Subcritical approximation and concentration phenomena in the critical Folland–Stein embedding

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Abstract. Over the past few decades, the study of nonlinear problems and geometric inequalities in the Heisenberg group $\mathbb{H}^n$ has attracted increasing attention. In this talk, we discuss some effects of the lack of compactness in the critical Folland–Stein–Sobolev embedding within the Heisenberg group. In this context, we employ several results in PDEs and Calculus of Variations, which also serve as general and independent contributions. In particular, we obtain extensions of the concentration–compactness principle of P.–L. Lions in the homogeneous setting for Folland–Stein spaces and for fractional horizontal Sobolev spaces. We furthermore extend the celebrated Global Compactness result in the Heisenberg framework via a completely different approach from the one originally proposed by Struwe in [3]. The results of the talk are based on joint works [1, 2] with M. Piccinini (Univ. Pisa) and G. Palatucci (Univ. Parma).

References

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[2] G. Palatucci, M. Piccinini, L. Temperini, Global Compactness, *Struwe's Global Compactness and energy approximation of the critical Sobolev embedding in the Heisenberg group*, preprint, 27 pp.

[3] M. Struwe, *A global compactness result for elliptic boundary value problems involving limiting nonlinearities*, Math. Z. **187** (1984).