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Report on the doctoral dissertation

Stochastic Volatility in Selected Models of Financial Markets

by Zofia Miśkiewicz

submitted for the degree of Doctor of Philosophy

The thesis by Zofia Miśkiewicz addresses the issue of numerically efficient approaches to pricing of financial derivatives, with the special emphasis on mathematical methods and numerical algorithms developed for the purpose of option pricing in models of stochastic volatility and Markov-switching models. Before making more detailed comments, let me observe that the thesis is well written, gives a fairly comprehensive analysis of theoretical and numerical approaches, and proposes several original generalisations of methods known from the existing literature. The author demonstrated very good knowledge of the existing literature and proved her familiarity with both theoretical and numerical aspects of the considered models and problems arising in option pricing. The technical quality and the style of the thesis are very good. Since the main emphasis of the research presented in the thesis is on numerical schemes, it is worth stressing that theoretical results are supported by numerical experiments.

Chapters 1 and 2 are devoted to an overview of issues related to financial modelling with the special emphasis on options pricing in stochastic volatility models.

In Chapter 3, the author focuses on option pricing for an extension of the classical Stein and Stein (1991) model of stochastic volatility, which is obtained by relaxing the assumption made in the original paper that the Brownian motions driving the dynamics of the stock price and its volatility are independent. The main tools for computation of prices of European options are formulas for the moments of the stock price and Laplace or Mellin transform. Closed-form expressions for the moments of the stock price are obtained in Theorems 3.2.1, 3.2.3 and 3.2.6 under various assumptions about model parameters. The crucial observation used in the proofs is that the square of the volatility process is a squared radial Ornstein-Uhlenbeck process. The next step is the computation of the Mellin transform where the result from the paper by Schöbel and Zhu (1999) is extended in Theorem 3.2.8. Subsequently, the author applies her results in Section 3.3.1 to option pricing through three alternative methods and presents in Section 3.3.2 results of several numerical experiments.

Chapter 4 deals with a completely different modelling approach, which hinges on inhomogeneous time change technique for Markov chains (as opposed to general Markov processes). After first establishing in Theorem 4.2.3 the existence and uniqueness of a solution to time change equation, the author studies in some detail the properties of the time-changed Markov chain.

In my opinion, the most interesting problem examined in Chapter 4 is the question of preservation of the strong Markov consistency property under a time change, which is studied in Section 4.3. In essence, the strong Markov consistency property of a multi-dimensional Markov process means that each of its coordinate processes enjoys the Markov property with respect to the filtration of the whole process. The main results are here Theorem 4.3.1 and its Corollary 4.3.5, which provide equivalent conditions for the time-changed process to enjoy the preservation of its strong consistency property. The author introduces also a novel concept of a *quasi Markov consistent* process, that is, a Markov chain that gains a strong Markov consistency property after a non-trivial time change and gives in Proposition 4.3.8 an explicit sufficient condition for quasi consistency of a Markov chain.

Finally, the author studies in Section 4.4 various applications of time change technique to modelling of stock prices as a regime-switching diffusion (see, in particular, Theorem 4.4.4), as well as to the mathematical problem of representing coordinates of a two-dimensional regime-switching diffusion through a time-changed diffusion. Sufficient conditions for the existence of a solution to the latter problem, which is also important in practical applications, are provided in Theorem 4.4.7. Similarly to Chapter 3, theoretical results obtained in Chapter are illustrated in Section 4.5 by means of Monte Carlo simulations of sample paths of time-changed asset prices and option prices.

Summary

Mathematical contributions of the research presented in the thesis by Zofia Miśkiewicz are twofold. First, she examines several existing stochastic volatility models and provides a detailed analysis of existing pricing methods for complex financial derivatives. Second, she proposes and studies in detail a large variety of original methods and argues that in many instances they outperform previously known approaches. In my opinion, in both respects the author succeeded to provide a sufficiently detailed analysis of numerical techniques and their applications to specific pricing problems in the framework of stochastic volatility and regime-switching diffusion models and her original contributions are clearly presented and of a high standard.

Conclusions

The results derived in the thesis are likely provide sound foundations for further modelling work in the area of modelling of stochastic volatility and regime-switching financial models. I note that some of the results presented in Chapters 3 and 4 have already been published in international journals. The thesis is written very clearly and represents an important contribution to the areas of financial mathematics and financial applications. The research of Zofia Miśkiewicz presented in her doctoral dissertation is valuable from the mathematical viewpoint and it brings essential contributions to solving practical problems arising in the area of pricing and hedging of financial derivatives.

Therefore, I recommend Zofia Miśkiewicz be awarded the degree of Doctor of Philosophy for her doctoral dissertation *Stochastic Volatility in Selected Models of Financial Markets.*

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