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Report for the submitted PhD thesis entitled "Tame the Infinite - Simplification Problems For Infinite-State Systems" by Mr Radosław Piórkowski

Scientific background

When modeling computer programs using a finite presentation a central trade-off problem arises. On the one hand the underlying model should be as expressive as possible in order to model the actual system as precisely as possible. On the other hand, the model itself should be algorithmically manageable such that algorithmic verification tasks, such as for instance safety verification, are performable by computer programs themselves. Indeed, the more expressive a mathematical model is the less likely it is to be suitable for algorithmic manipulation — conversely, the easier it is to algorithmically deal with a given mathematical model, the less likely it is to be highly expressive.

Infinite-state systems comprise a successful mathematical means to generically model computer programs. Different infinite-state models have been introduced in the last forty years with different expressiveness and algorithmic properties. Different sources of infinity arise. For instance, in order to model the behavior of real-time systems automata that are equipped with clocks (so-called timed automata) have turned out to be extremely useful and sucessful. A second prominent example of infinite-state systems are automata that are equipped with counters ranging over the integers — these so-called vector addition systems (with states) have turned out to have a plethora of applications for modeling various phenomena in computer science.

This dissertation studies *simplification problems* for infinite-state systems, a very simply-stated algorithmic question. It asks, whether a given infinite-state system admits a behavior that is of simpler nature. It is worth pointing out that on the one hand, such questions are already quite classic, dating back to Rice's theorem or various regularity problems stated in the 1960's and 1970's. On the other hand, and unfortunately, it seems fair to say that simplification problems are not that well understood to date. Indeed, some such early-stated simplification problems are still not known to be decidable. The submitted thesis of Radosław Piórkowski concretely focusses on the following more concrete instances of the simplification problem:

1. given a complex specification, does there exist a simpler device realizing it (synthesis problem)?
2. can disjoint languages of complex models be distinguished (separated) in a simpler way?
3. can a complex device be equivalently described in simpler terms?
4. is a complex system bound to contain a simpler, more structured one?

The investigated infinite-state systems that are studied in this thesis are various classes of register automata, various classes of timed automata, and two-dimensional vector addition systems with states.

Content of the thesis

The submitted thesis consists of ten chapters.

In the first chapter the different infinite-state models are introduced, namely register automata, timed automata and vector addition systems with states. Moreover, the above-mentioned problems are introduced in more detail, but not yet fully formally. The chapter is intended to give the reader an overview what this thesis will be about. Based on these more detailed descriptions, the main results are listed and brought into context with the existing literature.

Chapter two introduces the general mathematical notation, most notably labeled transition systems. In a formally rigorous manner both the underlying infinite labeled transition systems and the studied decision problems are formally introduced.

Extending Church's synthesis problem, the *generalized synthesis game* is content of chapter three: it extends Alice's possibilities in that her moves now consist of choosing letters over an infinite alphabet, whereas Bob's moves remain to be letters ranging over a finite alphabet — accordingly, Alice's winning condition is specified by an ω-language over a suitably chosen infinite alphabet. The *generalized synthesis problem* asks whether for a given such game Bob has a winning strategy that can described by simple controller, i.e. a deterministic transducer of a specific kind. Different instantitiations of the generalized synthesis game and according computation problems are introduced: register synthesis game/problem and timed synthesis game/problem with various requirements on the controller.

Chapter four introduces and motivates separability problems via a reduction to the above-mentioned generalized synthesis problem. The separability problem asks whether the languages of two given nondeterministic register automata can be separated by a language of some determinisitic register automaton using k registers, where k itself is also part of the input. A related question that is studied in the chapter is whether the languages of two given timed automata can be separated by a deterministic timed automaton using k clocks, where again k is part of the input. A final variant of the latter problem is the one, where in addition to the maximal number of clocks k a number m is given an input to the problem, where m is an upper bound for the numbers that may appear in guards of the sought deterministic timed automaton.

Chapter five shows decidability of the question, if for a given $k \in \mathbb{N}$ and a given register synthesis game over a homogeneous structure of atoms, Bob has a winning strategy that can be specified by a controller in terms of a deterministic register automaton using at most k registers. Moreover, such a controller is shown to be computable in case it exists. Via a sequence of reductions the problem is reduced to the well-known Büchi-Landweber theorem (stating that the Church synthesis problem is decidable).

An analogous result as the one for register automata in chapter five is solved in chapter six: for a given $k \in \mathbb{N}$ and a given timed synthesis game the question whether there exists a controller for Bob (and computing one if it exists) in terms of a deterministic timed automaton using k clocks is decidable. The approach again reduces the problem to the Church synthesis problem, this time however requiring a technically more involved machinery involving several reduction steps. A suitably tailored variant of the region construction for timed automata is developed.

The main result of chapter seven states that it is decidable, given some $k \in \mathbb{N}$ and a (nondeterministic) register automaton using one register only, whether its language can be accepted by some deterministic register automaton using at most k registers.

The proof is shown when the underlying infinite domain is the set of naturals numbers with equality. A discussion is given in that the result can be extended to further relational structures of atoms, as for instance to well-quasi ordered structures. Moreover, the complexity is shown to be Ackermann-complete. The upper bound proof uses techniques from the theory of atoms and makes use of orbit-finiteness constructions.

A similar result is shown for timed automata: it is shown that given some $k \in \mathbb{N}$ and some nondeterministic timed automaton over a single clock, it is decidable if its language is already recognizable by a deterministic timed automaton using at most k clocks — also if an upper bound on the maximally possible number appearing in guards is additionally given as part of the input or not: both problems are decidable. Again orbit-finiteness observations are a crucial ingredient to the proof, this time however combined with an interplay with a carefully tailored region construction. The upper bound result is complemented by a matching Hyper-Ackermannian lower bound.

Matching lower bounds are proven. In particular, it is shown that slight generalizations of the problem (when the timed automaton allows ε-transitions or has two clocks) are undecidable. Via reduction from the undecidable universality problem, generic conditions are developed for a general class of timed languages satisfying some natural effective closure properties.

Various undecidability results are shown in chapter eight. Concretely it is shown that given some nondeterministic register automaton with one register, the question if its language is already the language of some deterministic register automaton is undecidable. So is the register synthesis problem already undecidable when Alice's winning condition is given by a register automaton with two registers.

When deciding if a nondeterministic register automaton with one register already accepts a language of by some deterministic register automaton undecidability already holds when the register automaton does not have any ε-transitions and also if a bound for the maximal number appearing in a guard is additionally part of the input to the problem. Finally, it is shown that the timed synthesis problem is undecidable in case Alice's winning condition is given by a nondeterministic register automaton with only two registers. These undecidability results all complement the previous upper bound results and give a clear decidability frontier.

Chapter nine shows a structural theorem on runs of two-dimensional vector addition systems with states. A dichotomy is shown in that for some suitably calculated exponential bound B, any run is either B-thin or B-thick, where the notions of B-thick and B-thin are ones that are newly being introduced here, to the best of my knowledge. The proof takes several steps and involves arguments that are both of geometrical and of combinatorial flavor.

Applications of this result are given. Firstly, a pumping lemma for two-dimensional vector addition systems with states is shown: in every sufficiently long run starting and ending in configurations in which both counters are zero one can find cycles that can simultaneously be pumped and still constitute a valid run from the same source and target configurations. A second application is given, namely that in any two-dimensional vector addition system with states, in case there is a run between two configurations, there already a short run. The whole section makes use of carefully defining suitable geometric notions.

Chapter ten finally concludes with a a short overview open problems.

Assessment of the submitted thesis

It was a pleasure and an honor to read this PhD thesis.

The subject of this thesis, namely simplication problems of infinite-state sytems, is both a classical subject and a subject that is very much in the center of current research. The studied problems arise naturally and yield clean extensions of existing problems, such as the Church synthesis problem or the separability problem. The studied models are all models of high interest: timed automata, register automata and vector addition systems with states are currently all under the high focus of the automata/games/formal language community. Therefore I assess the motivation of this thesis as excellent.

The results of this thesis are technically very demanding. Indeed, the techniques developed and applied in this thesis come from various areas: theory of atoms, theory of games, formal languages theory, geometry, and combinatorics. This is very impressive and indeed constitutes an unusual breadth for a PhD thesis. Equally impressive is the fact that this thesis gives a beautiful and complete complexity and decidability landscape — in particular the decidability borders have carefully been worked out. Put in one sentence: this thesis conveys a clear story.

The writing style is truly excellent and mathematically absolutely rigorous and mature. I liked the fact that the author has found a perfect balance between high-level explanation and mathematical precision, sometimes using repetitions, but repetitions that perfectly yield a nice reading flow — a reading flow that is, unfortunately, often missing in PhD theses. The typesetting is a real feast for the eyes and very inspiring. I am convinced that my few typesetting remarks can easily be implemented in a short period of time.

Wrapping up, I assess this thesis to be sufficient to grant a PhD. I propose to award a PhD degree with honorary distinction.

Kassel, August 26, 2022

Prof. Dr. Stefan Göller