On nonlinear critical problems involving the Grushin Operator: bifurcation and multiplicity results

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Given a generic point $z = (x, y) \in \mathbb{R}^N = \mathbb{R}^m \times \mathbb{R}^\ell$, the Grushin Operator is defined as

$$\Delta_\gamma u(z) = \Delta_x u(z) + |x|^{2\gamma} \Delta_y u(z),$$

where Δ_x and Δ_y are the classical Laplace operators and $\gamma \in \mathbb{R}_+$.

In [1] we consider the nonlinear critical problem

$$\begin{cases} -\Delta_\gamma u = \lambda u + |u|^{2^*_\gamma - 2} u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

We prove a multiplicity and bifurcation result for this problem, extending the results of Cerami, Fortunato and Struwe in [2] and of Fiscella, Molica Bisci and Servadei in [3].

This is a joint work with S. Secchi (Milano Biccoca, Italy) and G. Molica Bisci (Urbino, Italy).

References

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[3] A. Fiscella, G. Molica Bisci, R. Servadei, *Multiplicity results for fractional Laplace problems with critical growth,* Manuscripta Math., 2018.