Semi-classical analysis for nonlinear Choquard equations in possibly degenerate settings

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We study existence of semi-classical states for the nonlinear Choquard equation:

$$-\epsilon^2 \Delta v + V(x)v = \frac{1}{\epsilon^\alpha}(I_\alpha * F(v))F'(v) \quad \text{in } \mathbb{R}^N,$$

where $N \geq 3$, $\alpha \in (0, N)$, $I_\alpha(x) = \frac{A_\alpha}{|x|^{N-\alpha}}$ is the Riesz potential, $F \in C(\mathbb{R}, \mathbb{R})$ and $\epsilon > 0$ is a small parameter. We give a new variational approach to show the existence of a family of solutions concentrating to a local maximum or a saddle point of the potential $V(x)$ under general conditions on $F(s)$. Our results extends the results of Moroz and Van Schaftingen (2015) for local minima and Wei and Winter (2009) for non-degenerate setting.

If time allows, we also give related results on other singular perturbation problems.

This is a joint work with Silvia Cingolani.

References

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