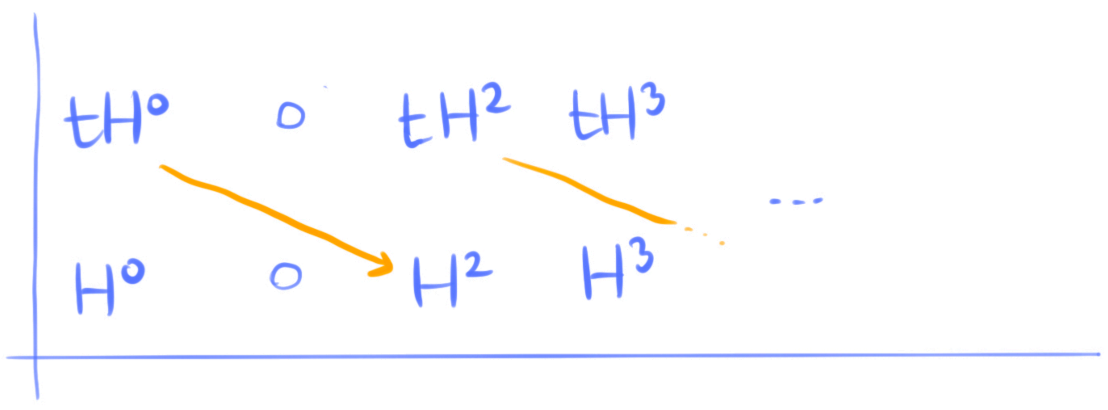
$$\underbrace{H^q(S^1)}_{\mathbb{Z}[t]/(t^2)} \otimes \underbrace{H^p_T(X)}_{H^p} \Rightarrow H^{p+q}(X)$$

$$\begin{tikzcd} S^1 \arrow[r] \arrow[d] & S^\infty \times X \arrow[r] \arrow[d] & S^\infty \times_{S^1} X \arrow[d] \\ S^1 \arrow[r] & S^\infty \arrow[r] & \mathbb{CP}^\infty \end{tikzcd}$$



$dt = a \in H^2$

$\forall b \in H^i \quad db = 0$

$d(tb) = bdt \pm tdb = ba$

So d is multiplication by a.

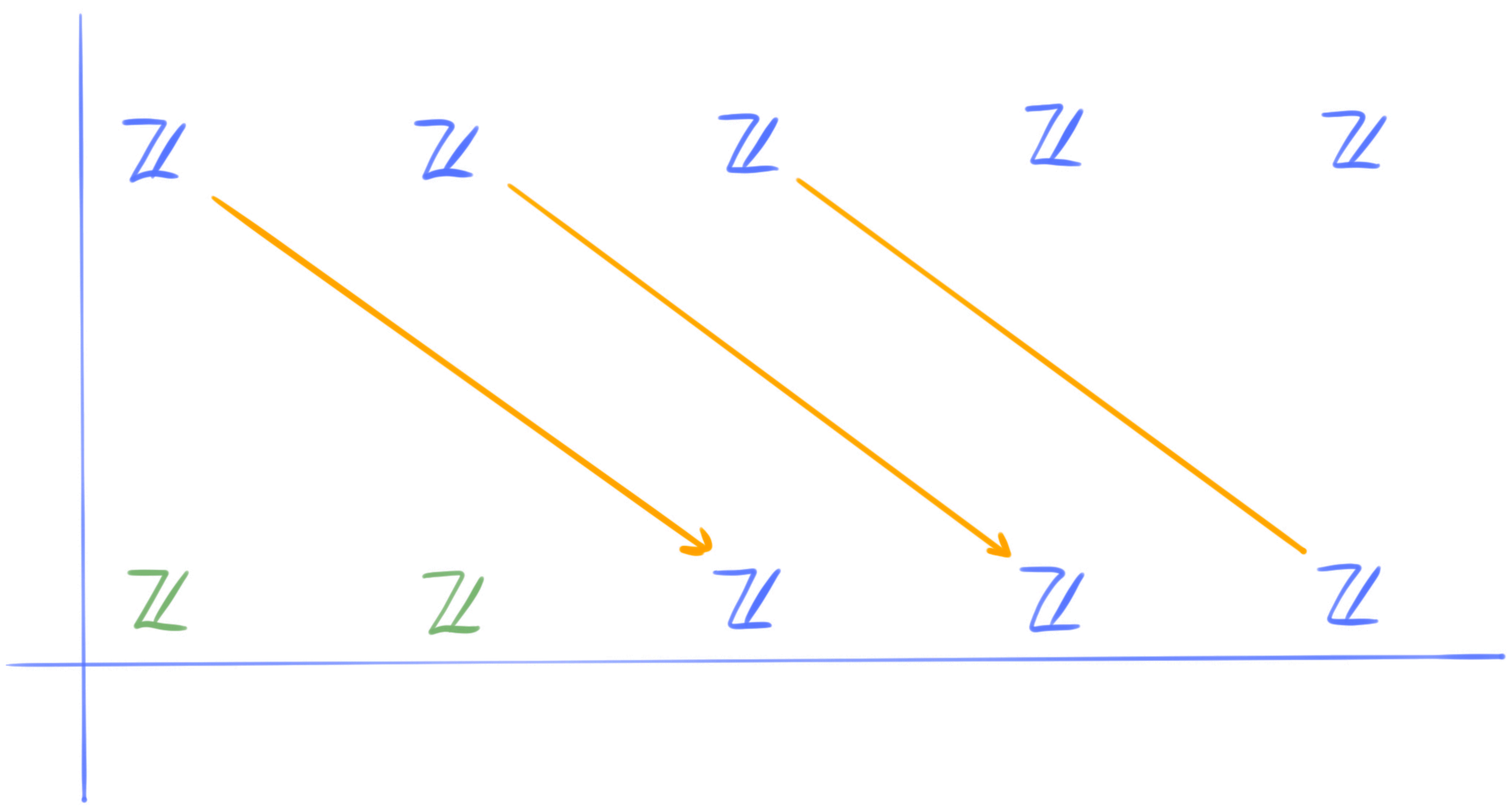
But $a = dt$ also under pullback of $X \to pt$,

so can be identified with non-zero element of $H^2(BT)$

as in sequence of $X = pt$ this map is $\simeq$ $(E^\infty_{p,q} = 0$ for $(p,q) \neq (0,0))$

$$EV \Rightarrow d \text{ non-zero} \Rightarrow H^{odd} = 0$$

3. $$S^3 \longrightarrow \mathbb{E} \times^T S^3 \longrightarrow \mathbb{CP}^\infty$$



action free $\Rightarrow H^\bullet_T(S^3) \simeq H^\bullet(S^3/S^1) \simeq H^\bullet(S^2)$ (Hopf fibration)

so all drawn arrows must be $\simeq$ to get convergence to $H^\bullet(S^2)$

4) The magic square is equivalent to diagram of fibrations

$$\begin{tikzcd} X \times G \arrow[r, "\eta"] \arrow[d, "\pi"] & X \arrow[d] \\ X \arrow[r] & {[X/G]} \end{tikzcd}$$

$$\begin{tikzcd} T \times X \arrow[r] \arrow[d, "\eta"] & \mathbb{E} \times X \arrow[r] \arrow[d] & \mathbb{B} \arrow[d] \\ X \arrow[r] & \mathbb{E} \times^T X \arrow[r] & \mathbb{B} \end{tikzcd}$$

$$\begin{tikzcd} X \times T \arrow[d, "\eta"] & \mathbb{E} \times^T (X \times T) \simeq (\mathbb{E} \times^T T) \times X \simeq \mathbb{E} \times X \arrow[d] \\ X & \mathbb{E} \times^T X \end{tikzcd}$$

so the transgressions are induced by Gysin pushforward of η

$$H^i_T(X \times T) \longrightarrow H^{i+2}_T(X)$$