The Dirichlet problem for the one-dimensional Rudin-Osher-Fatemi functional

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We study minimizers of the following functional,

$$\Phi_{f,\lambda,\varphi}(u) = \begin{cases} \int_0^L |Du| + \frac{\lambda}{2}\int_0^L (u-f)^2\,dx, \quad \gamma u_{f,\lambda,\varphi} = \varphi & u \in BV(\Omega), \\ +\infty & \text{otherwise.} \end{cases}$$

where $\Omega = (0, L)$ and $\gamma : BV(\Omega) \to L^1(\partial\Omega) \equiv \mathbb{R}^2$ is the trace operator. The problem we face is the lack of the lower semicontinuity of $\Phi_{f,\lambda,\varphi}$. As a result minimizers need not exist.

We offer a few sets of sufficient conditions for the existence of minimizers of $\Phi_{f,\lambda,\varphi}$ satisfying the Dirichlet boundary conditions in the trace sense. In one case we rely on the methods specific for the total variation flow. We also present a few counter-examples.