On a Schrödinger system with shrinking regions of attraction

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We consider the elliptic system of 2 equations

$$\begin{cases} -\Delta v_1 + v_1 = \mu_1 Q_\varepsilon(x-y_1)|v_1|^{2p-2}v_1 + \lambda|v_2|^p|v_1|^{p-2}v_1, \\ -\Delta v_2 + v_2 = \mu_2 Q_\varepsilon(x-y_2)|v_2|^{2p-2}v_2 + \lambda|v_1|^p|v_2|^{p-2}v_2, \\ v_1, v_2 \in H^1(\mathbb{R}^N), \quad v_1, v_2 \neq 0 \end{cases}$$

where $N \geq 2$, $y_i \in \mathbb{R}^N$, $\mu_i > 0$, $\lambda < 0$, $1 < p < 2^*/2 := N/(N-2)$, $\varepsilon > 0$ and $Q_\varepsilon(x) = 1$ if $|x| < \varepsilon$, $Q_\varepsilon(x) = -1$ if $|x| \geq \varepsilon$. The solutions v_i may represent some particles or species that are attracted to the small region $B_\varepsilon(y_i) = \{x \in \mathbb{R}^N : |x - y_i| < \varepsilon\}$ and repelled from its complement. Since $\lambda < 0$, the system is competitive, i.e. different species repel each other. We show that this system has infinitely many solutions and that one of them is nonnegative and of least energy. Then we discuss the limit profile and the concentration behaviour of least energy solutions as $\varepsilon \to 0$. The same results hold for a similar system of $\ell \geq 2$ equations but for simplicity we only consider the case $\ell = 2$.

This is joint work with Mónica Clapp and Alberto Saldaña.