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## Review of doctoral thesis by Jakub Koncki

In his PhD dissertation “Motivic Chern classes and stable envelopes” the author studies cohomological K-theoretic invariants of BB-cells. This subject has a very long history and is an important part of contemporary mathematics. The author introduces a notion of *twisted motivic Chern class*, which generalizes the definition of motivic Chern classes studies previously by other mathematicians. In addition to the previously studied classes, the new construction depends on a choice of a fractional Cartier divisor. The twisted classes coincide with the usual motivic Chern classes for the trivial choice of the divisor.

The definition of the twisted classes involves a resolution of singularities. One of the important results in the thesis is that the constructed classes are, in fact, well defined and are independent on the involved choices of resolutions.

The main goal of the thesis is to prove that the defined twisted Chern classes coincide with the K-theoretic stable envelope classes introduced by A. Okounkov and his coauthors. The choice of the divisors, i.e., the twist parameter for the twisted Chern classes plays the role of the slope parameter in the Okounkov’s construction. The proof of the equivalence given by the author is by a direct check that the twisted motivic Chern classes satisfy the axioms of stable envelopes, which determine this classes uniquely.

In addition the thesis contains an introductory section on equivariant K-theory and an appendix section on stable envelopes where the necessary technicalities are reviewed.

To conclude, this work meets all standards of academic work and is sufficient to grant a PhD degree. Moreover, I see this thesis a mature work and an important contribution to the field. The results of the thesis have been already published in a refereed mathematical journals. I think the present work exceeds the expectation of a PhD thesis and I’m happy to recommend it for an **honorary distinction award**.

Yours very truly,  
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