Dynamical Systems Working Seminar 2017-2018

Indecomposable continua in exponential dynamics

20/10/17

Krzysztof Lech

The presentation was about a paper by Łukasz Pawelec and Anna Zdunik, "Indecomposable continua in exponential dynamics - Hausdorff dimension". We discussed a general theorem which states that the Hausdorff dimension for certain dynamically defined continua has to be equal to 1. A sketch of the proof was presented, along with some intuition behind the geometry of it. Since both authors of the paper were present, they enthusiastically helped with developing a better understanding of such problems, without diving too deep into technical details. We briefly mentioned some applications of the theorem that were presented in the paper and the interesting fact that the one dimensional Hausdorff measure of continua in the plane is in fact not even sigma-finite. The seminar ended with some remarks from (among others) Anna Zdunik about related open problems and work done by other people in this field.

Random interval transformations I

3/11/17

Bartłomiej Żak

On the first seminar we've analysed a skew product system which consisted of two diffeomorphisms applied randomly on unit interval satisfying condition for interval interior $f_1(x) > x > f_2(x)$. We discussed what is a Lapunov Exponent on the interval borders. We distinguished 3 different behaviours of system depending on the Lapunov Exponents' relation with zero and shown probabilistic intuition behind it. We defined a stationary measure for the system and we claimed that it exists in the case of Lapunov Exponents being greater than 0 at both ends of the interval. To find ńontrivialśtationary measures we constructed a set of measures that don't have atoms in 0 or 1 with intent of finding interesting measures there.

How projections affect the dimension spectrum of fractal measures

10/11/17

Reza Mohammadpour Bejargafsheh

We presented 'How projections affect the dimension spectrum of fractal measures' by Brian R Hunt and Vadim Yu kaloshin.We started by defining t-Energy and Box dimension, Hausdroff dimension, local dimension and q-dimension(D_q) and gave some examples for them. Then for $0 \leqslant q < 1$ and $q > 2$ we gave examples for which D_q is not preserved by any linear transformation into $\mathcal{R}^m$.Finally we proved D_q is preserved by any linear transformation into $\mathcal{R}^m$ for $1 \leqslant q < 2$.

Random interval transformations II
17/11/17
Bartłomiej Żak

In the second seminar we showed that in a constructed set there exists a stationary measure, and we discovered how it looks. Thanks to its existence we proved that when applying functions with positive Lapunov Eponents, when the number of functions applied goes to infinity, all points except from one tend to 0 or 1. Finally we looked at the simple example where f_1, f_2 were homeomorphisms with Lapunov Exponents being greater then 0 and checked what is a stationary measure in this case.

Ergodicity and mixing for stationary processes: point processes approach
24/11/17
Łukasz Treszczotko

We presented a point-process approach to the study of the dependence structure for some stationary infintely divisible processes. The method is based on stydying the underlying structure of randome measures (Lévy noises) driving these processes. Lévy noises are continuum superpositions of Poisson noises and for some well known processes , e.g., linear fractional stable motion, this gives as a tool for checking mixing properties of the processes in question.

Mathematical problems of blood cells dynamics
1/12/17
Rafał Tryniecki

We considered a model of blood cells dynamics provided by Maria Wazewska-Czyzewska and Andrzej Lasota. We began with a general description of a hematopoietic system model with delayed feedback and deriving a system of differential partial equation describing this model. Then we analysed stationary solutions of this system of equations. In the next part, we analysed reduced model and have shown that in asymptotical solutions for healthy organisms and people with extremally strong hematophilia, it is not possible for long-term oscillations of the number of erytrocytes to appear. It is, however possible in case of strong hematophilia with extended time of blood cells maturing, but in this case oscillations cannot be permanent.

Synchronization properties of random piecewise isometries
8/12/17
Piotr Miszczak

We will investigate two results regarding synchronization properties in the case of random double rotations on the circle and a lack of it for its higher dimensions analogue, due to A. Gorodetski and V. Kleptsyn. We will explain what a synchronization means and give a proof of synchronization phenomena, at least in one dimension.

Multifractal analysis of the Birkhoff sums of Saint-Petersburg potential

15/12/17

Reza Mohammadpour Bejargafsheh

Let $((0,1], T)$ be the doubling map in the unit interval and ϕ the Saint-Petersburg potential defined by $\phi(x) = 2^n$ if $x \in (2^{-n-1}, 2^{-n}]$ for all $n \geqslant 0$. We considered asymptotic properties of the Birkhoff sum $S_n(x) = \phi(x) + \cdots + \phi(T^{n-1}(x))$. With respect to the Lebesgue measure, the Saint-Petersburg potential is not integrable and it is known that $\frac{1}{n \log n} S_n(x)$ converges to $\frac{1}{\log 2}$ in probability. We determined the Hausdorff dimension of the level set $\{x : \lim_{n\to\infty} S_n(x)/n = \alpha\}(\alpha > 0)$, as well as that of the set $\{x : \lim_{n\to\infty} S_n(x)/\Psi(n) = \alpha\}(\alpha > 0)$, when $\Phi(n) = n \log n$, n^a, or 2^{n^γ}, for $a > 1, \gamma > 0$.

Box counting dimension of the flexed Sierpiński gasket with non-equal division

12/01/18

Jan Kwapisz

We calculate the box counting dimension for the flexed Sierpń ski gasket, where in each iteration the division of the sides of triangles is unequal. This work is an extension of the article "Multifractal analysis on the flexed Sierpiński gasket"by Krzysztof Barański.

On the subadditive ergodic theorem

19/01/18

Klaudiusz Czudek

This talk was devoted to two very important theorems in ergodic theory. The first of them, Birkhoff's Ergodic Theorem, was proved in 1931. Birkhoff's proof was actually valid only in the case of dynamical systems arising from differential equations on manifolds. Several other proofs in general case were given later, for example by Wiener, Yosida and Kakutani in 1939, where the authors showed so called Maximal Ergodic Theorem. In 1968 Kingman showed the Subadditive Ergodic Theorem, which is a generalization of the above theorem.The first proof of the Kingman's Subadditive Ergodic Theorem relied on the Birkhoff Ergodic Theorem. The first proof, which does not rely on it was provided in 'Kingman's subadditice ergodic theorem', by Steele. Among the simplest proofs of Birkhoff's and Kingman's theorems that can by found in the literature we have those of Keane and Petersen, and Steele. The proof presented here, by Avila and Bochi, is based on these proofs and also does not rely on the Birkhoff's Ergodic Theorem. The sketch of the proof is the following. We define two functions:

$$f_b := \liminf_{n\to\infty} \frac{f_n}{n}, \quad f_u := \limsup_{n\to\infty} \frac{f_n}{n},$$

and the number:

$$L := \lim_{n\to\infty} \frac{1}{n} \int f_n = \inf_n \frac{1}{n} \int f_n,$$

and then we show that:

$$\int f_b \geqslant L \geqslant \int f_u.$$

The proof consists of two parts. In the first part we showed that $\int f_b = L$. This step is sufficient to show the Birkhoff's Ergodic Theorem but not to show the Kingman's. The idea is contained in the papers of Keane and Petersen, and Steele, and and this ingredient is not significantly new. Later we showed a tricky way how to deduce the inequality $\int f_u \leqslant L$ from the first part. This is essential step in the presented proof.

On the exceptional set for absolute continuity of Bernoulli convolutions

26/01/18

Łukasz Chomienia

In my talk I presented the paper by Pablo Schmerkin titled 'On the exceptional set for absolute continuity of Bernoulli convolutions' (arxiv:1303.3992), which was published in 2013. I began by showing the duality of the representation of self-similar measures. In my opinion, this could help the audience to develop better intuitions of the discussed objects. Next, I said a few words on the $\lambda \in (0, 1/2)$ case and I set out Riemann-Lebesgue lemma with some obvious corollaries. I also presented some less popular definitions and facts(chap.2.1). Then I looked closely to the first part of the important lemma(Lemma 2.1) about convolutions of measures whose Fourier transform has at least power decay. After that, I formulated Proposition 2.2 and proved Proposition 2.3. Later I moved on to the Hochman's theorem(Theorem 3.1) about dimension equality for analytically parametrized family of iterated function systems. Of course, I omitted the proof of that. Finally, equipped with all the necessary tools, I proved in details the main theorem (Theorem 1.2), which says that the set of exceptional parameters is of Hausdorff dimension zero.

Absolute continuity of non-homogeneous self-similar measures

02/03/18

Adam Śpiewak

During the talk I have presented results from a recent preprint by S. Saglietti, P. Shmerkin and B. Solomyak concerning properties of non-homogenous self-similar measures on the real line. The main result states that the invariant measure for the system of the form $(\lambda_1 x + t_1, ..., \lambda_k x + t_k)$ is absolutely continuous with respect to the Lebesuge measure for almost every $(\lambda_1, ..., \lambda_k)$ with similarity dimension strictly greater than one. The main difference with the homogenous case ($\lambda_1 = ... = \lambda_k$) is lack of the convolution structure of measures under consideration.

Commuting functions and the Fatou set

09/03/18

Vasiliki Evdoridou

It is known since 1920's that if f and g are rational functions which commute, i.e. $f \circ g = g \circ f$, then $F(f) = F(g)$. This was proved independently by Fatou and Julia. In this talk we discussed the same problem in the case where f and g are transcendental entire functions. In this case, this remains an open question. However, significant progress has been made in the past few years. We looked at earlier results by Baker, and Bergweiler and Hinkkanen which led to a partial answer to the question. We focused on the recent result of Benini, Rippon and Stallard, according to which if f and g are commuting transcendental entire functions that have no simply connected, fast escaping wandering domains, then $F(f) = F(g)$. Finally, we sketched the proof of this result, which uses previous results of Bergweiler, Rippon and Stallard on multiply connected wandering domains.

The differentiability of the hairs of $\exp(z)$
16/03/18
Łukasz Pawelec

We investigated a paper by M. Viana da Silva proving that any hair (a well-known object in the complex transcendental dynamics) of the map λe^z is a smooth curve. All the necessary definitions were given, together with the main ideas of the proof. This proof uses a natural construction of a series of homeomorphisms converging to the required parametrization. Then it was shown that the derivatives of any order of those maps converge uniformly.

Julia sets of random iterations of $z^2 + c_n$
23/03/18
Krzysztof Lech

At the seminar we discussed a paper by Bruck, Buger and Reitz, in which they present various conditions for the connectedness of non autonomous Julia sets of quadratic polynomials. The necessary and sufficient condition for connectedness was presented, followed by a discussion of what is known about total disconnectedness. We also looked at various examples from the paper. An example of a sequence of (c_n) such that the Julia set is disconnected but not totally disconnected was presented, and after that and example in which the Julia set is indeed totally disconnected. We finished by having a look at an open problem left by the authors: Is it true that if we randomize (c_n) from a large enough disk, then the Julia set will almost always be totally disconnected?

On the Hausdorff dimension of invariant measures of weakly contracting on average measurable IFS
6/04/18
Reza Mohammadpour Bejargafsheh

In this talk we give a contribution to the study of the multifractal properties of measures which are invariant for iterated function systems. In a paper by J. Myjak, T. Szarek the systems contracting on average and having Dini-continuous, separated from zero probabilities were considered and the upper bound of the Hausdorff dimension of the unique invariant distribution was given. A. H. Fan, K. Simon, H. R. Toth, have shown Hausdorff dimension of any of the possibly uncountably many invariant measures which the system is contracting on average in a sense which is wide enough to permit the existence of a common fix point at which some functions of the system are expanding and perhaps none of them are contracting. We are presenting an article by J. Jaroszewska and M. Rams who study iterated function systems without uniqueness of invariant distributions in this respect.

N-expansive homeomorphisms with the shadowing property
13/04/18
Welington Cordeiro

The dynamics of expansive homeomorphisms with the shadowing property may be very complicated but it is quite well understood (see Aoki and Hiraide's monograph, for example). It is known that these systems admit only a finite number of chain recurrent classes (Spectral Decomposition Theorem). In 2012, Morales introduced a generalization of the expansivity property, called N-expansive property. For every $N \in \mathbb{N}$, we will exhibit an N-expansive homeomorphism, which is not $(N-1)$-expansive, has the shadowing property and admits an infinite number of chain-recurrent classes. Therefore they cannot have a Spectral Decomposition theorem. We discuss some properties of the local stable (unstable) sets of N-expansive homeomorphisms with the shadowing property and use them to prove that some types of the limit shadowing property are present. This deals some direction to the problem of non-existence of topologically mixing N-expansive homeomorphisms that are not expansive.

Self-similar sets, entropy and additive combinatorics
20/04/18 *Łukasz Chomienia*

During the seminar I was presenting Micheal Hochman's paper śelf-similar sets, entropy and additive combinatorics", which was written to explain in more accessible way the ideas established by the author in his previous paper. I began with recalling some basic definitions like box dimension, covering number, etc. Then I derived the estimation for the box dimension by similarity dimension, introduced the measure of the closeness of cylinders and posed the weakened form of dimension drop conjecture in terms of asymptotic behaviour of cylinders. As an example of connection between this weakened form and original one, I showed that in the family of rational IFSs we have equivalence of both forms of conjecture. Here, I had planned to define some notions from binary tree theory, but due to the lack of time I decided to omit this part, draw a picture and relay on the audience intuition. I formulated the so called 'inverse theorem'(Theorem 2.3.) and then I moved to the third chapter of the paper. I defined discretization of self-similar set for given IFS and said how we can code it on the binary tree. Now I wanted to process by contradiction, so I assumed that there is strict inequality in dimension formula and that there is lower bound for concentration of cylinders. With such assumptions I discussed lemmas 3.1 and 3.2 and said about intuitive meaning of Propositions 3.3 and 3.4. Later, assuming existence of single interval which complishes both propositions, I wrote down

’Proposition’3.5. Now I could demonstrate that conclusion of ’Proposition’3.5 is false. I applied inverse theorem to discretized self-similar set and used ’Proposition’3.5. At that point I focused on controlling the branching of associated trees, so after I had mentioned Lemma 3.6. I used it to give the lower bound on the fully-branched nodes. I realized that I have only a few minutes left so I formulated Proposition 3.7, shortly said how we obtain the estimation on the probability of finding the node with many descendants and finally obtained lower bound on number of leaves which gave contradiction. At the end I said that to obtained the fully valid proof we need to replace sets with measures, covering numbers with entropy and follow analogous way in such new setting.

Continuity of the Hausdorff measure of continued fractions and countable alphabet iterated function system

27/04/18 *Rafał Tryniecki*

I will be talking about a paper written by A. Zdunik and M. Urbanski about Hausdorff measure of continued fractions. I will show that Hausdorff measure of the set Jn(G) of all numbers in [0,1], whose infinite continued fraction expansion have all entries in finite set 1,?,n satisfies limn??Hhn(Jn(G))=1, where hn is the Hausdorff dimension of Jn(G), and Hhn is corresponding Hausdorff measure. I will also show that this property is not too common, by constructing a class of IFS, such that upper limit of Hausdorff measure in corresponding Hausdorff dimension is smaller than the measure of the limit set.

Random interval transformations

4/05/18

Bartłomiej Żak

On the third seminar we’ve analysed the case of zero Lyapunov Exponent at point 0. We’ve shown, that there exist a neighbourhood of 0 that each trajectory of a random walk defined by our random iterates of our diffeomorphisms with starting in this neighbourhood, leaves it in finite time. Then we took a assumption that Lapunov Exponent is greater than zero at point 1 and proved that in that case, for any neighbourhood of zero expected time of leaving it is infinite and expected time of re-entering it is finite.

Weak convergence to stable Lévy processes for nonuniformly hyperbolic dynamical systems

18/05/18

Łukasz Treszczotko

We consider weak invariance principles (functional limit theorems) in the domain of a stable law. A general result is obtained on lifting such limit laws from an induced dynamical system to the original system. An important class of examples covered the result are Pomeau?Manneville intermittency maps, where convergence for the induced system is in the standard Skorohod J1 topology. For the full system, convergence in the $J1$ topology fails, but we prove convergence in the $M1$ topology

Eternal cosmological inflation, dynamical systems approach

25/05/18

Jan Kwapisz

Inflation started its career in the 80ties when it turned out that classical cosmology picture struggles with fine tuning of matter density i.e. very uniform distribution of matter at the time of last scattering in circa 1084 disconnected regions. The Universe must have started its history with a very special initial state. Guth [1] and Starobinsky [2] and argued that the possible solution to this problem exists without fine-tuning the initial conditions. If one assumes that at the very early stage of expansion of the Universe there was an accelerated expansion era, which eventually went into the decelerated FLRW epoch, then the fine tuning problem is solved. This solution of the problem is called inflation but it introduced an issue of its own initial conditions what is under active investigation nowadays. There were many mechanisms proposed and discussed to generate the $\ddot{a} > 0$ epoch just after the Big Bang. In my talk, which will be based on [3], I will discuss one special case: eternal inflation within dynamical systems approach.

References:
[1] A. H. Guth, *Inflationary Universe: A possible solution to the horizon and flatness problem*, Physical Review D, 23 (1991). access on 29.12.2016:
http://journals.aps.org/prd/pdf/10.1103/PhysRevD.23.347.
[2] A. Starobinsky, *A new type of isotropic cosmological models without singularity*, Physics Letters B, 91 (1980), pp. 99 - 102.
[3] V. Vanchurin, *Dynamical systems of eternal inflation: a possible solution to the problems of entropy, measure, observables and initial conditions*, Phys. Rev., D86 (2012), p. 043502.

Invariant measures for a class of cocycles

1/06/18

Michał Rams

We consider a step one cocycle of two homeomorphisms of interval $[0,1]$ with the following properties: $f_0(0) = 0,\ f_0(1) = 1,\ f_0(x) > x$ for $x \in (0,1)$ and $f_1(0) = 1,\ f_1(1) = 0$. We investigate the space of ergodic invariant measures for this system. Clearly, it consists of two subsets: measures supported on $\{0,1\} \times \Sigma_2$ and measures supported on $(0,1) \times \Sigma_2$, and we want to know which measures in the former set can be weak* aproximated by measures in the latter set. We will also investigate the uniqueness of the measure of maximal entropy. This is a joint work with L. Diaz, K. Gelfert, T. Marcarini.

On the derivative of the Hausdorff dimension of the quadratic Julia sets

8/06/15

Ludwik Jaksztas

Let $d(c)$ denote the Hausdorff dimension of the Julia set $J(z^2 + c)$. We will investigate the derivative $d'(c)$, for real c converging to a parabolic parameter c_0. First, we will prove that $d'(c)$ tends to infinity, when $c \nearrow \frac{1}{4}$. Next, we will see that $d'(c)$ tends to a constant or minus infinity depending on the value $d(c_0)$, where c_0 is a parabolic parameter with two petals.

Hausdorff dimension of the escaping set for a family of meromorphic maps
15/06/18
Piotr Gałązka

Escaping set is a set of points which tend to infinity under iterates of a map. During the talk we will see how large the Hausdorff dimension of the escaping set is for maps from the family $R \circ \exp$, where R is a non-constant rational map.