Report on
"Coherent distributions on the square"

The paper studies extreme points of coherent distributions on $[0,1]^2$. A distribution μ is called coherent if there is a probability space (Ω, F, P), an event E, and sub-sigma-algebras F_1 and F_2 such that μ is the distribution of (X_1, X_2), where $X_i = P[E \mid F_i]$. Such distributions have attracted interest in probability theory and economics as they capture the opinions of two experts with access to different information.

Arieli et al. (2021) and Zhu (2022) initiated the study of extreme points of coherent distributions. The paper under review makes substantial progress in this direction by characterizing the extreme points with finite support. Based on this characterization, the authors develop a new technique showing that such finite-support extreme points are enough to understand the coherent distributions maximizing a certain linear functional. As an illustration, the limiting behavior of the maximal value of $\int |x-y|^\alpha d\mu(x,y)$ over coherent μ is studied for $\alpha \to \infty$.

This paper is a rare case where I have nothing to complain about as a reviewer. The question studied is interesting and is also motivated by the current literature. The analysis is elegant and insightful. The paper is written engagingly and clearly. Moreover, I could not even spot any typos. I recommend **acceptance**.

Minor comments

- The reader would benefit from a discussion of which results of the paper have the potential to be generalized to $n \geq 2$ experts and/or different linear functionals. For example, Theorem 1.8 seems to extend to arbitrary n straightforwardly. Proposition 4.4 seems to extend beyond $|x-y|^\alpha$ to arbitrary $f(x,y)$ under some convexity assumptions on f. Rather than reshaping the already written parts, adding a short extra section with a discussion of the potential extensions would be enough. This is a minor comment, and my acceptance recommendation is not contingent on it.

- The definition of minimality on page 3 is clear but not very intuitive. Adding a short discussion of the intuition behind it and/or an illustrative example where it is satisfied/violated would be helpful.

- At the beginning of Section 4, say that X_n and Y_n take at most n values. The reader can guess it from the context, but it is better to be explicit.