Elliptic sytems related to self-similar solutions of Boussinesq system

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I shall present recent results obtained jointly with Lorenzo Brandolese [1] on an existence of solutions

$$U = \big(U_1(x), U_2(x), U_3(x)\big), \quad \Theta = \Theta(x) \quad \text{with} \quad x \in \mathbb{R}^3$$

to the system

$$\begin{aligned} -\Delta U - U - (x\cdot\nabla)U + (U\cdot\nabla)U + \nabla P &= \Theta\,\nabla(|\cdot|^{-1}) + F,\\ \nabla\cdot U &= 0,\\ -\Delta\Theta - \Theta - (x\cdot\nabla)\Theta + \nabla(\Theta U) &= 0. \end{aligned}$$

These solutions, via the formula

$$u(x,t) = \frac{1}{\sqrt{2t}}U\Big(\frac{x}{\sqrt{2t}}\Big), \qquad \theta(x,t) = \frac{1}{\sqrt{2t}}\Theta\Big(\frac{x}{\sqrt{2t}}\Big),$$

correspond to *self-similar solutions* of the well-known Boussinesq system from the fluid mechanics. The construction is based on the the Leray–Schauder theorem and compactness arguments

References

[1] Lorenzo Brandolese, Grzegorz Karch, *Large self-similar solutions to Oberbeck-Boussinesq system with Newtonian gravitational field*, (2023), arXiv:2311.01093.