Concentration phenomena for nonlinear fractional relativistic Schrödinger equations

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In this talk, we focus on the following class of fractional relativistic Schrödinger equations:

$$\begin{cases} (-\Delta + m^2)^s u + V(\varepsilon x)u = f(u) + \gamma u^{2_s^*-1} \text{ in } \mathbb{R}^N, \\ u \in H^s(\mathbb{R}^N), \quad u > 0 \text{ in } \mathbb{R}^N, \end{cases}$$

where $\varepsilon > 0$ is a small parameter, $s \in (0,1)$, $m > 0$, $N > 2s$, $\gamma \in \{0,1\}$, $2_s^* = \frac{2N}{N-2s}$ is the fractional critical exponent, $(-\Delta + m^2)^s$ is the fractional relativistic Schrödinger operator, $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous potential satisfying a local condition, and $f : \mathbb{R} \to \mathbb{R}$ is a superlinear continuous nonlinearity with subcritical growth at infinity. Utilizing the extension method and penalization techniques, we first show that there exists a family of positive solutions $u_\varepsilon \in H^s(\mathbb{R}^N)$, with exponential decay, that concentrates around a local minimum of V as $\varepsilon \to 0$. Finally, by combining the generalized Nehari manifold method with the Ljusternik-Schnirelman theory, we relate the number of positive solutions to the topology of the set where the potential V attains its minimum value. We emphasize that in the critical case, that is, $\gamma = 1$, a more refined analysis is required to obtain these results. The talk is based on the papers [**?**, **?**].

References

[1] V. Ambrosio, *The nonlinear fractional relativistic Schrödinger equation: existence, multiplicity, decay and concentration results*, Discrete Contin. Dyn. Syst. **41** (2021), no. 12, 5659–5705.

[2] V. Ambrosio, *Concentration phenomena for a fractional relativistic Schrödinger equation with critical growth*, Adv. Nonlinear Anal. **13** (2024), no. 1, Paper No. 20230123, 41 pp.