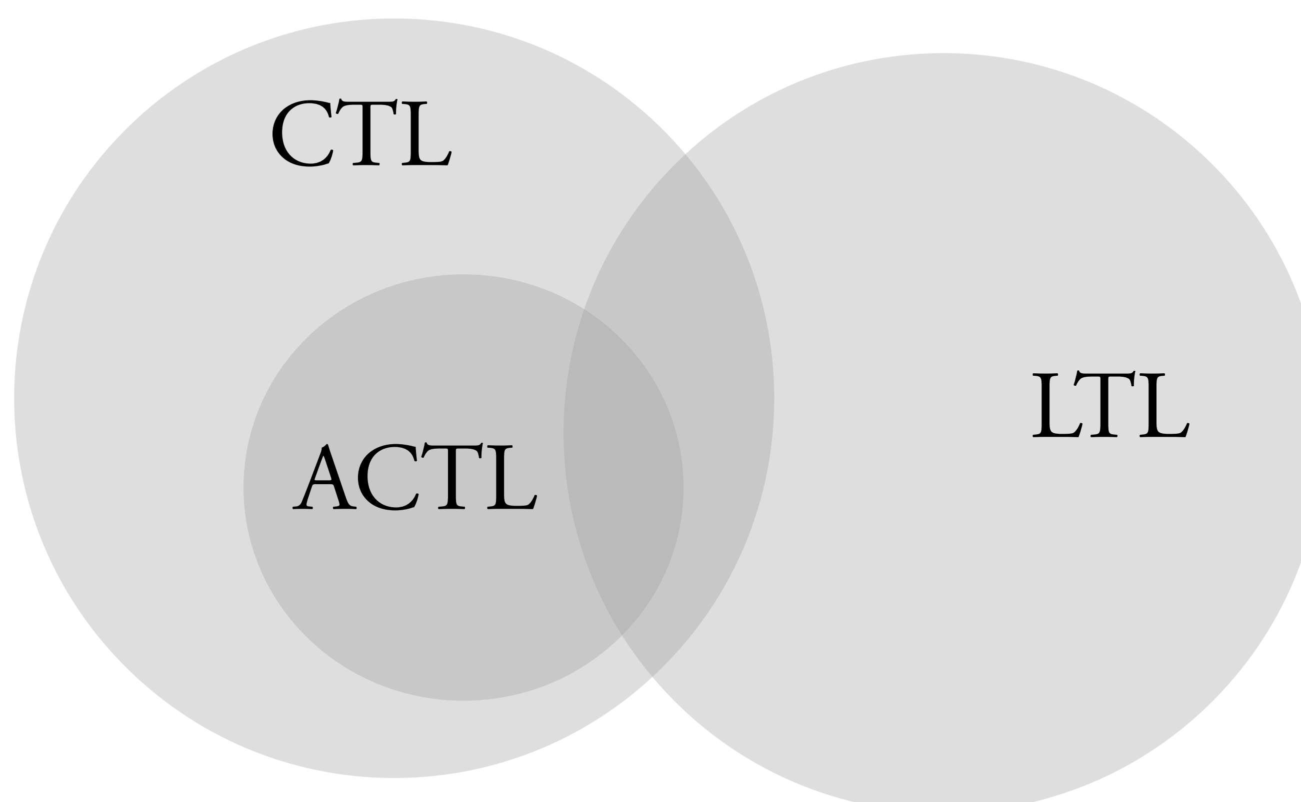
Main contribution.

$\mathrm{ACTL} \cap \mathrm{LTL} \quad \subsetneq \quad \mathrm{CTL} \cap \mathrm{LTL}$

decidable membership



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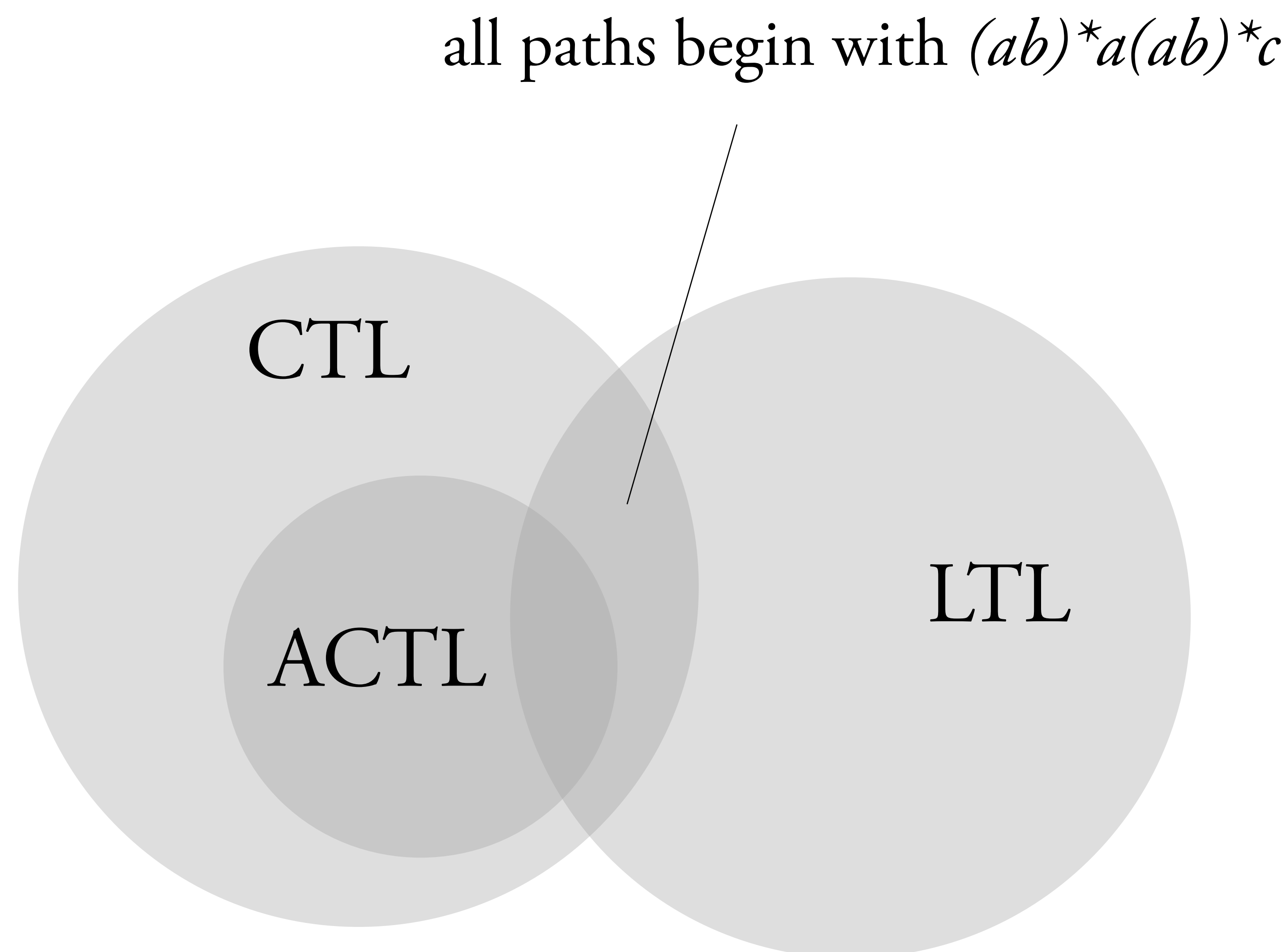

ACTL

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Thm. [Maidl 00] Let L be a set of infinite words.

The tree language "all paths in L" is definable in ACTL

iff

L is definable in $\Pi_2(<)$

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$$\forall x_1 \cdots \forall x_n \quad \exists y_1 \cdots \exists y_m \quad \overbrace{\varphi(x_1, \ldots, x_n, y_1, \ldots, y_m)}^{\text{quantifier-free, using labels and } <}$$

On every path, every *a* is followed by some *b*.

$$\forall x \exists y \quad a(x) \Rightarrow \big(x < y \ \wedge \ b(y)\big)$$

$$\mathsf{AG}\big(\mathsf{a} \Rightarrow \mathsf{A}\ \mathsf{a}\ \mathsf{U}\ \mathsf{b}\big)$$

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Contribution 1. Not the same thing as CTL $\cap$ LTL.

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Contribution 2. Effective criterion.

$\in$ CTL

ACTL $\not\ni$ all paths begin with *(ab)*a(ab)*c*

$\in$ LTL

words that begin with *(ab)*a(ab)*c* $\notin \Pi_2(<)$

Obvious

$\in$ CTL

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Obvious

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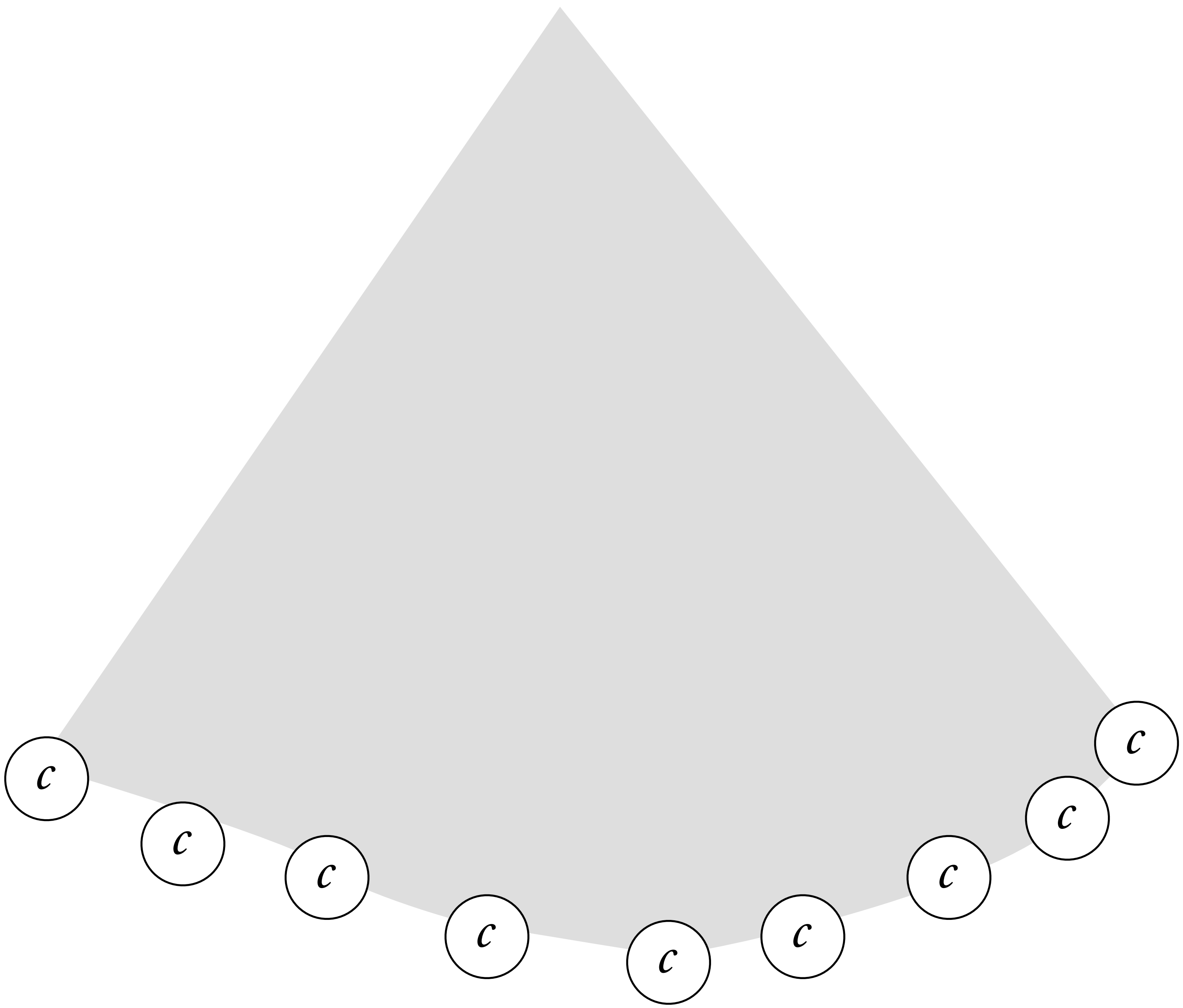

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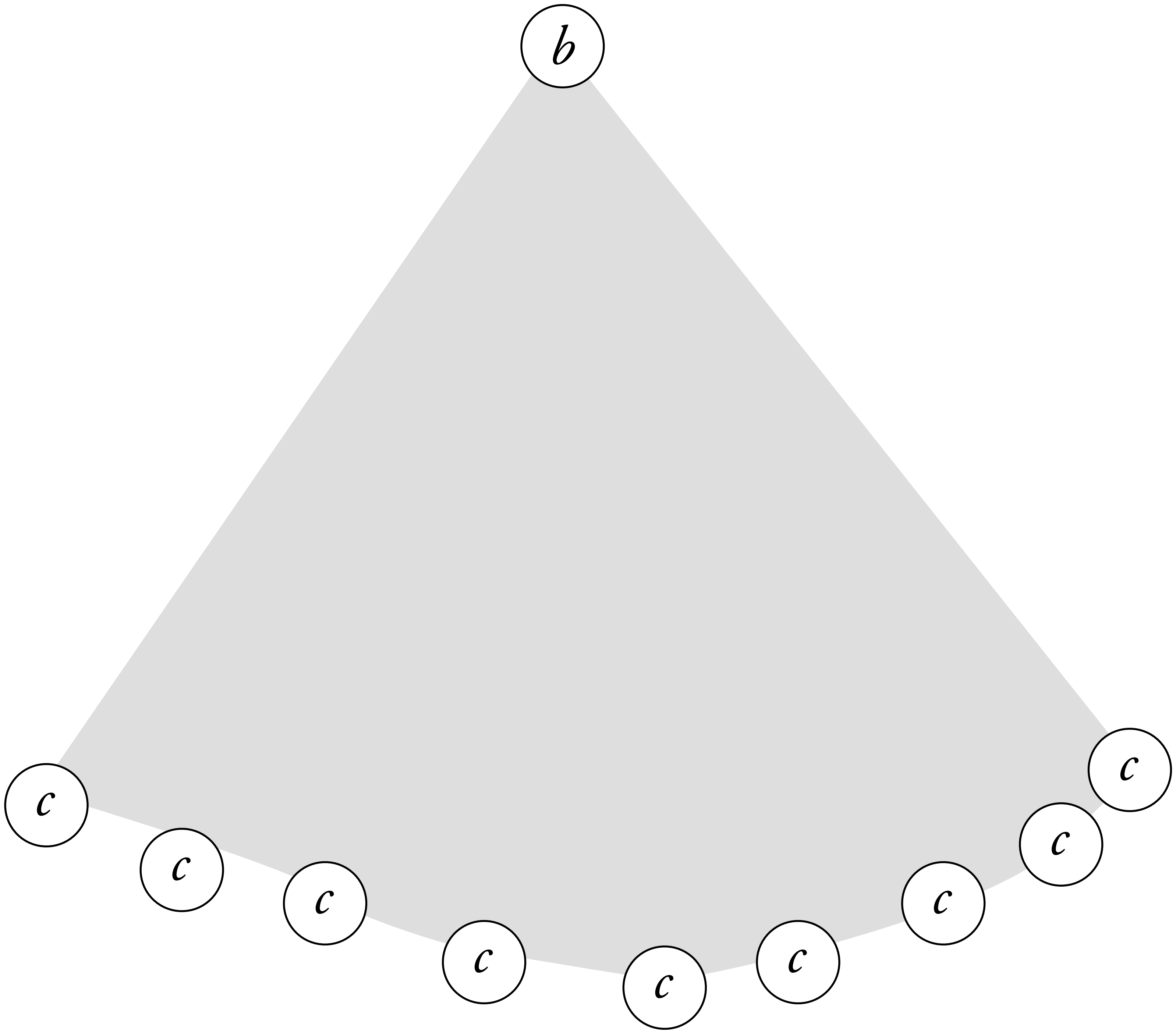

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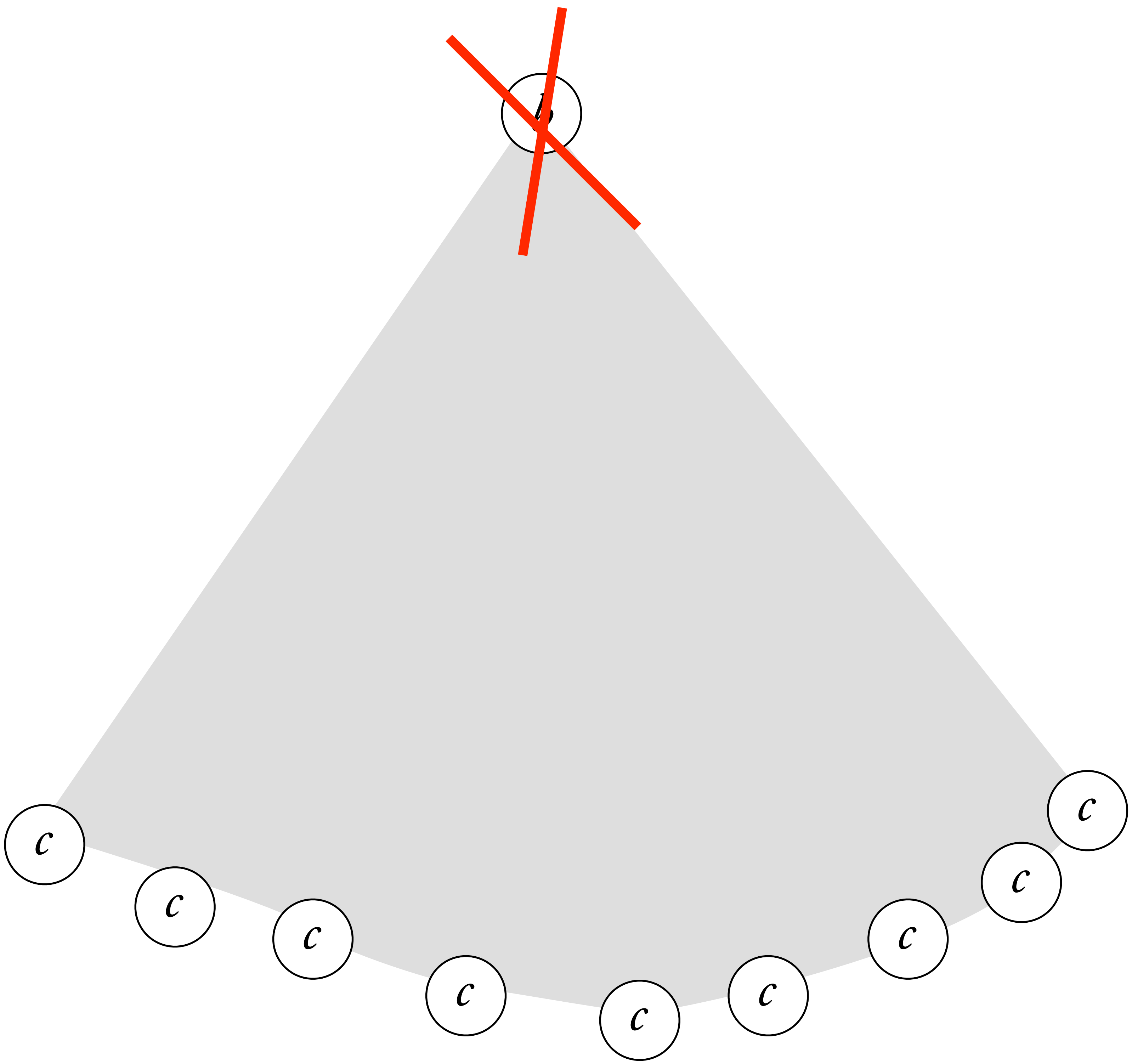

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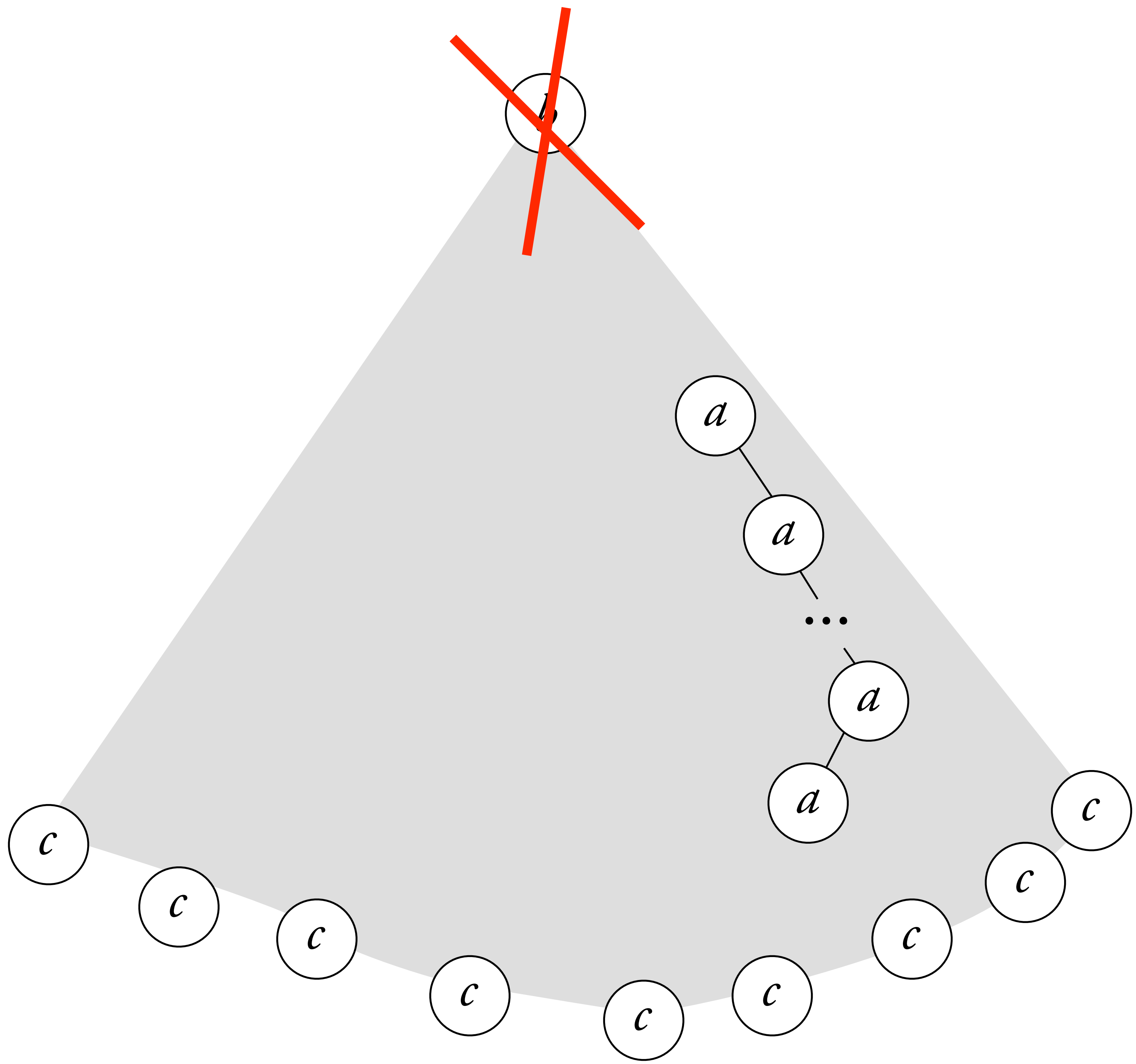

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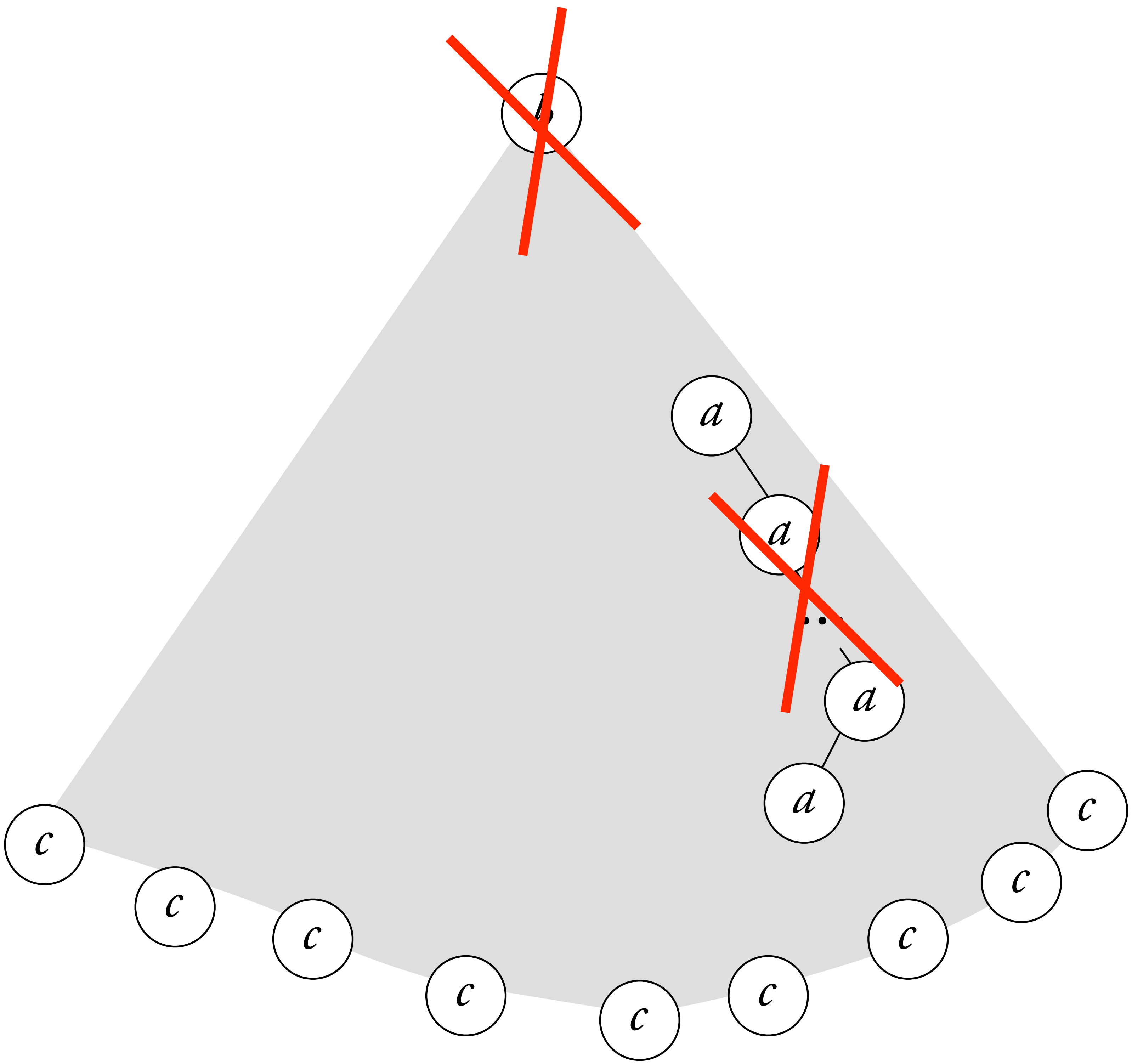

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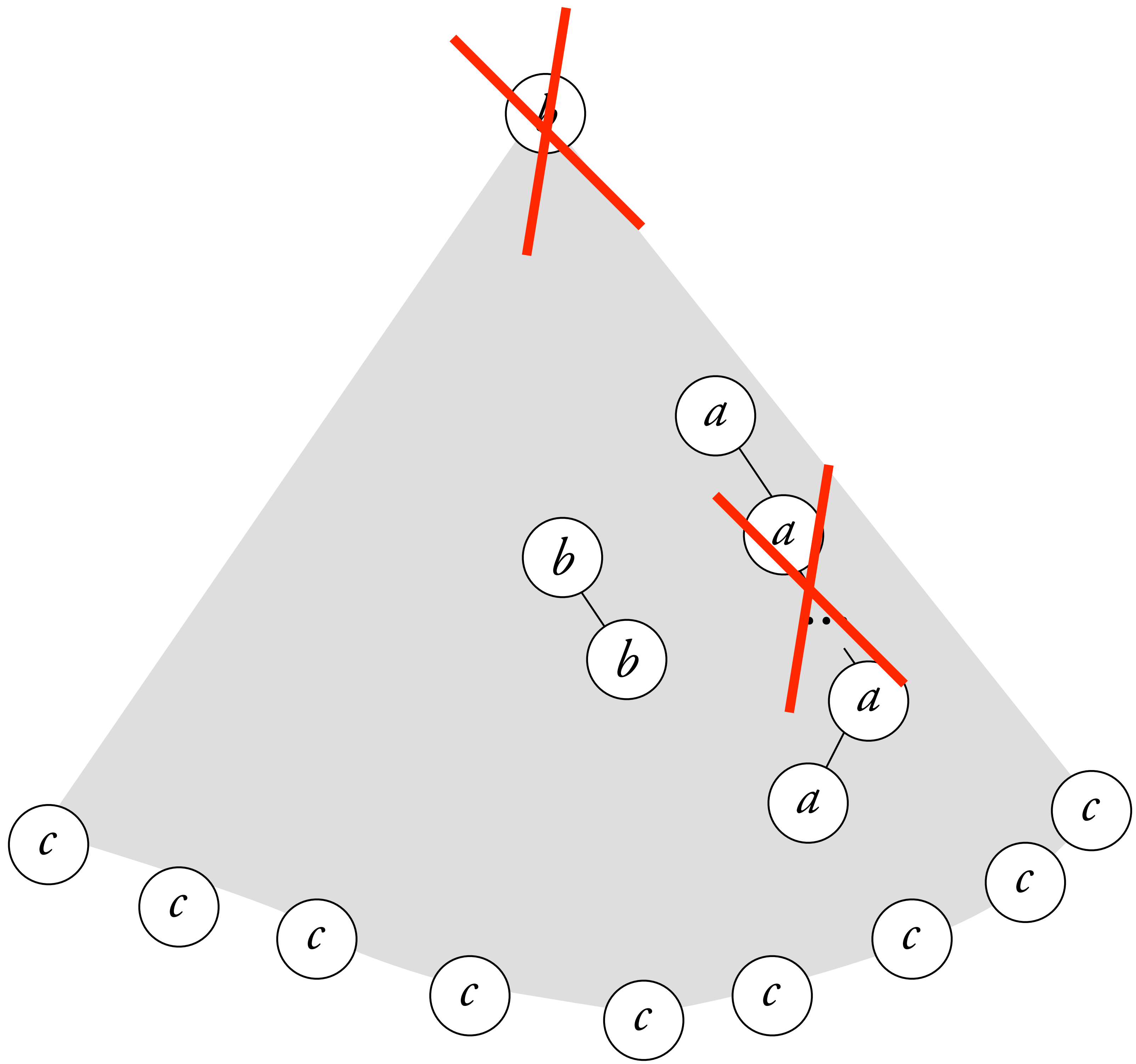

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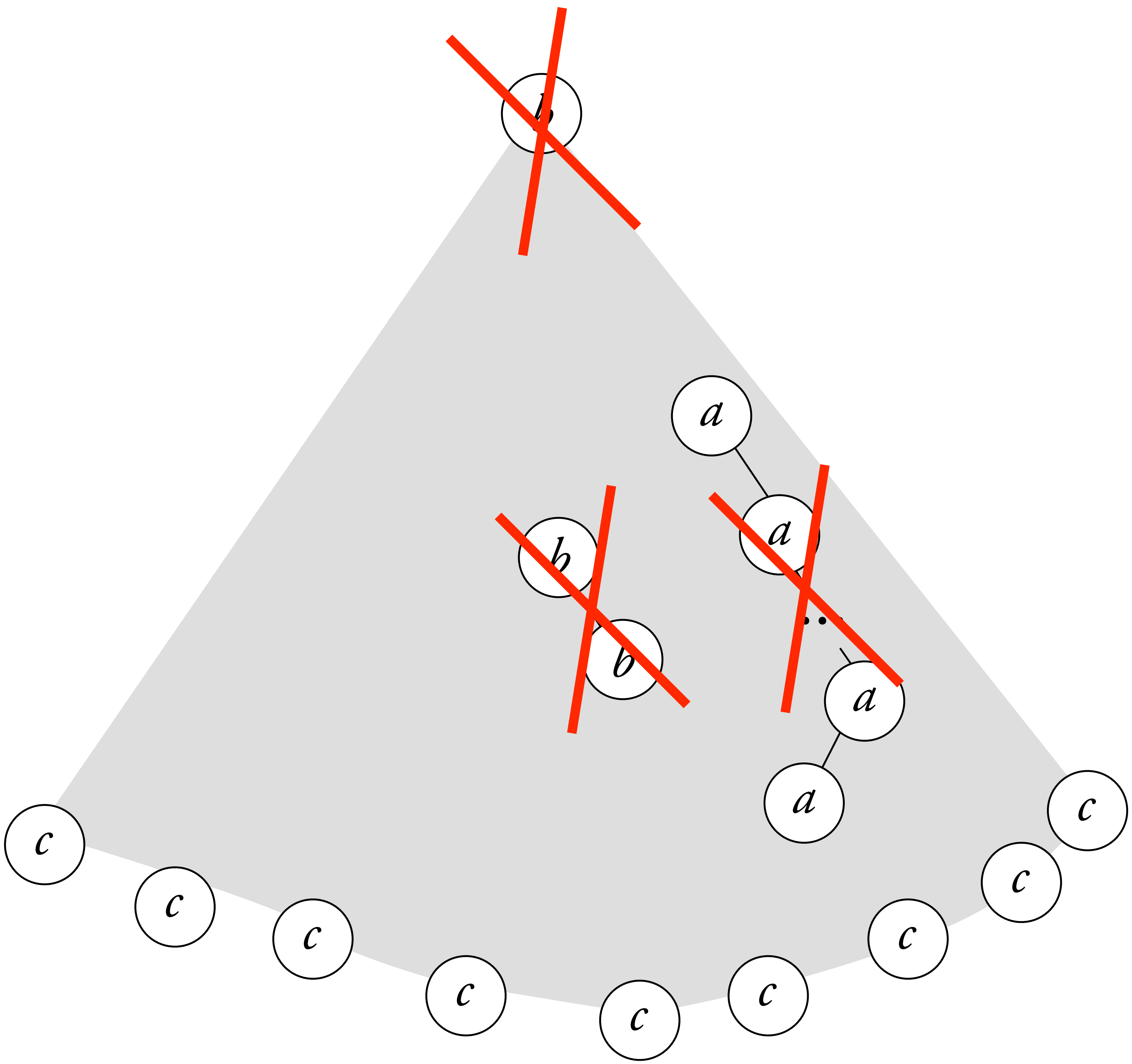

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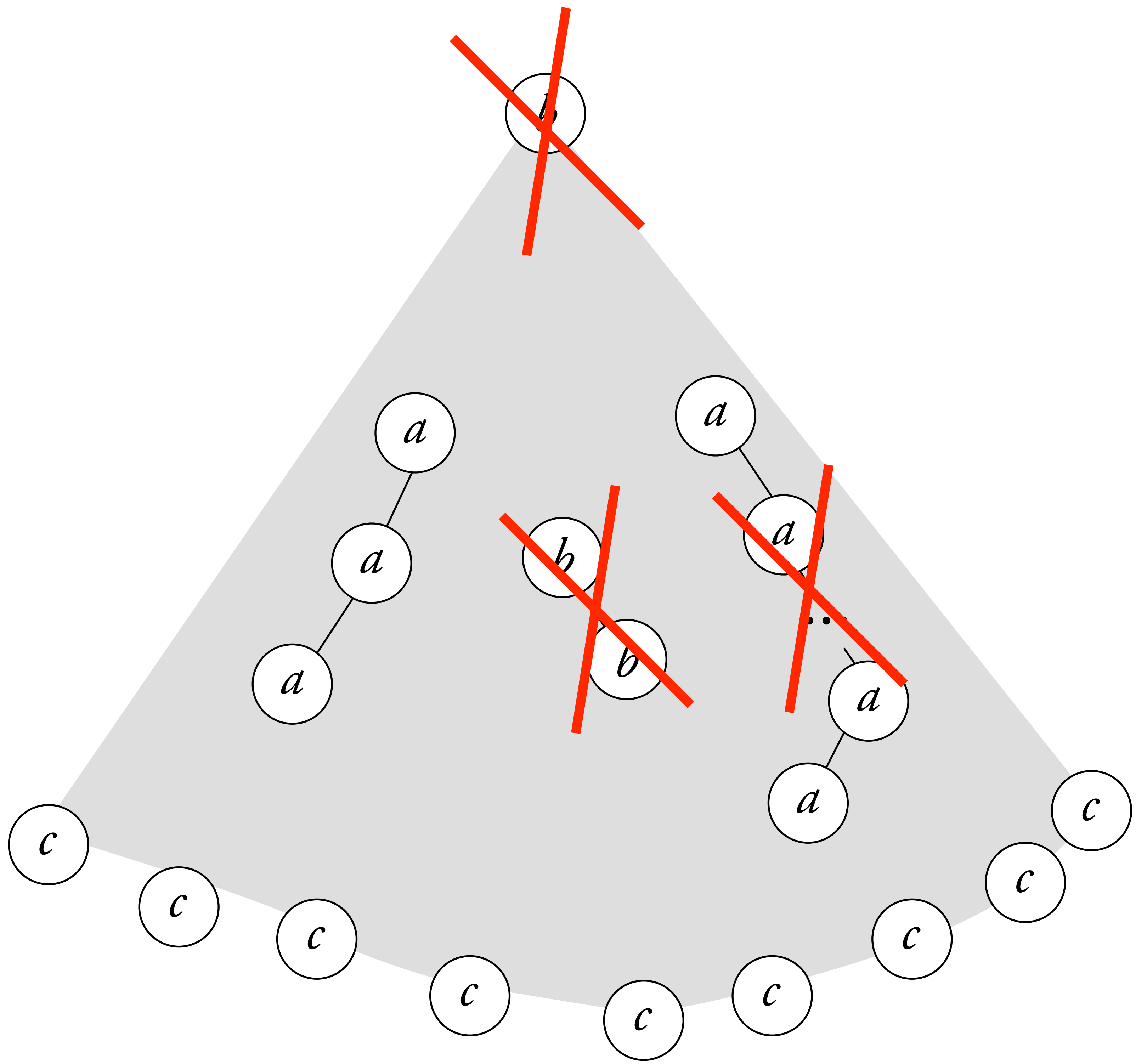

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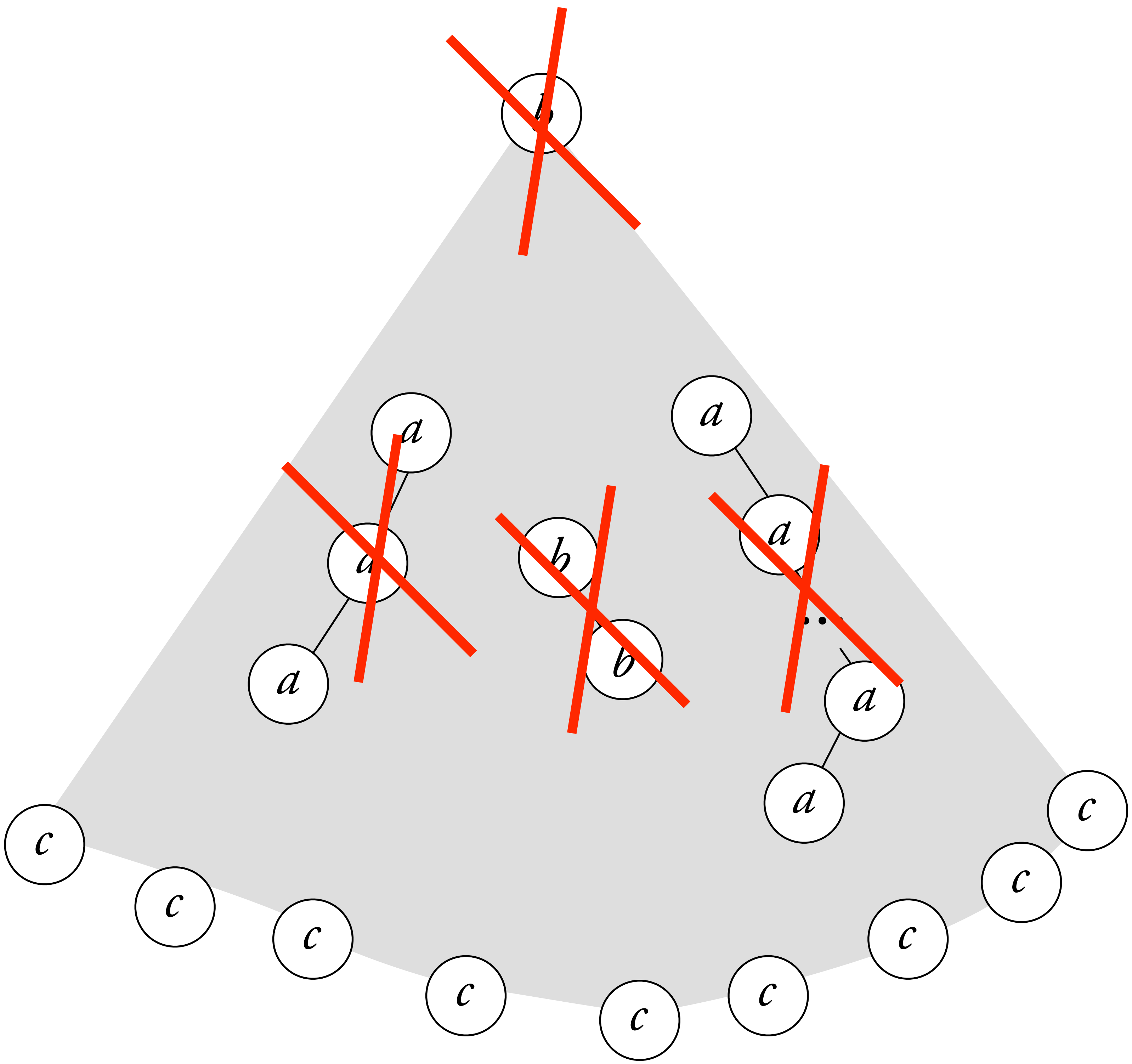

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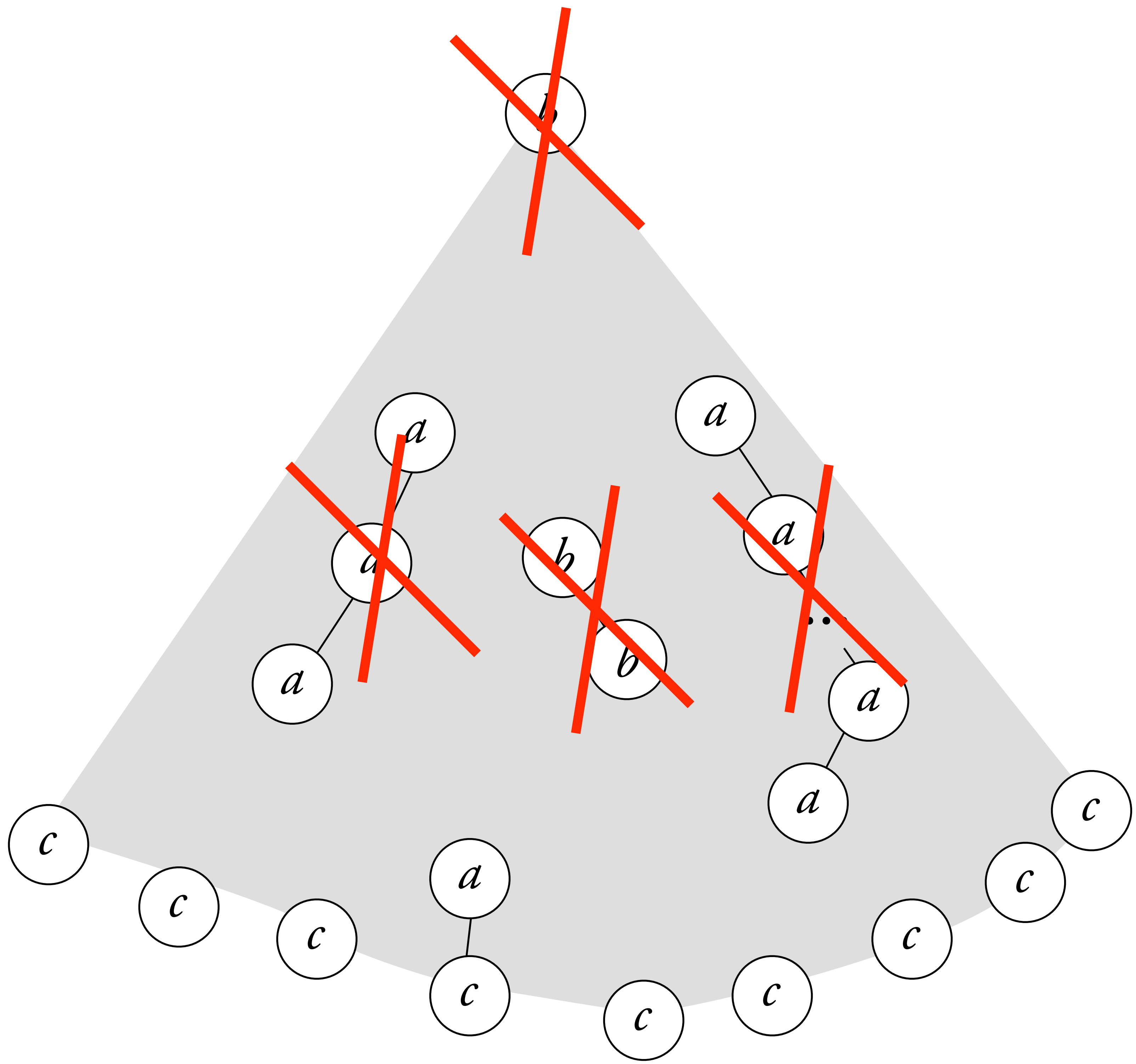

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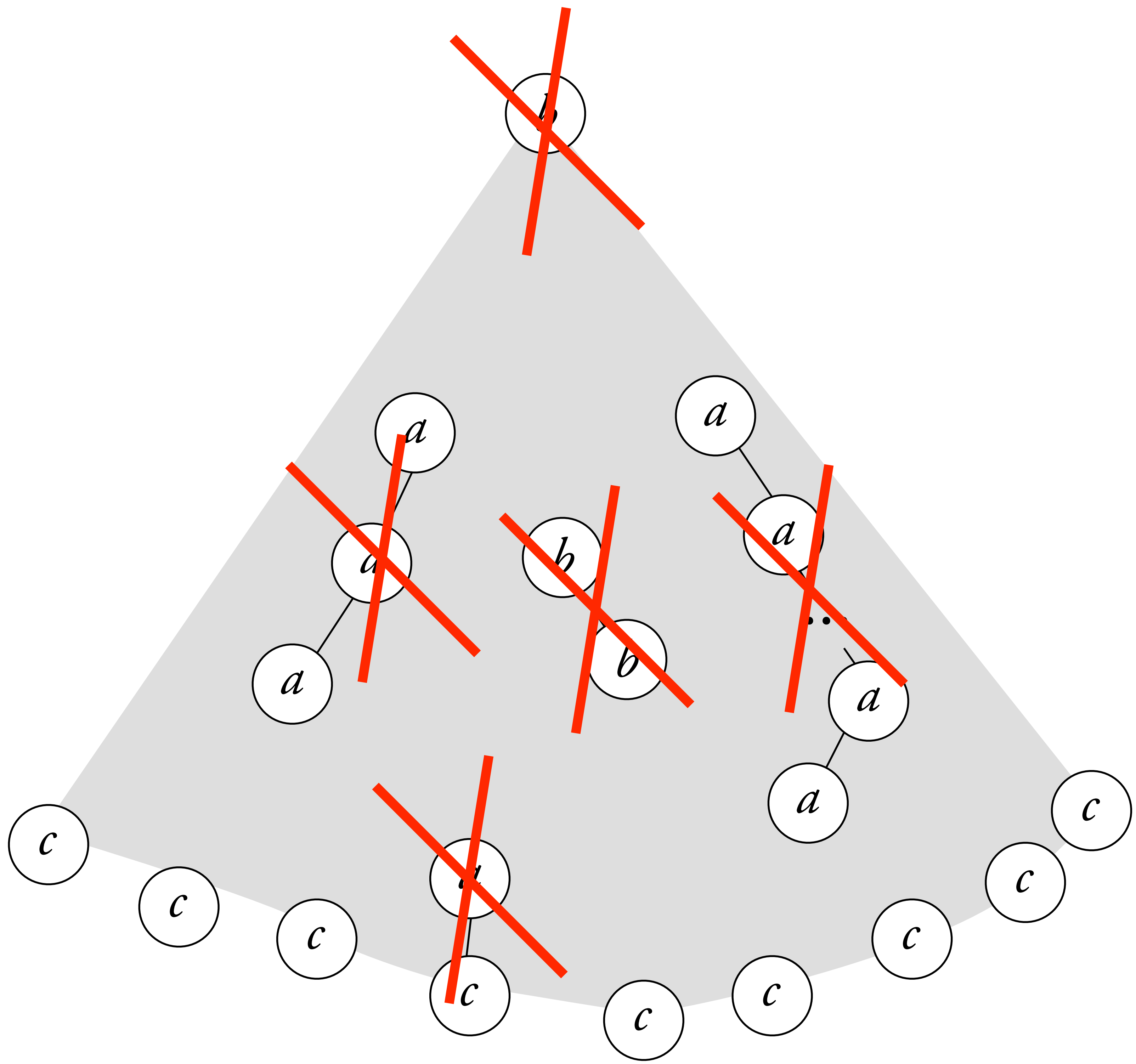

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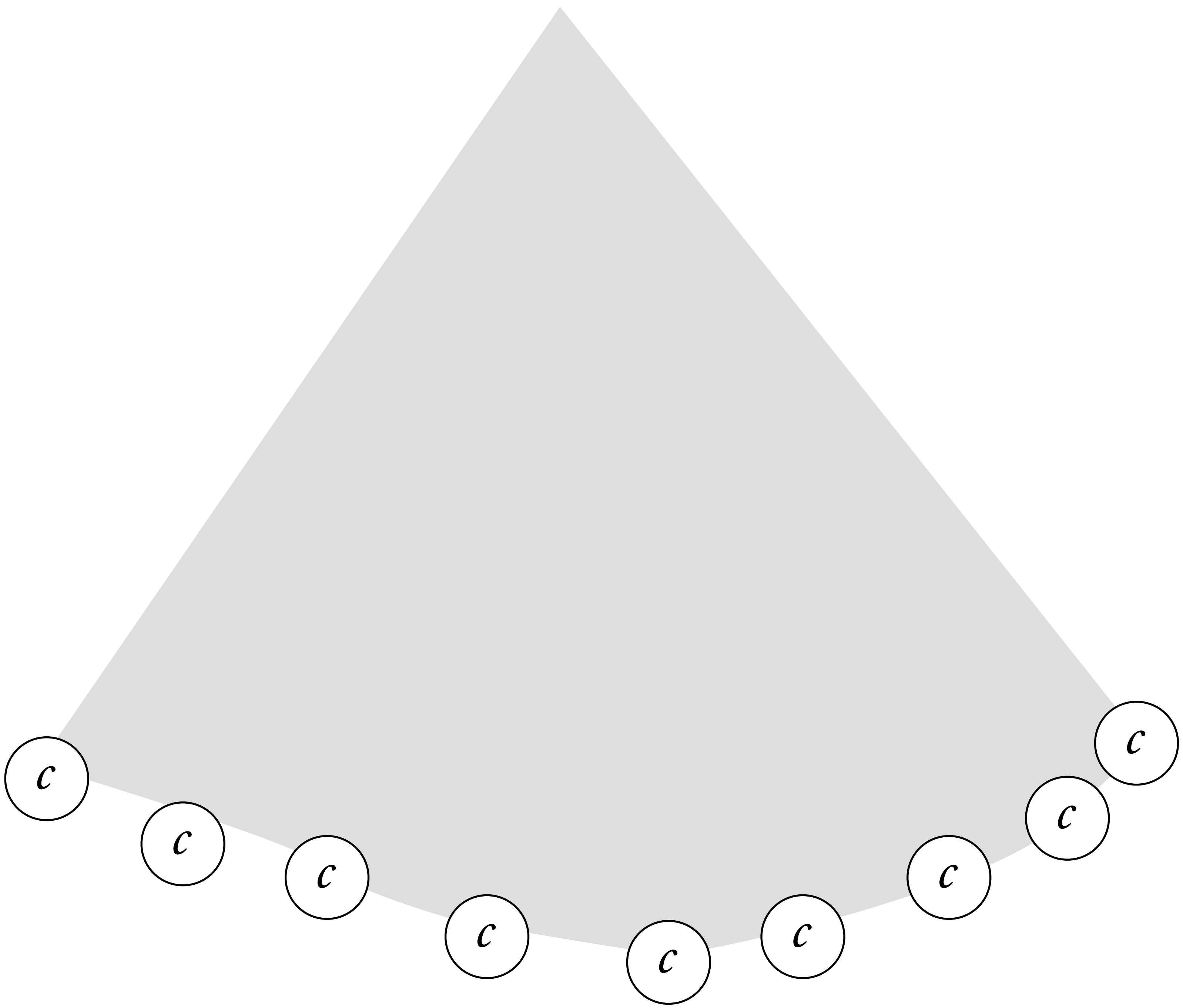

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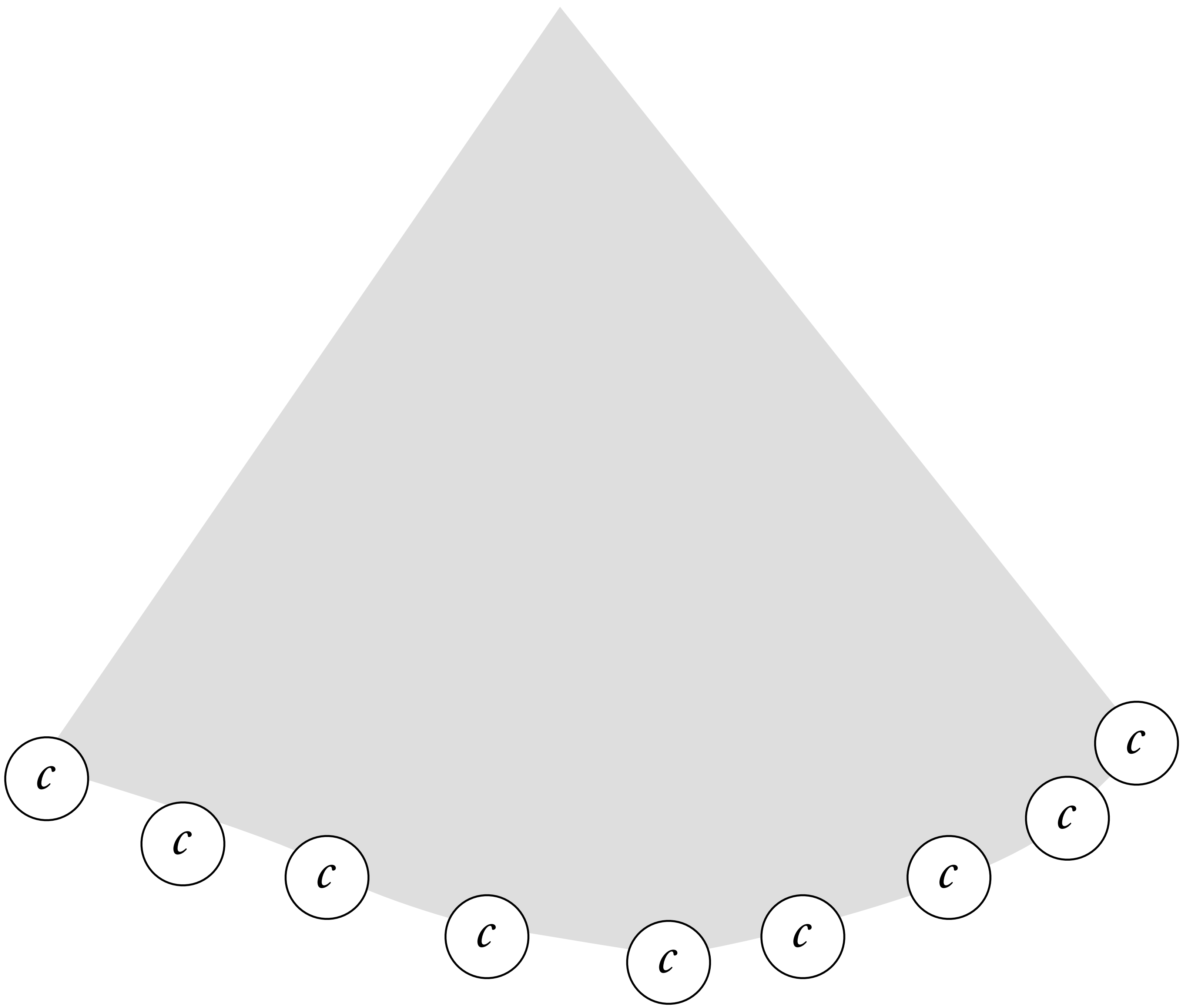

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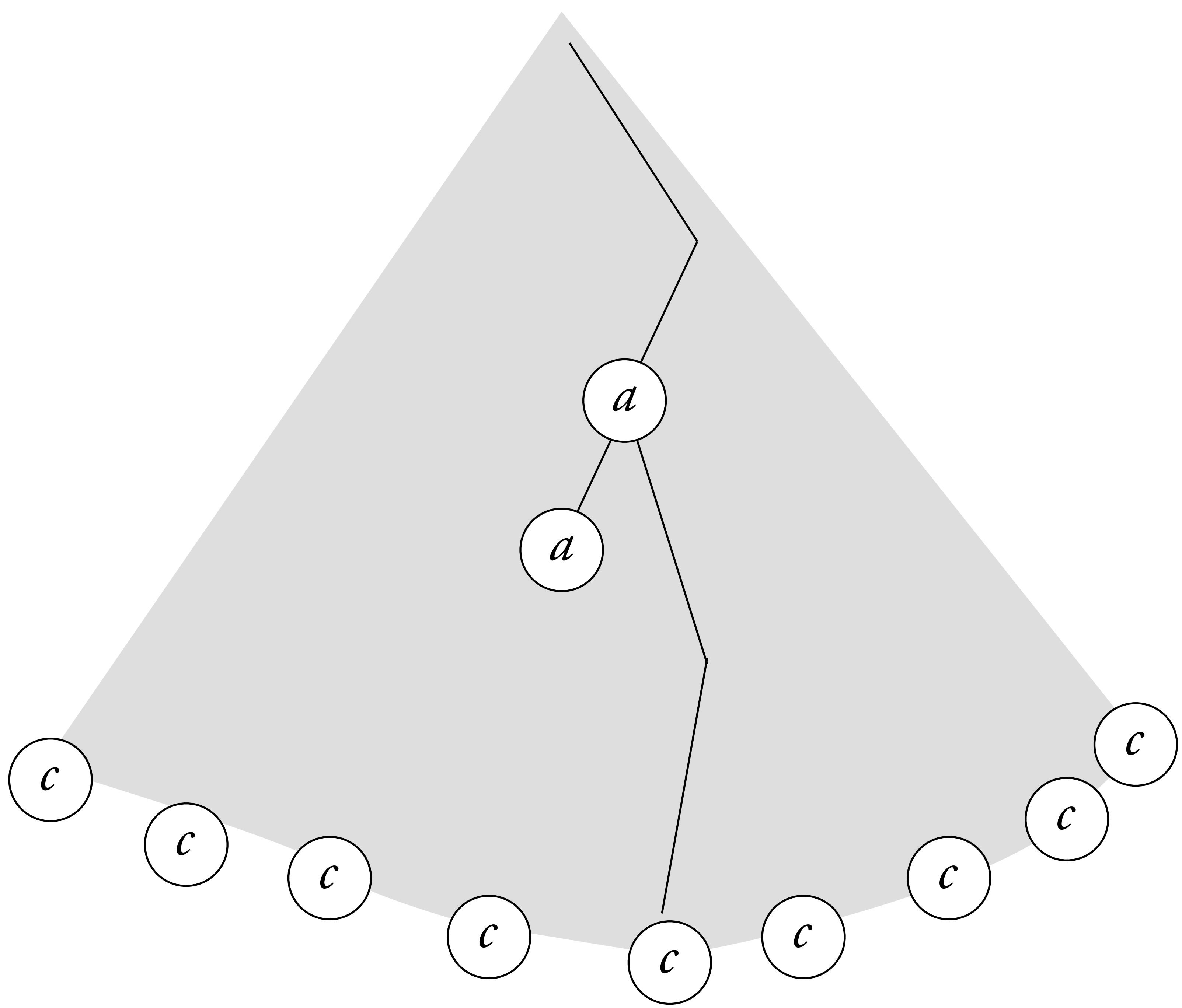

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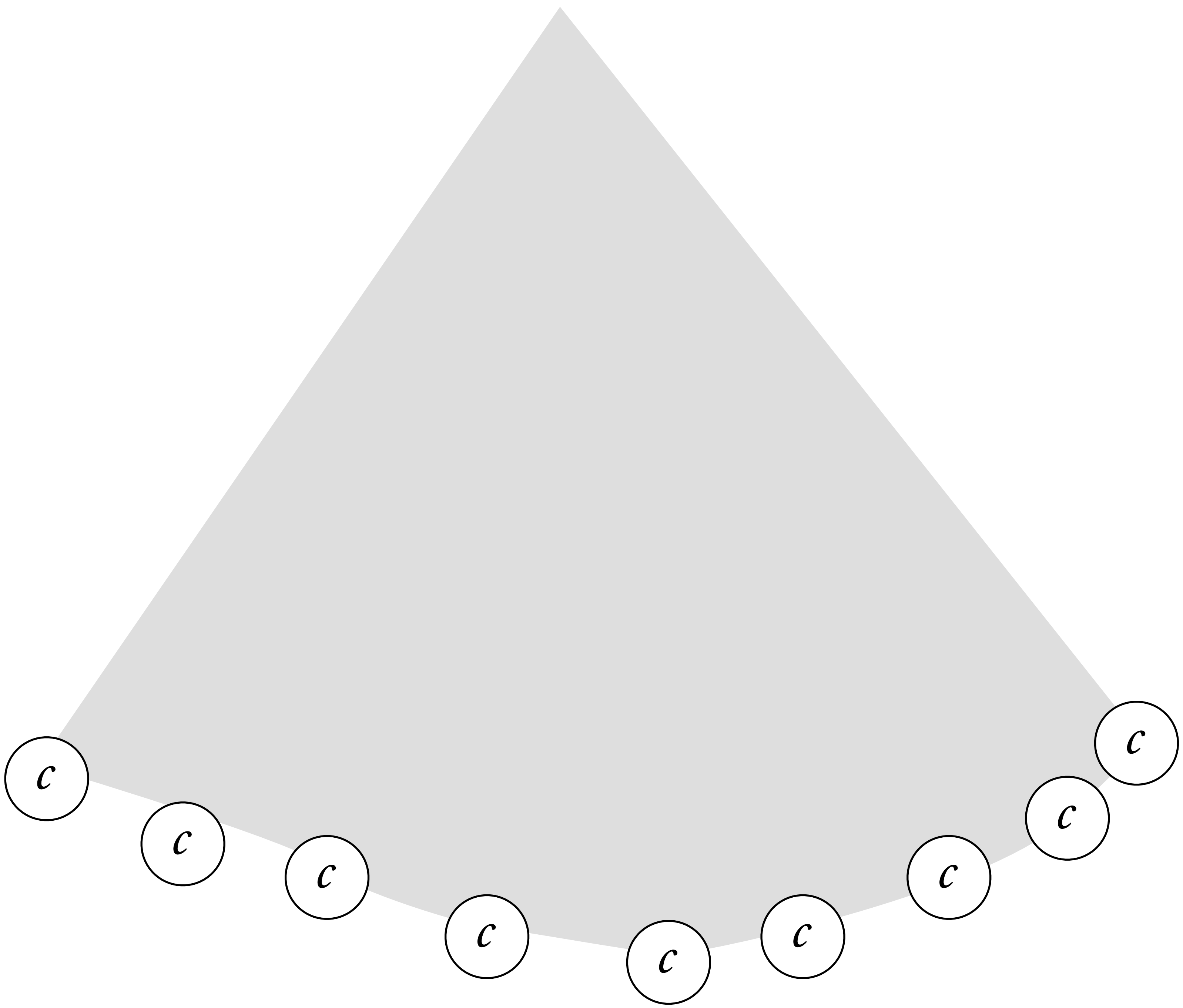

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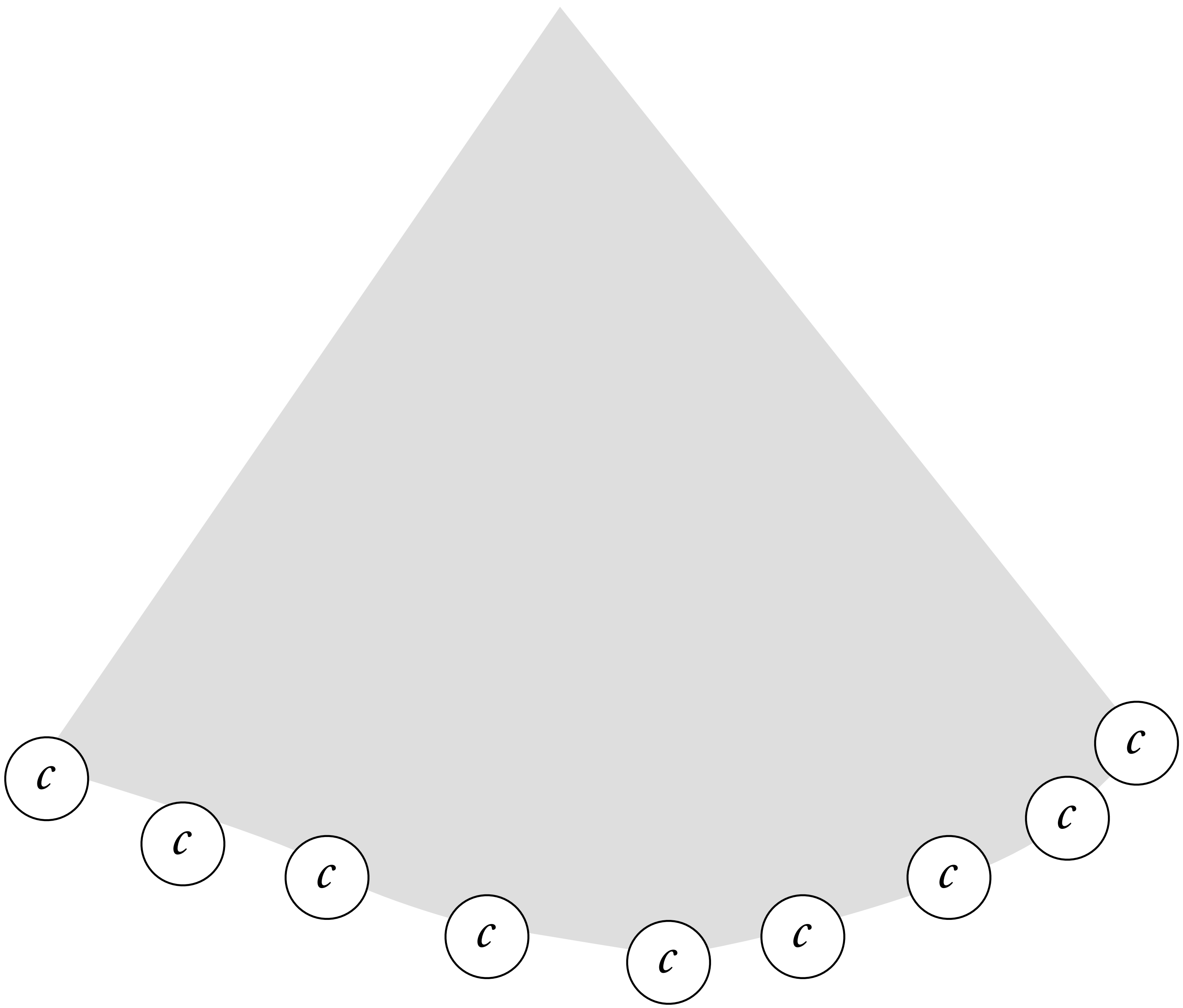

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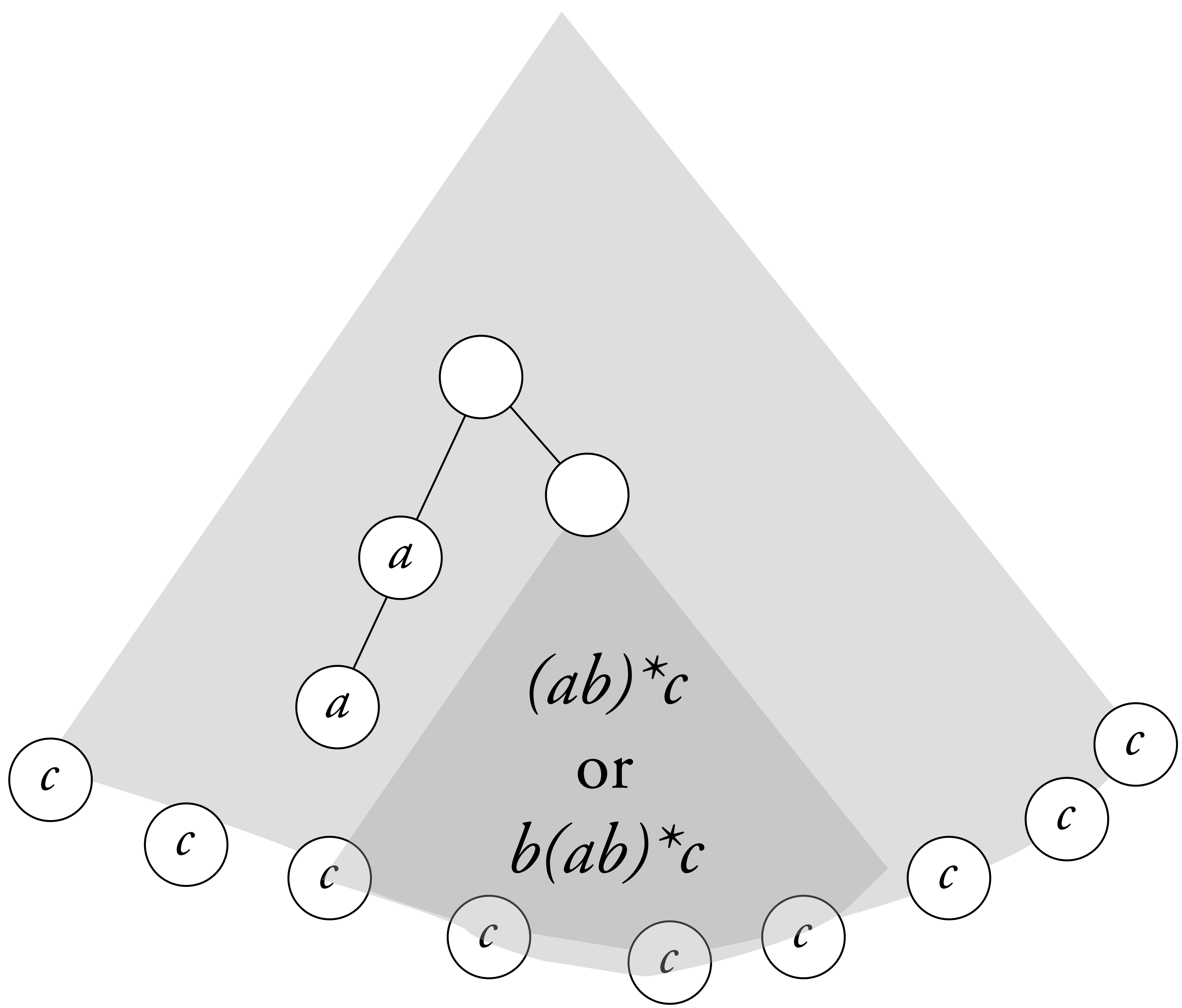

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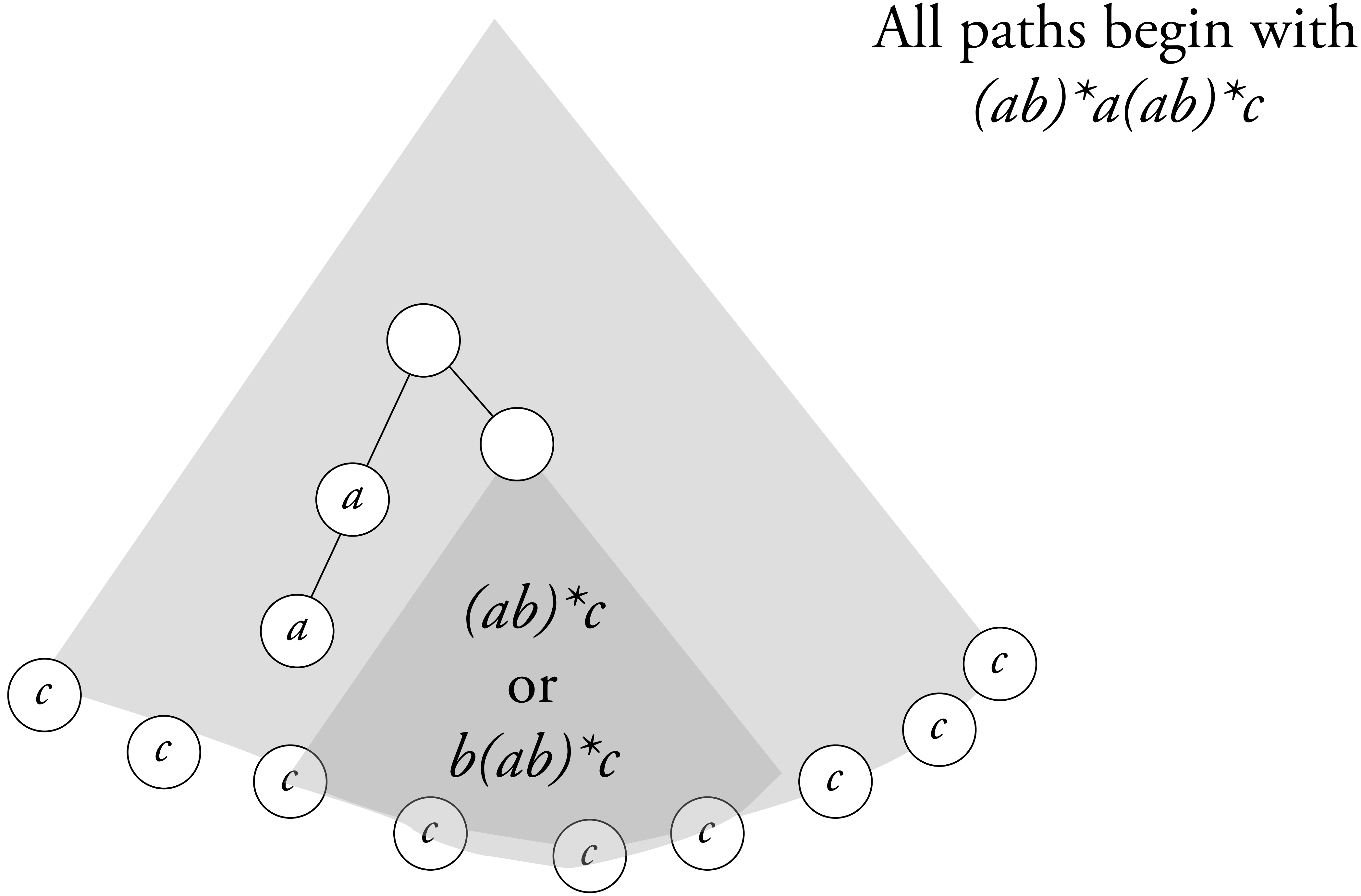

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$\Uparrow$

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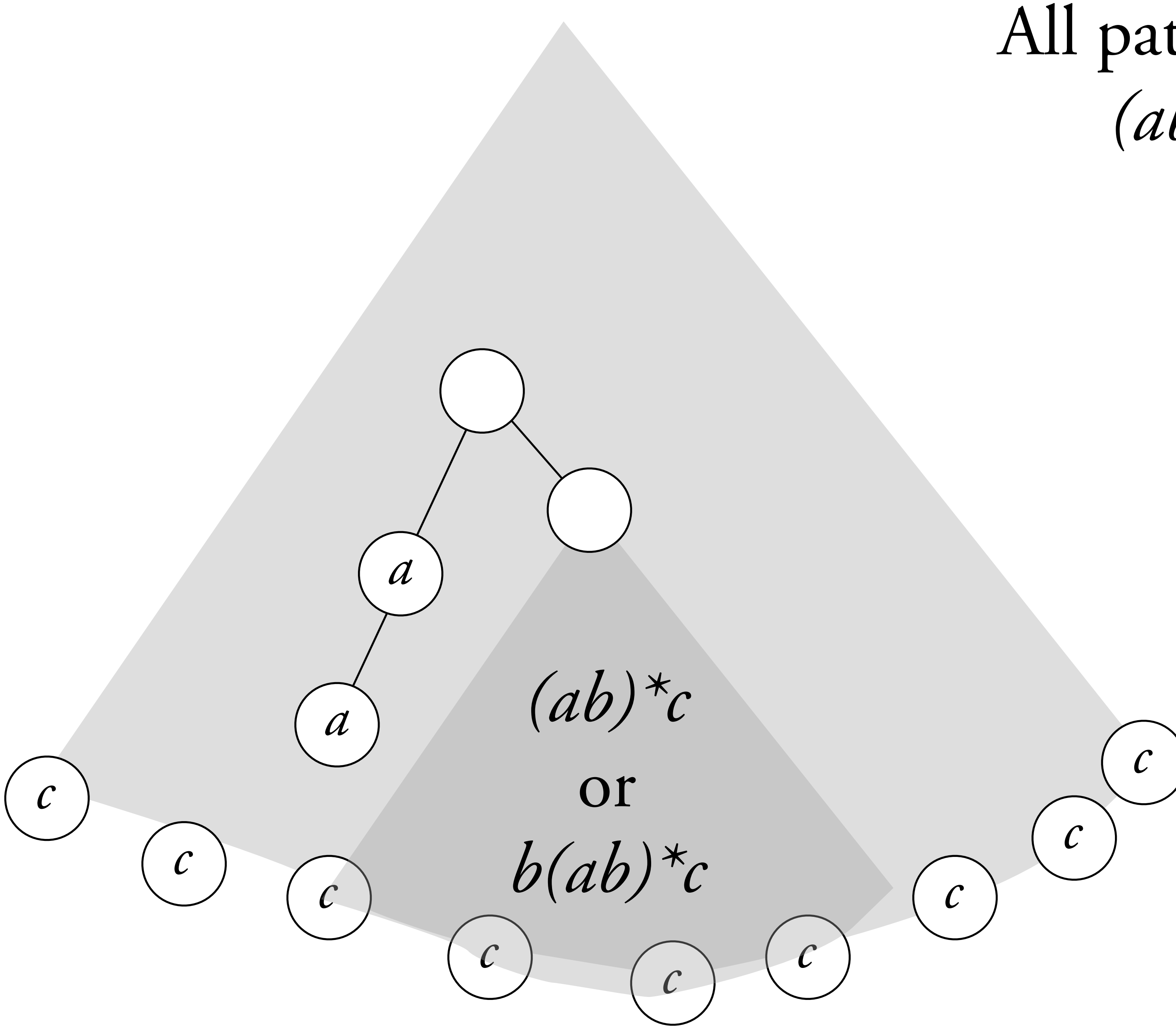

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$\Uparrow \Downarrow$

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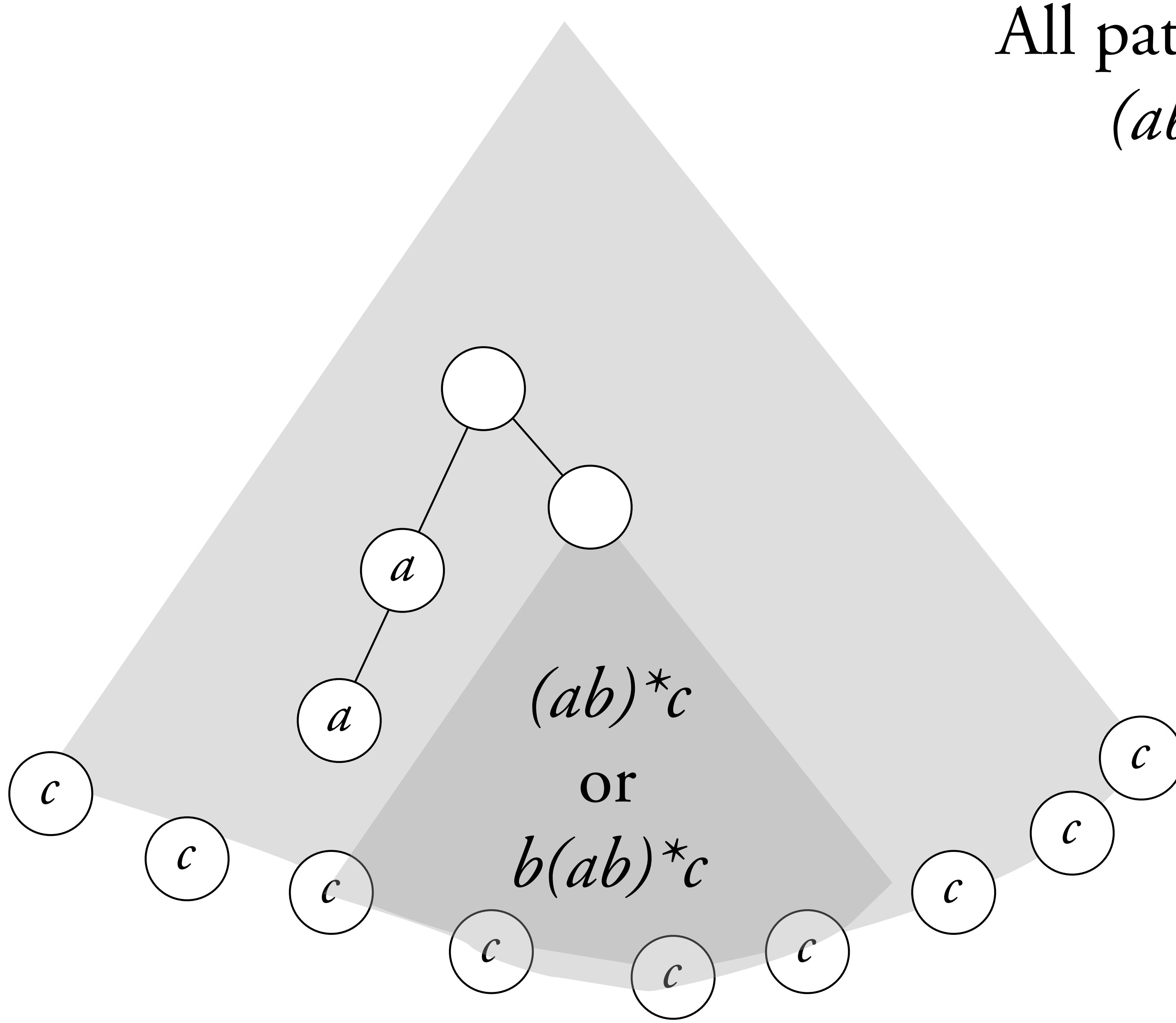

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$\Updownarrow$

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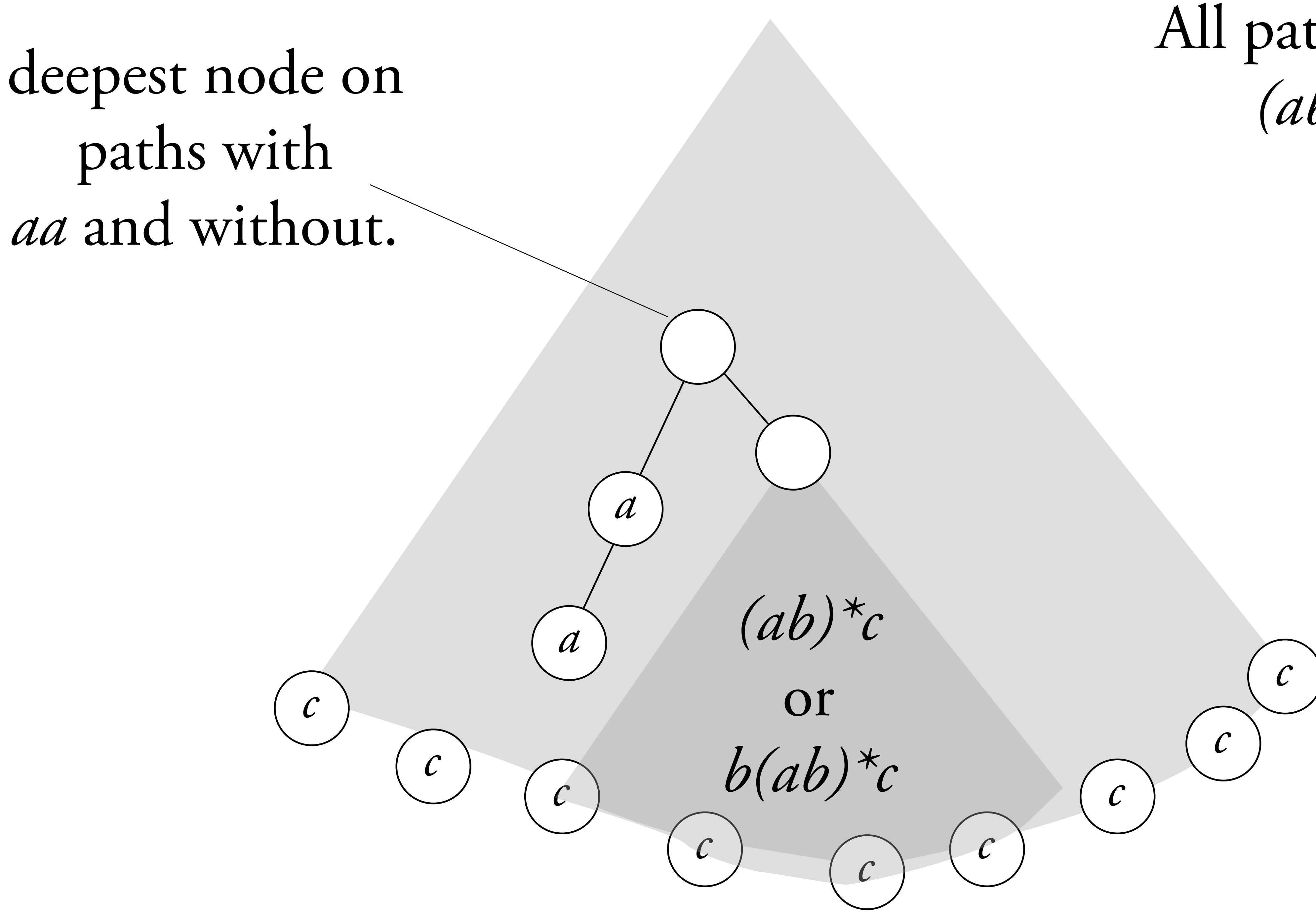

Thm. [Maidl 00] Let L be a set of infinite words.

The tree language "all paths in L" is definable in ACTL

iff

L is definable in $\Pi_2(<)$

Thm. It is decidable if a language is definable in $\Pi_2(<)$.

generalization to infinite words
of a result of Arfi '91.

$$\exists x_1 \cdots \exists x_n \quad \forall y_1 \cdots \forall y_m \quad \varphi(x_1, \ldots, x_n, y_1, \ldots, y_m)$$

A language definable in $\Sigma_2(<)$ is closed under the following rewriting rules.

$$\exists x_1 \cdots \exists x_n \quad \forall y_1 \cdots \forall y_m \quad \varphi(x_1, \ldots, x_n, y_1, \ldots, y_m)$$

A language definable in $\Sigma_2(<)$ is closed under the following rewriting rules.

$$w^k \longrightarrow w^k\, v\, w^k$$

if k is large, and v is a subword of w.

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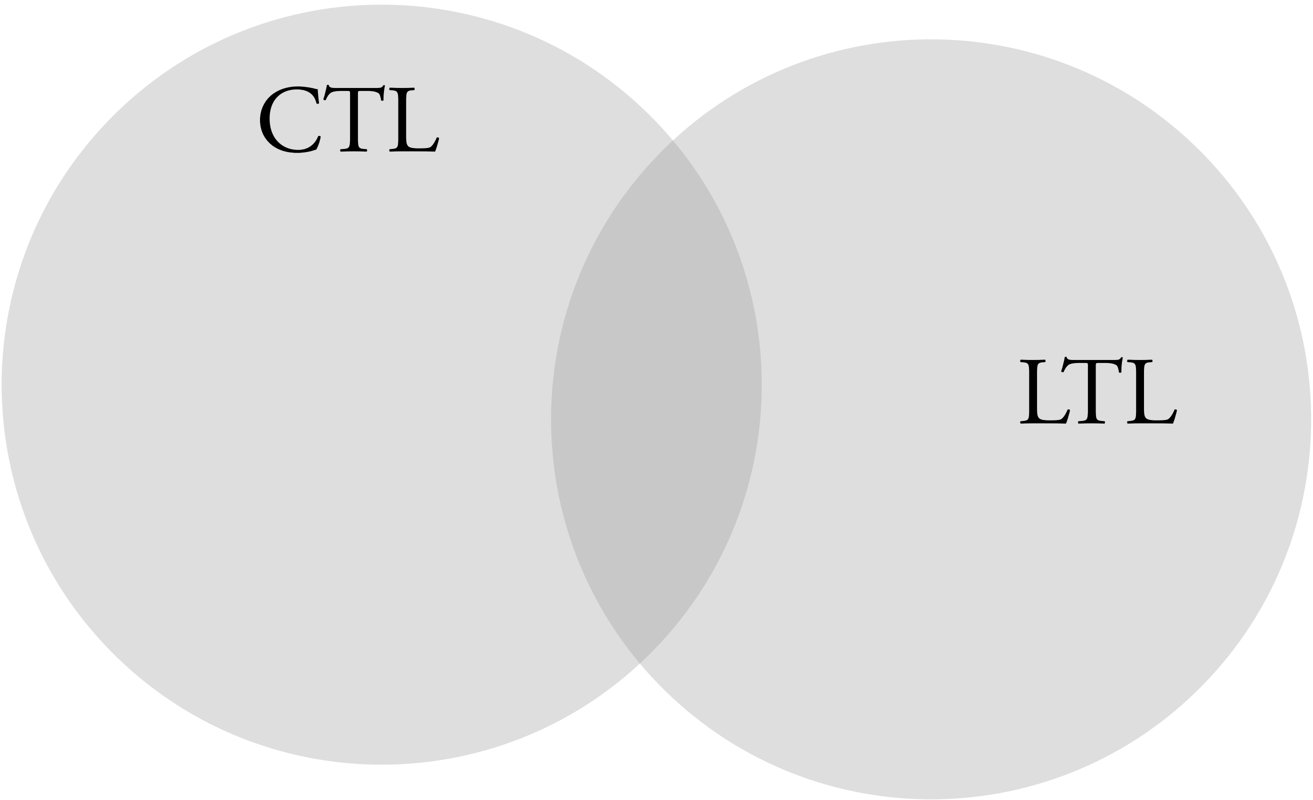
$$w^\omega \longrightarrow w^k\, v\, u^\omega$$

if k is large, and v, u are subwords of w.

Contribution

- ACTL $\cap$ LTL is a proper subset of CTL $\cap$ LTL.
- An effective characterization of $\Pi_2(<)$ for infinite words.
- A simpler characterization of $\Pi_2(<)$ for finite words.



Future work

- Effective characterization of CTL...
- ...or at least the common fragment of CTL and LTL.