Parikh One-Counter Automata (POCA)

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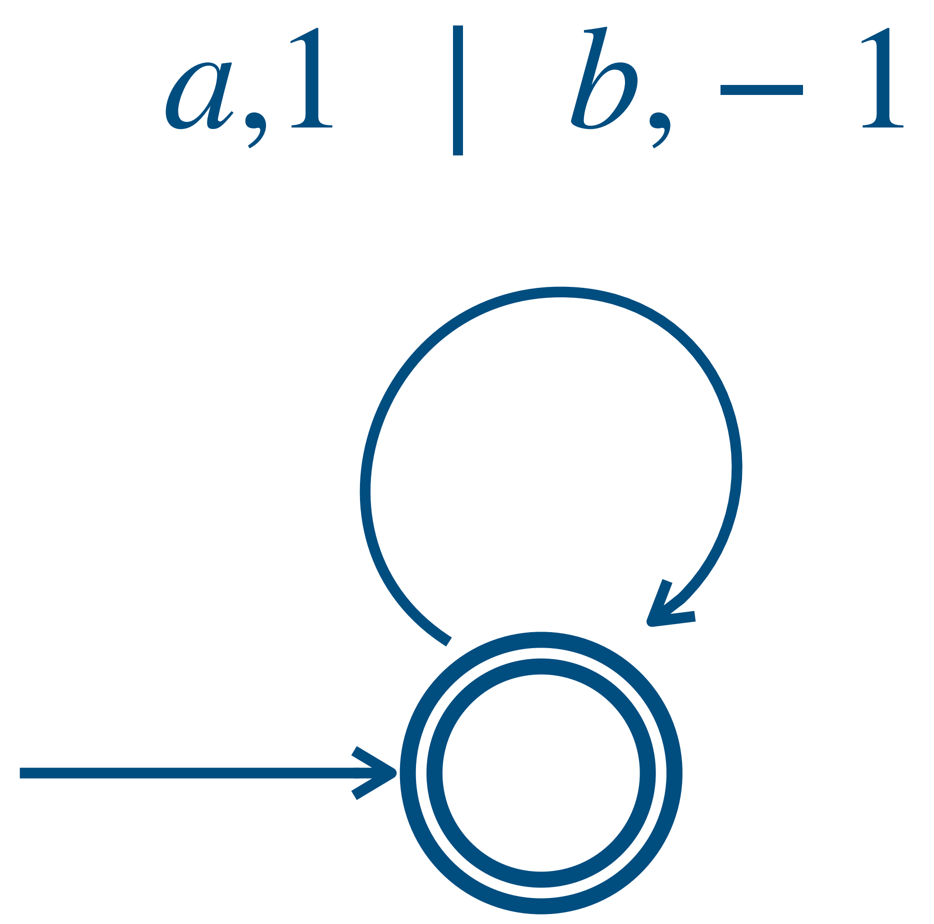
Example

$$L = \{x \mid \#_a(y) > \#_b(y) \text{ for any prefix } y \text{ of } x, \text{ and } \#_a(x) = 2 \cdot \#_b(x)\}$$

$A =$



$$\varphi = (t_a = 2 \cdot t_b)$$

$$M = (A, \varphi)$$

one-counter automaton

Presburger formula with variables T = set of transitions of A

accepting run of M = accepting run of A satisfying φ

expressivity vs complexity

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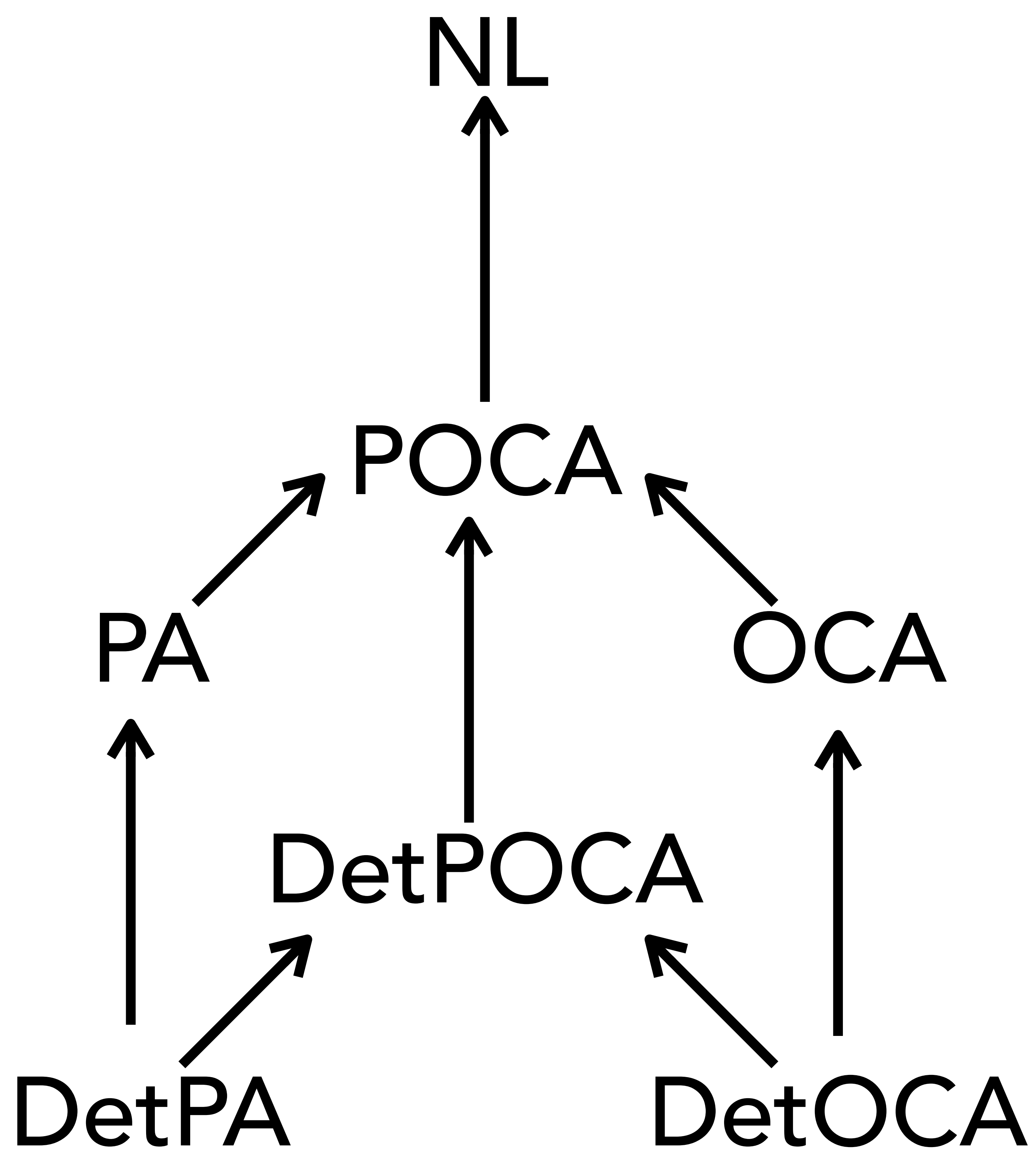

PA = Parikh automata

OCA = one-counter automata

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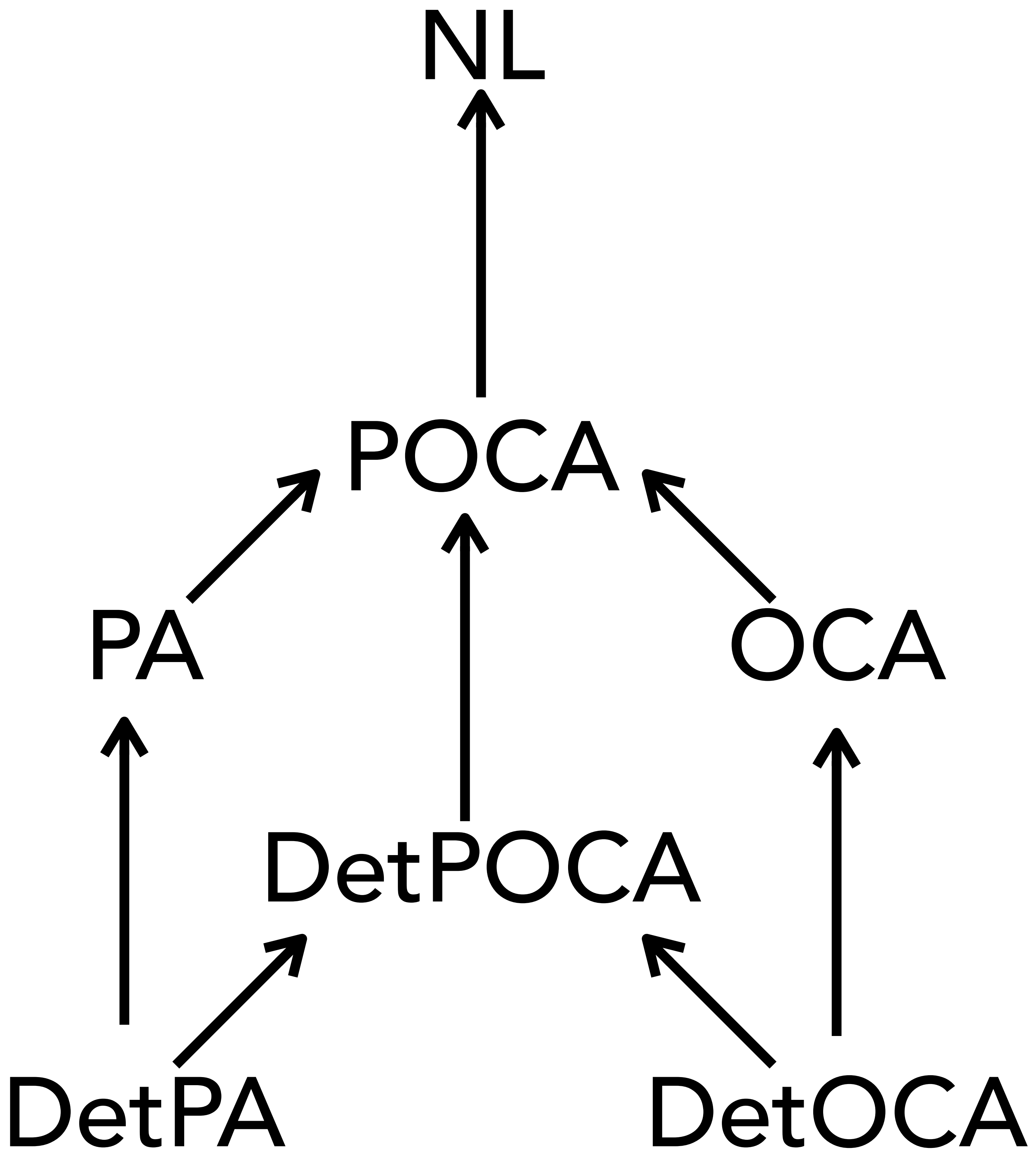

	$L \neq \emptyset$	$L \neq \Sigma^*$	$L_1 \subseteq L_2$	$L_1 = L_2$
DetPA	NP-c	coNP-c	coNP-c	coNP-c
DetOCA	NL-c	NL-c	Undec	NL-c
DetPOCA	NP-c	coNP-c	Undec	?
PA	NP-c	Undec	Undec	Undec
OCA	NL-c	Undec	Undec	Undec
POCA	NP-c	Undec	Undec	Undec

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Closure properties

	$\cup$	$\cap$	^{c}	$\cdot$	h	h^{-1}
DetPA	Y	Y	Y	N	N	Y
DetOCA	N	N	Y	N	N	Y
DetPOCA	N	N	Y	N	N	Y
PA	Y	Y	N	Y	Y	Y
OCA	Y	N	N	Y	Y	Y
POCA	Y	N	N	Y	Y	Y

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Depumping lemma :

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$L \subseteq \Sigma^*$, a POCA language

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there exists words $u, u_1, \ldots, u_n, v, v_1, \ldots, v_{m+1}, m, n \geq N$ such that

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1. $x = (u_1 u) \cdot (u_2 u) \cdot \ldots \cdot (u_n u) \cdot (v_1 v) \cdot (v_2 v) \cdot \ldots \cdot (v_m v) \cdot v_{m+1} \in K$

2. $uv \neq \varepsilon$

3. there exists $w_1 w_2 \in \Sigma^*$ such that $w_1 x' w_2 \in L$ where,

$$x' = u_1 \cdot \ldots \cdot u_n \cdot v_1 \cdot \ldots \cdot v_{m+1}$$

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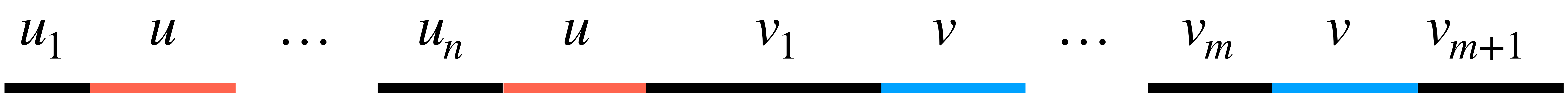

$m, n \geq N$

$\in$

K

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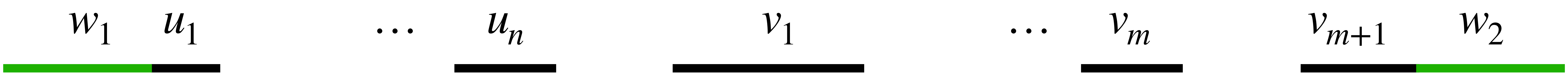

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Application of the depumping lemma :

$\{\#a^n b^n c^n\# \mid n \in \mathbb{N}\}^*$ is not a POCA language

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Take $K = \{\#a^n b^n c^n \# \mid n \in \mathbb{N}\}$ and $N = 3$

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Parameters can be used to describe behaviours of the POCA

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$\forall$ valuations of the parameters the resulting POCA is non-empty

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Next steps

1. Parikh pushdown automata
2. Decidability of equivalence of detPOCA
3. Decidable restrictions of parametric universality

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Depumping lemma for detPOCA:

$L \subseteq \Sigma^*$, a POCA language

For any infinite $K \subseteq [\varepsilon]_L$ and $N \in \mathbb{N}$

$$u_1 \, u \; \dots \; u_n \, u \, v_1 \, v \; \dots \; v_m \, v \, v_{m+1}$$

$$\Cap$$

$$K$$

$m, n \geq N$

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