Anisotropic nonlinear Schrödinger equations on the plane

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In this talk we present some results, contained in a recent joint work with Amin Esfahani and Hichem Hajaiej, on the following anisotropic nonlinear Schrödinger equations in the plane

$$\begin{cases} i\partial_t \Phi + \partial_{xx}\Phi - (-\partial_{yy})^s \Phi + |\Phi|^{p-2}\Phi = 0, & \text{for } (t,x,y) \in \mathbb{R}\times\mathbb{R}^2, \\ \Phi(0,x,y) = \Phi_0(x,y), & \text{for } (x,y) \in \mathbb{R}^2, \end{cases}$$

where $p > 2$ and $(-\partial_{yy})^s$ denotes the fractional Laplacian with $0 < s < 1$.

We first study the existence of normalized solutions to this equation in the mass subcritical, critical, and supercritical cases. To this aim, regularity results and a Pohozaev type identity are necessary. Then, we determine the conditions under which the solutions blow up. Furthermore, we demonstrate the existence of boosted travelling waves when $s \geq 1/2$ and their decay at infinity. Additionally, for the delicate case $s = 1/2$, we provide a non-existence result of boosted travelling waves and we establish that there is no scattering for small data. Due to the nature of the equation, we do not impose any radial symmetry on the initial data or on the solutions.