Existence and order of the self–trapping transition in the non–local NLSE

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We study the Pekar–Hartree functional with regular, radially symmetric, and positive kernels, whose stationary points lead to a non–linear and non–local Schrödinger equation, known as the inhomogeneous Choquard equation. It arises as an effective model of particles moving in deformable media, and has recently resurfaced in the study of ions in Fermi gases. Our main object of interest is the infimum of the functional over the unit ball in $L^2(\mathbb{R}^d)$ for $d = 2$ and $d = 3$. We prove that minimizers exist as long as the coupling is large enough, both in dimensions two and three, which is in contrast with the linear case, where L^2–normalized minimizers exist in $2d$ for arbitrarily small couplings. The dimensionalities do exhibit different behavior also in the non–linear case, as we show that no minimizers exist for $d = 2$ at the critical coupling while they do for $d = 3$. This difference is reflected in the effective mass, a key quantity in the problem from the physical point of view, thus providing the motivation for several open problems for consideration by the community.

References

[1] J. Ginibre and G. Velo, Math. Z. **170**, 109–136 (1980)

[2] E.H. Lieb, Stud. Appl. Math. **57**, 93-105 (1977)

[3] H. Berestycki and P. Lions, Arch. Rat. Mech. Anal. **82**, 313–345 (1983)

[4] P. Lions, Nonlinear Analysis. Theory, Methods & Applications, Vol. 4 No. 6, 1063-1073 (1979)

[5] K. Myśliwy and K. Jachymski, Phys. Rev. B **109**, 214208 (2024)