Topological paradoxical decompositions and partial actions

Pere Ara

Universitat Autònoma de Barcelona

Email: para@mat.uab.cat

Abstract: The *Banach-Tarski Paradox* asserts that the unit ball of $\mathbb{R}^3$ is paradoxical with respect to the action of the group of all the isometries of $\mathbb{R}^3$. I will recall the general notion of paradoxicality for an action of a (discrete) group G on a set X. An important result in the area is Tarski's Theorem, that establishes a dichotomy result for an action $G \curvearrowright X$ as above: Given a subset E of X, either E is paradoxical or there is an invariant finitely additive measure μ on $\mathcal{P}(X)$ such that $\mu(E) = 1$. The question of whether such a dichotomy holds for actions of groups by homeomorphisms on topological spaces was raised in [3] and [4]. I will recall a result from [1] showing that Tarski's dichotomoy does not extend to this setting. This uses, amongst other things, the theory of partial actions, see [2].

References

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