Asymptotics for eigenfunctions of the laplacian in domains with small holes

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Let us consider the following eigenvalue problem in $\mathbb{R}^N$, $N \geq 2$,,

$$\begin{cases} -\Delta u_\epsilon = \lambda_\epsilon u_\epsilon & \text{in } \Omega_\epsilon = \Omega \setminus B(P,\epsilon), \\ u_\epsilon = 0 & \text{on } \partial\big(\Omega \setminus B(P,\epsilon)\big). \end{cases} \tag{1}$$

where $B(P,\epsilon)$ is the ball centered at $P \in \Omega$ and small radius ϵ. It is well known that the previous problem "converges as $\epsilon \to 0$" in a suitable sense to

$$\begin{cases} -\Delta u = \lambda u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{2}$$

In this talk we want to give additional information on the convergence of the pair $(\lambda_\epsilon, u_\epsilon)$ to (λ, u) as $\epsilon \to 0$. In particular we try to give an answer to the following questions,

- If $u(P) \neq 0$ we have that u_ϵ does not converge uniformly to u near $\partial\Omega_\epsilon$. Is it possible to describe more precisely the behavior of u_ϵ near $\partial\Omega_\epsilon$?
- If λ is a multiple eigenvalue is it true that the eigenvalues λ_ϵ are simple (Uhlenbeck's property) for ϵ small?
- What about the nodal region of u_ϵ near $\partial B(P,\epsilon)$?

This is a joint paper with Laura Abatangelo (Politecnico di Milano, Italy).