

A twisting of $C(K)$ -spaces scrapbook

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A twisted CK-space is a Banach space X admitting a subspace isomorphic to some space $C(K)$ of continuous functions on a compact space such that the corresponding quotient is isomorphic to some space $C(S)$. Namely, the middle space in an exact sequence

$$0 \longrightarrow C(K) \longrightarrow X \longrightarrow C(S) \longrightarrow 0$$

We will also say that X is a twisting of $C(K)$ and $C(S)$. Twisted CK spaces inherit many properties of true $C(K)$ spaces, even if they do not have to be even Lindenstrauss. During the talk we will explore:

- ▶ The basic properties of twisted CK spaces.
- ▶ The appearance of twisted CK spaces in nature.
- ▶ How to twist $C(K)$ and $C(S)$? If the twisting is $C(M)$, how to recover M from K and S ?
- ▶ How far can twisted CK spaces be from true $C(K)$ -spaces?
- ▶ The CCKY problem, Koszmider's problems, the Complemented Subspace Problem, ...