Spectral optimization problems arising in logistic model with anisotropic diffusion

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We will discuss some recent results concerning weighted eigenvalue problems in bounded Lipschitz domains $\Omega \subset \mathbb{R}^N$, $N \geq 1$, under Robin boundary conditions in the presence of an anisotropic diffusion.
First, we show the existence of two positive principal eigenvalues $\lambda^{\pm}$ respectively associated with a positive and a negative eigenfunction. We analyze the minimization of $\lambda^{\pm}$ with respect to the sign-changing weight, showing that the optimal eigenvalues $\Lambda^{\pm}$ are equal if the domain has a centre of symmetry and the optimal weights are of bang-bang type, namely piece-wise constant functions, each one taking only two values. As a consequence, the problem is equivalent to the minimization with respect to the subsets of Ω satisfying a volume constraint. The optimization problem is completely solved in dimension one, in the case of homogeneous Dirichlet or Neumann conditions, showing new phenomena induced by the presence of the anisotropic diffusion.
The analogous study in higher dimension is open in its generality even in the case of the Laplacian operator. With this respect, we will present some recent results.
This class of optmization problems naturally arises in the study of the optimal spatial arrangement of resources for a species to survive in an heterogeneous habitat.

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References

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