Singular Liouville equations in high and low dimensions

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In this talk, I will discuss existence, classification and non-degeneracy results for solutions to singular Liouville-type equations of the form

$$(-\Delta)^{\frac{n}{2}}u = |x|^{n(\alpha-1)}e^{nu} \quad \text{in } \mathbb{R}^n. \tag{1}$$

In dimension one, the problem has applications in mathematical modelling of galvanic corrosion phenomena for ideal electrochemical cells consisting of an electrolyte solution confined in a bounded domain with an electrochemically active portion of boundary. In higher dimension, Lioville equations have applications to prescribed curvature problems in conformal geometry: solutions correspond to constant Q-curvature metrics on the Euclidean space, with a singular point at the origin.

After a general overview of the existing literature, I will focus on the 1-d case and I will prove that solutions of (1) are non-degenerate for $\alpha \in (0,1) \cup (1,2)$. Namely, the space of solutions in $H^{\frac{1}{2}}(\mathbb{R})$ to the linearized equation

$$(-\Delta)^{\frac{1}{2}}\varphi = |x|^{\alpha-1}e^{u}\varphi \quad \text{in } \mathbb{R} \tag{2}$$

has dimension one. The proof relies on the use of harmonic extensions and conformal transformations to rewrite the linearized equation (2) as a Steklov eigenvalue problem on either an intersection or a union of two disks, depending on the values of α. These results are contained in joint works in collaboration with A. DelaTorre, A. Pistoia, A. Hyder and L. Martinazzi (see [1, 2]).

References

[1] A. DelaTorre, G. Mancini, A. Pistoia, *Non degeneracy of the bubble for a fractional singular 1d Liouville equation*, preprint arxiv 2404.14119.

[2] A. Hyder, G. Mancini, L. Martinazzi, *Local and nonlocal singular Liouville equations in Euclidean spaces*, International Mathematics Research Notices 2021, no. 15, 11393–11425.