OPERATOR SEMIGROUPS IN THE CALKIN ALGEBRA (PART 1)

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Let $\mathcal{H}$ be an infinite-dimensional separable Hilbert space; $\mathcal{B}(\mathcal{H})$ and $\mathcal{K}(\mathcal{H})$ stand for the algebras of all bounded (respectively: compact) linear operators on $\mathcal{H}$. In the first part of the talk, we shall explain the interplay between operator semigroups in the Calkin algebra $\mathcal{Q}(\mathcal{H}) = \mathcal{B}(\mathcal{H})/\mathcal{K}(\mathcal{H})$ and the Brown–Douglas–Fillmore theory of extensions. Recall that the central notion of the BDF theory is the extension group $\mathrm{Ext}(X)$, where X is a compact metric space, consisting of (equivalence classes of) short exact sequences in the category of C*-algebras of the form

$$0 \longrightarrow \mathcal{K}(\mathcal{H}) \stackrel{\iota}{\longrightarrow} \mathcal{A} \stackrel{\varphi}{\longrightarrow} C(X) \longrightarrow 0.$$

We show that with any normal C_0-semigroup $(q(t))_{t\geq 0}$ in $\mathcal{Q}(\mathcal{H})$ one can associate an extension $\Gamma \in \mathrm{Ext}(\Delta)$, where Δ is the inverse limit of certain compact metric spaces defined purely in terms of the spectrum $\sigma(A)$ of the infinitesimal generator of $(q(t))_{t\geq 0}$. We call inverse limits arising in this manner *admissible*, and we study their topological properties from the point of view of strong/uniform continuity of the semigroups they generate. Observe that when deciding whether a semigroup $(q(t))_{t\geq 0} \subset \mathcal{Q}(\mathcal{H})$ is a C_0-semigroup (i.e. continuous in the strong operator topology), we obviously need to pick a concrete faithful *-representation of $\mathcal{Q}(\mathcal{H})$ on a Hilbert space $\mathbb{H}$ (necessarily of density at least $\mathfrak{c}$). Our choice is to focus on 'canonical' Calkin's representations induced by limits with respect to ultrafilters. We will provide a characterization of admissible compact metric spaces X that correspond to C_0-semigroups, as well as a sufficient condition on X guaranteeing that all semigroups $(q(t))_{t\geq 0} \subset \mathcal{Q}(\mathcal{H})$ corresponding to X satisfy the automatic continuity phenomenon: whenever they are strongly continuous, they must be uniformly continuous.

The talk will be mostly based on the preprint: *Compact perturbations of operator semigroups*, `arXiv:2203.05635v2`

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