A Schiffer-type problem for annular domains

David Ruiz
IMAG, Depto. Análisis Matemtico
University of Granada
Facultad de Ciencias, Granada, Spain
daruiz@ugr.es

The so-called Schiffer conjecture is a widely open problem that can be stated as follows:

If a nonconstant Neumann eigenfunction u of the Laplacian on a smooth bounded domain in $\mathbb{R}^N$ is constant on the boundary, then the domain is a ball.

In this talk we will consider a version of such question for planar domains with disconnected boundary. Specifically, we consider Neumann eigenfunctions that are locally constant on the boundary (that is, constant on each connected component of the boundary) and we wonder if the domain has to be necessarily a disk or an annulus. We will show that this question shares some rigidity features inherent to the original Schiffer conjecture.

Despite, the answer is negative: we are able to build nonradial Neumann eigenfunctions which are locally constant on the boundary of the domain. The proof uses a local bifurcation argument together with a suitable reformulation of the problem by Fall, Minlend and Weth that avoids a problem of loss of derivatives. As a byproduct we obtain an example of a continuous, weak solution of the steady Euler 2D equations with compact support and noncircular streamlines.

This is joint work with A. Enciso, A. J. Fernández and P. Sicbaldi.