

# FAIRNESS IN RESOURCE ALLOCATION

**Nidhi Rathi**

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# Fair division: some indicative problems

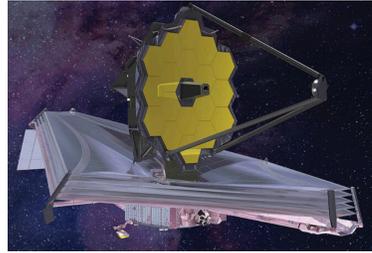
Goal: To **divide** the resource among agents in a **fair** manner

# Fair division: some indicative problems



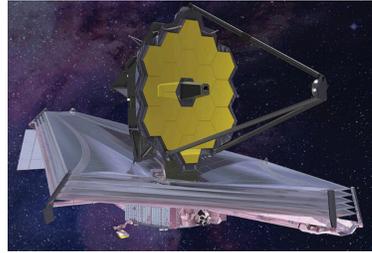
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Divisible



Indivisible

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# Fair division in Resource Allocation

					
	4	1	2	2	2
	1	0	5	1	1
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Q2. How to **compute a fair allocation**?

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[IJCAI'19, AAAI'25, TOCS'24, AAMAS'25]

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[SODA'19, WINE'19, AAAI'19, EC'20,  
MOR'22, WINE'23, AAAI'25, AAMAS'25]

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Computational Social choice (COMSOC): *Collective* decision-making based on *individual* preferences

					
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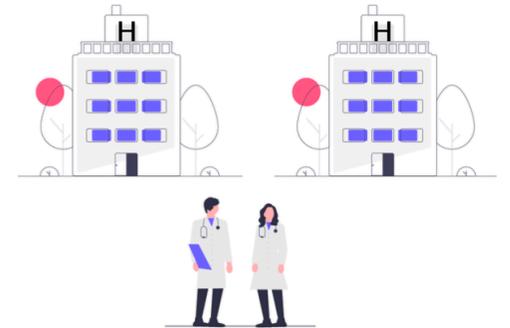
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Fair Division



Participatory Budgeting



Stable Matchings

Other related areas

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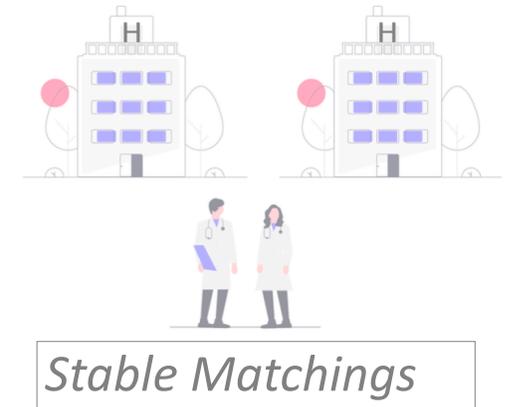
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**Fair Division**



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Two **interpretations of fairness**:

- **Comparative:**
  
  
  
  
  
  
  
  
  
  
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- **Comparative**: every agent compares the share allocated to her to the parts allocated to other agents

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Foley [1967]

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- **In absolute terms**: each agent defines a **threshold value** based on her view and deems an allocation as *fair* if it gives her a value  $\geq$  threshold

**Proportionality**: for all agents  $i$ ,  $v_i(A_i) \geq \frac{1}{n} v_i(G)$

Steinhaus, Knaster and Banach [1948]

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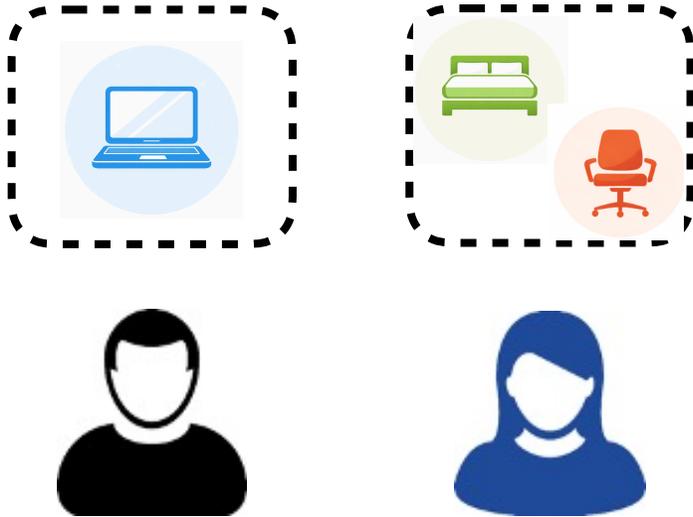


# Fair Cake Division (Divisible Setting)

- **Proportional cake division** exists and can be computed in **polynomial time**
- **Envy-free cake division** exists! (beautiful fixed-point theory)



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Hypothetically removing **any** item, makes the envy disappear!

(there is envy, but not-so-much)

# Biggest Open Problem in Fair Division



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makes the envy disappear!

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[Procaccia, Communications of ACM 2020]

Question: Do EFX allocations exist for four or more agents?

... via relaxations and approximations

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  - maintaining fair allocations under dynamic updates
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**Thank you!**