Monotonicity trick in nonsmooth critical point theory and its application

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A motonicity trick due to Struwe [St88] and Jeanjean [Je99] is a powerful tool for functionals of class C^1 when it is hard to check the Palais–Smale condition. However, some functionals corresponding to equations appearing in physics or geometry are not of class C^1. In [Sz86], Szulkin extended the mountain pass and symmetric mountain pass theorem due to Ambrosetti and Rabinowitz [AR73] into nonsmooth functionals. The aim of this talk is to provide an extension and an application of the monotonicity trick for nonsmooth functionals in a setting which is close to the one in [Sz86]. In particular, we consider Born–Infeld type equations and prove the existence of infinitely many solutions. This talk is based on joint work [BIMM] with Jaeyoung Byeon, Andrea Malchiodi and Luciano Mari.

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